A business-cycle model with money-in-utility (MIU) and government sector: the case of Bulgaria (1999-2020)

Aleksandar Vasilev*

April 1, 2022

Abstract

Purpose: We augment an otherwise standard business cycle model with a richer government sector, and add money-in-utility (MIU) considerations to study economic fluctuations.

Design/methodology/approach: More specifically, real money balances enter in a non-separable way with consumption and leisure. This specification is then calibrated to Bulgarian data after the introduction of the currency board (1999-2020), gives a role to money in accentuating economic fluctuations.

Findings: This novel mechanism allows the framework to reproduce - better than the RBC model - the observed variability and correlations among model variables, and those characterizing the labor market in particular. In addition, money is non-neutral and affects aggregate economic activity.

Originality: This is the first micro-founded monetary-DSGE model on Bulgaria trying to explain the role of money for economic fluctuations.

Keywords: business cycles, money-in-utility (MIU) considerations, Bulgaria

JEL Classification: E32

*Senior Lecturer, University of Lincoln, UK. E-mail: avasilev@lincoln.ac.uk.
1 Introduction and Motivation

It is a well-known fact, e.g. Prescott (1986), that the perfectly-competitive (Walrasian) approach to modelling labor markets in real-business-cycle (RBC) context - that is, without money in the setup - does not fit data well, and thus creates a "puzzle" for neoclassical economists. More specifically, in the standard RBC model the fluctuations in employment are due to movements in labor supply.\(^1\) Instead, if an RBC model is to fit data better (and along the labor market dimension in particular), even for a small economy like Bulgaria, shocks to labor demand would be much better candidates to explain the observed fluctuations in the wage rate, aggregate hours and employment. Instead, a model with money may introduce new wedges, and thus potentially improve the model fit, even within a Walrasian environment. In addition, the presence of money will shed a light whether it is neutral, or important for real aggregate activity even in the long-run.

The next question faced by modelers is then how exactly to introduce money within the general equilibrium paradigm, when we model the Bulgarian economy. Among several alternatives existing in the literature,\(^2\) we start with the simplest one that has implications for macrodynamics, i.e., by assuming money is a direct source of utility.\(^3\) In this paper we thus augment an otherwise standard business cycle model with a richer government sector, and add money-in-utility (MIU) considerations.\(^4\) We adopt the approach followed in Walsh

\(^1\)In other words, households increase hours in the face of a raise in the return on labor, the wage, driven by shocks to technology.

\(^2\)The monetary economics literature has utilized several modeling approaches, such as money-in-utility, cash-in-advance, shopping-time, monetary-search models, models with nominal price- and wage rigidities. For a recent treatment and discussion on the relative merits of each approach, the interested reader is referred to Nosal and Guillaume (2011), Gali (2016), Walsh (2017).

\(^3\)This method is also a way to solve "Hahn’s problem," as outlined in Hahn (1965) and Bewley (1983). In other words, with money present in the utility function, money becomes immediately valuable, and thus the economy is a monetary one, as the optimal quantity of real money balances held by the household in equilibrium is positive (subject to the utility featuring "sufficient curvature" in real money balances).

\(^4\)This modelling approach goes back to Patinkin (1965) and Sidrauski (1967). On a similar note, Samuelson (1958) and Sims (2013) use this approach as a simple way to treat money like an asset used to transfer resources inter-temporally. Lastly, the MIU approach is connected to money-search models in Nosal and Guillaume (2011).
(2017) and incorporate a modified money-in-utility (MIU) considerations in RBC models in order to investigate the quantitative effect of money on business cycle fluctuations in aggregate variables in Bulgaria, and whether it is able to address the "labor market puzzle," and validate certain labor market facts, while at the same time retain technology as the only shock process.\textsuperscript{5} In particular, real money balances enter in a non-separable way with consumption. This specification creates interesting interactions; the setup is then calibrated to Bulgarian data after the introduction of the currency board (1999-2020), gives a role to money in accentuating economic fluctuations.\textsuperscript{6} Bulgaria is the poorest EU member state, and a former transition economy. After the banking and financial crisis in 1996-97, the economy was stabilized via an extreme fixed-exchange-rate regime ("currency board"). Together with the privatization of the banking sector, and the entry of foreign banks in Bulgaria, the financial development was put back on track, and the banking sector was re-monetized via credit lines from the headquarters of the foreign banks.

The MIU approach in this paper is a way to model money demand, starting from microfoundations.\textsuperscript{7} In addition, the implementation of the Currency Board arrangement in Bulgaria achieved stabilization of the monetary base after the economy was monetized (with the help of the headquarters of the foreign-owned banks), and the trust in the local currency was restored. Lastly, the modified MIU approach allows for a presence of a second asset in our model setup, which is a substitute for investment in physical capital. Therefore, MIU approach produces a mechanism that allows the framework to reproduce better-than the benchmark RBC model - the observed variability and correlations among model variables, and those characterizing the labor market in particular.\textsuperscript{8} This shows that money is an important model ingredient and should be present in DSGE setups. Furthermore, the

\textsuperscript{5}In addition, in contrast to the setups in Gali (2016), and later adaptations in Vasilev (2022), where a model with rigid (Calvo) pricing and rigid (Calvo) wages are explored, and where monetary policy has a real effect only due to the pre-set prices, here we prefer a model where money has an intrinsic role instead.

\textsuperscript{6}The model also yield a micro-founded money demand, so our work could be considered as a follow-up to the research in Slavova (2003).

\textsuperscript{7}If a money demand is to be consistent with theory, it has to feature consumption instead of output. Such a correct specification would then help policy makers evaluate better anticipated effects of different reforms.

\textsuperscript{8}With the introduction of the inflation tax in the MIU framework, the improvement may be similar to the role that fiscal policy has in improving model fit.
improvement in the model fit is due to the non-neutrality of money in the framework, which was an artifact of the non-separable utility function in consumption and money. This feature is essential for the result in this paper, and sheds an important light on how money demand should be modeled in Eastern European economies.\footnote{Most of the authors in the field pre-assume separability of money and consumption in utility, which leads to neutrality of money, e.g., Ivanov et al. (2014). We implicitly show in our paper that their assumption was wrong. Others who model money demand in Central and Eastern European countries are Babic (2000), Brzoza-Brzenina (2011), Buch (2001), Dobnik (2011), Dreger at el (2007), Cziraky and Gillman (2006), Petrevski and Jovanovski (2010), Siliverstovs (2007, 2008), Weber (1994), Skrabic and Tomic-Plazibat (2009), Kjosevski (2013), Kollarova and Carsky (2007), Komarek and Melecky (2001), Waller (1999, 2005).}

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

\section{Model Setup}

There is a representative household, which derives utility out of consumption, real money balances and leisure. The time available to households can be spent in productive use or as leisure. The households use cash for their purchases. The government taxes consumption spending and levies a common tax on all income, in order to finance purchases of government consumption goods, and government transfers. The monetary authority follows an endogenous money supply rule, and redistributes all seigniorage back to the household. On the production side, there is a representative firm, which hires labor and utilized capital to produce a homogeneous final good, which could be used for consumption, investment, or government purchases.
2.1 Household problem

Each household maximizes expected discounted utility, which is non-separable in consumption and money, as in Walsh (2017):^{10}

\[
U = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln (ac_t^b + (1-a)m_t^b) + \gamma \ln (1-h_t) \right\},
\]

where $E_0$ is the expectation operation conditional on information available as of $t = 0$, $0 < \beta < 1$ is the discount factor, $c_t$ is individual household consumption in period $t$, $m_t$ are real money balances, and $h_t$ are hours worked. Parameters $0 < a, 1-a < 1$ reflect the weights attached to consumption and money, while $1/b < 0$ denotes the elasticity of substitution between consumption and real money balances. Lastly, parameter $\gamma > 0$ is the relative weight attached to the utility of leisure.

The household starts with a positive endowment of physical capital, $k_0$, in period 0, which is rented to the firm at the nominal rental rate $R_t$, that is, before-tax capital income equals $R_t k_t$. Therefore, each household can decide to invest in capital to augment the capital stock, which evolves according to the following law of motion:

\[
k_{t+1} = i_t + (1-\delta)k_t,
\]

where $0 < \delta < 1$ is the depreciation rate of physical capital.

In addition to the rental income, the household owns the firm, and thus has a legal claim to the firm’s nominal profit, $\Pi_t$. Lastly, the household works a certain number of hours, which are remunerated at the spot nominal wage rate $W_t$, producing a total nominal labor income of $W_t h_t$ in period $t$.

^{10}We need a departure from the Cobb-Douglas specification, in order for money to have non-super-neutrality (otherwise the model exhibits a real-monetary dichotomy. In addition, the substitutability between consumption and money in the utility function below creates substitutability between money and capital, and generates the so-called negative Tobin effect, as pointed in Tobin (1965), and expanded in Stein (1969) and Fischer (1972). More specifically, a higher inflation rate discourages investment, and lowers capital accumulation and growth.
The budget constraint of the aggregate household, expressed in real terms, is then

\[(1 + \tau^c)c_t + k_{t+1} - (1 - \delta)k_t + \frac{M_{t+1} P_{t+1}}{P_t} = (1 - \tau^y)[w_t h_t + r_t k_t] + \frac{M_t}{P_t} + g_t + \frac{\Pi_t}{P_t}, \quad (3)\]

where \(\tau^c\) is the tax rate on final consumption, \(\tau^y\) is the proportional rate on labor and capital income, \(P_t\) is the aggregate price level. \(M_t\) denote the nominal quantities of money holdings in period \(t\). Money stock is treated like a consumption good, it stores wealth over time. That is why real money balances in period \(t\) are \(m_t = M_t/P_t\) in period \(t + 1\) only buy \(M_t/P_{t+1}\) (next period purchasing power).\(^{11}\) Similarly, \(w_t = W_t/P_t\), and \(r_t = R_t/P_t\) are the real wage and the real interest rate.

Real money balances are valued per se, and thus they enter directly the household’s utility function, the money-in-utility (MIU) approach is taken. Despite being ad hoc modeling, this exogenous specification introduces a role for money. In this paper, this shortcut introduces a tractable representation of money, and is to be interpreted as money providing valuable transaction services.\(^{12}\)

Next, we set up the Lagrangian of the household’s problem:

\[
L = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln[ac_t^b + (1 - a)m_t^b] + \gamma \ln(1 - h_t)ight.
\]

\[-\lambda_t \left[(1 + \tau^c)c_t + k_{t+1} - (1 - \delta)k_t + m_{t+1}(1 + \pi_{t+1})
\]

\[-(1 - \tau^y)[w_t h_t + r_t k_t] - m_t - g_t - \frac{\Pi_t}{P_t} \right\} \quad (4)\]

\(^{11}\)Note that we are using the timing convention in Lucas (1982) and Carlstrom and Fuerst (2001). For alternative timing assumptions, see Walsh (2017).

\(^{12}\)Importantly, the MIU approach produces a positive quantity of money in equilibrium, despite money being a zero-return asset.
The first-order optimality conditions (FOCs) are as follows:

\[ c_t : \frac{abc_t^{b-1}}{ac_t^b + (1-a)m_t^b} = (1+\tau^c)\lambda_t \]  
\[ (5) \]

\[ h_t : \frac{\gamma}{1-h_t} = \lambda_t(1-\tau^y)w_t, \]  
\[ (6) \]

\[ k_{t+1} : \lambda_t = \beta E_t \lambda_{t+1}[1-\delta + (1-\tau^y)r_{t+1}], \]  
\[ (7) \]

\[ m_{t+1} : \frac{(1-a)bm_{t+1}^{b-1}}{ac_t^b + (1-a)m_t^b} = \lambda_{t-1}(1+\pi_t) - \beta \lambda_t, \]  
\[ (8) \]

where \( \pi_t \) is the inflation rate between periods \( t-1 \) and \( t \). Lastly, the boundary (transversality) conditions for capital, and real money balances are as follows:

\[ TVC_k : \lim_{t \to \infty} \beta^t \lambda_t k_{t+1} = 0 \]  
\[ (9) \]

\[ TVC_m : \lim_{t \to \infty} \beta^t \lambda_t m_{t+1} = 0 \]  
\[ (10) \]

The interpretation of the optimality conditions is standard. In the first, the household equates the marginal utility of consumption, to the VAT adjusted shadow price of wealth and the CIA constraint. The second FOC determines optimal number of hours worked, by balancing at the margin the cost and benefit from working. The remaining equations from the original FOCs are standard: for example, the Euler equation for capital stock describes how capital is allocated across any adjacent periods in order to maximize household’s utility. Similarly, the other describes the rule for optimal real money balances. The transversality conditions (TVCs) for real cash holdings, and physical capital are imposed to rule out explosive solutions.

### 2.2 Stand-in firm’s problem

There is a stand-in firm in the economy, which uses homogeneous capital and labor to produce a final good, which can be used for consumption, investment, or government purchases, through the following production function:

\[ y_t = A_t k_t^\alpha h_t^{1-\alpha}, \]  
\[ (11) \]

where \( A_t \) denotes the level of total factor productivity in period \( t \), \( h_t \) are total hours used, and \( \alpha \) and \( 1-\alpha \) are the share of capital and labor, respectively. The firm’s problem, expressed
in real terms, is to

$$\max_{(k_t, h_t) \geq 0} A_t k_t^\alpha h_t^{1-\alpha} - r_t k_t - w_t h_t$$  \hspace{1cm} (12)$$

The first-order optimality conditions determining optimal capital, and labor use are

$$k_t : \frac{y_t}{k_t} = r_t,$$  \hspace{1cm} (13)$$

$$h_t : (1 - \alpha) \frac{y_t}{h_t} = w_t.$$  \hspace{1cm} (14)$$

Given the results above, it follows that profit is zero in all periods.

### 2.3 Monetary Authority

In this paper the monetary authority (central bank) supplies the money aggregate, $M_t$, endogenously. In other words, the money supply responds to the demand for currency for transaction purposes. All money created (seigniorage) in period $t$ is then distributed to the government, and then to the households in a lump-sum fashion

$$M_{t+1} - M_t = T_t,$$  \hspace{1cm} (15)$$

where $T_t$ is the lump-sum nominal transfer to the household. In the government budget constraint below, we assume that the central bank distributes the seigniorage to the Ministry of Finance, which in turn passes it to the household as part of the overall government lump-sum transfer, $g^t_t$.

### 2.4 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption in order to finance spending on government purchases and government transfers. The government budget constraint is as follows:

$$\tau^c c_t + \tau^y (w_t h_t + r_t k_t) = g^t_t + g^c_t$$  \hspace{1cm} (16)$$

Tax rates and government consumption-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually.
2.5 Stochastic process

Total factor productivity, $A_t$, is assumed to follow AR(1) processes in logs, in particular

$$\ln A_{t+1} = (1 - \rho_a) \ln A_0 + \rho_a \ln A_t + \epsilon_{t+1}^a,$$

where $A_0 > 0$ is steady-state level of the total factor productivity process, $0 < \rho_a < 1$ is the first-order autoregressive persistence parameter and $\epsilon_{t}^a \sim iidN(0, \sigma_a^2)$ are random shocks to the total factor productivity process. Hence, the innovations $\epsilon_{t}^a$ represent unexpected changes in the total factor productivity process.

2.6 Dynamic Competitive Equilibrium (DCE)

Given the stochastic process $\{A_t\}_{t=0}^{\infty}$, average tax rates $\{\tau^c, \tau^y\}$, endowments $(k_0, m_0)$, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t, k_t, h_t, m_t\}_{t=0}^{\infty}$, a sequence of government purchases and transfers $\{g^c_t, g^f_t\}_{t=0}^{\infty}$, and real input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget constraint is balanced in each period; (iv) all markets clear.

3 Data and Model Calibration

To calibrate the model to Bulgarian data, we focus on the period after the introduction of the currency board (1999-2020). Annual data on output, consumption and investment was collected from National Statistical Institute (2021), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2021). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, the discount factor, $\beta = 0.982$, as in Vasilev (2017a), is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 3.491$. The labor share parameter, $\alpha = 0.429$, was obtained from Vasilev (2017b) as the average value of labor income in aggregate output over the period 1999-2014.

The relative weights attached to the utility out of consumption and money in the household’s utility function, $a = 0.997$, and $1 - a = 0.003$ is set as in Chari, Kehoe and McGrattan.
Similarly, as in Christiano, Eichenbaum and Evans (2005), elasticity of substitution is set to $1/b = -1$. Parameter $\gamma > 0$ is set to match the fact that in steady-state consumers would supply one-third of their time endowment to working. The money in the model corresponds to M2 money aggregate, and $M2/Y = 0.848$ on average over the period 1999-2018. Next, the average depreciation rate of physical capital in Bulgaria, $\delta = 0.05$, was taken from Vasilev (2015). It was estimated as the average depreciation rate over the period 1999-2014. Similarly, the average income tax rate was set to $\tau^y = 0.1$, and the tax rate on consumption is set to its value over the period, $\tau^c = 0.2$. Lastly, as in Vasilev (2017c), the process followed by total factor productivity is estimated from the detrended Solow residual series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Description</th>
<th>Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta$</td>
<td>0.982</td>
<td>Discount factor</td>
<td>Calibrated</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>0.429</td>
<td>Capital Share</td>
<td>Data average</td>
</tr>
<tr>
<td>$a$</td>
<td>0.997</td>
<td>Weight attached to consumption</td>
<td>Set</td>
</tr>
<tr>
<td>$b$</td>
<td>-1.000</td>
<td>Inverse elasticity of substitution: money vs consumption</td>
<td>Set</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.050</td>
<td>Depreciation rate on physical capital</td>
<td>Data average</td>
</tr>
<tr>
<td>$\tau^c$</td>
<td>0.200</td>
<td>VAT/consumption tax rate</td>
<td>Data average</td>
</tr>
<tr>
<td>$\tau^y$</td>
<td>0.100</td>
<td>Average tax rate on income</td>
<td>Data average</td>
</tr>
<tr>
<td>$\rho_a$</td>
<td>0.701</td>
<td>AR(1) parameter, total factor productivity</td>
<td>Estimated</td>
</tr>
<tr>
<td>$\sigma_a$</td>
<td>0.044</td>
<td>st.dev, total factor productivity</td>
<td>Estimated</td>
</tr>
</tbody>
</table>

13 This is done to attach a small, but positive, weight on money, and thus make the problem more interesting via the interaction between consumption and real money balances. With full separability, money is superneutral, i.e., then money does not affect either the levels of real variables, or their growth rates.

14 This parameter is not very important, as central banks use interest rate rules, and not money supply rules.
4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the ”big ratios” can be compared to their averages in Bulgarian data. The results are reported in Table 2 on the next page. (We approximate the economy around zero inflation.) The model matches consumption-to-output ratio by construction; The investment and government purchases ratios are also closely approximated. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. Lastly, the after-tax return, net of depreciation, \( \tilde{r} = (1 - \tau^y)r - \delta \), is also very closely captured by the model.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Description</th>
<th>Data</th>
<th>Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>( y )</td>
<td>Steady-state output</td>
<td>N/A</td>
<td>0.568</td>
</tr>
<tr>
<td>( c/y )</td>
<td>Consumption-to-output ratio</td>
<td>0.674</td>
<td>0.674</td>
</tr>
<tr>
<td>( i/y )</td>
<td>Investment-to-output ratio</td>
<td>0.201</td>
<td>0.175</td>
</tr>
<tr>
<td>( g^c/y )</td>
<td>Government cons-to-output ratio</td>
<td>0.159</td>
<td>0.151</td>
</tr>
<tr>
<td>( w/h )</td>
<td>Labor income-to-output ratio</td>
<td>0.571</td>
<td>0.571</td>
</tr>
<tr>
<td>( r^k/y )</td>
<td>Capital income-to-output ratio</td>
<td>0.429</td>
<td>0.429</td>
</tr>
<tr>
<td>( h )</td>
<td>Share of time spent working</td>
<td>0.333</td>
<td>0.333</td>
</tr>
<tr>
<td>( \tilde{r} )</td>
<td>After-tax net return on capital</td>
<td>0.056</td>
<td>0.057</td>
</tr>
</tbody>
</table>

5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second
moments of the model perform when compared against their empirical counterparts. Special focus is put on the cyclical behavior of labor market variables.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response function (IRFs) are presented in Fig. 1 below.

![Impulse Response Graphs](image)

Figure 1: Impulse Responses to a 1% surprise innovation in technology

As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so uses of output - consumption, investment, and government consumption also increase contempora-
neously.

At the same time, the jump in productivity increases the after-tax return on the two factors of production, labor and capital. The representative households then respond to the incentives contained in prices and start accumulating capital, and supplies more hours worked. In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. In the labor market, the wage rate increases, and the household increases its hours worked. In turn, the increase in total hours further increases output, again indirectly.

Over time, as capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households’ incentives to save. As a result, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. Notice that the dynamics of money is almost a mirror image to capital, as it is an imperfect substitute for capital. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

5.2 Simulation and moment-matching

We now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at annual frequency. To minimize the sample error, the simulated moments are averaged out over the computer-generated draws. For consistency purposes, we evaluate the MIU model against the benchmark RBC model. The MIU model matches quite well the absolute volatility of output. However, the model substantially overestimates the variability in consumption, and investment. Still, the MIU model is qualitatively consistent with the stylized fact that consumption is less volatile than output, and investment is more volatile than output. In addition, investment is less volatile in the MIU model exactly because there is a second asset (money), that despite carrying zero nominal return, provides

\footnote{The model-predicted 95\% confidence intervals are available upon request.}
valuable (transaction) services. By construction, government spending in the model varies as much as in data. With respect to the labor market variables, the variability of employment predicted by the model is too little than that in data, but the variability of wages in the MIU model is much closer to that in data, as compared to the RBC model.

Table 3: Business Cycle Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>RBC Model</th>
<th>MIU Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_y$</td>
<td>0.05</td>
<td>0.05</td>
<td>0.05</td>
</tr>
<tr>
<td>$\sigma_c/\sigma_y$</td>
<td>0.55</td>
<td>0.63</td>
<td>0.68</td>
</tr>
<tr>
<td>$\sigma_i/\sigma_y$</td>
<td>1.77</td>
<td>3.26</td>
<td>3.00</td>
</tr>
<tr>
<td>$\sigma_g/\sigma_y$</td>
<td>1.21</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$\sigma_h/\sigma_y$</td>
<td>0.63</td>
<td>0.63</td>
<td>0.37</td>
</tr>
<tr>
<td>$\sigma_w/\sigma_y$</td>
<td>0.83</td>
<td>0.52</td>
<td>0.77</td>
</tr>
<tr>
<td>$\sigma_{y/h}/\sigma_y$</td>
<td>0.86</td>
<td>0.52</td>
<td>0.77</td>
</tr>
<tr>
<td>$corr(c, y)$</td>
<td>0.85</td>
<td>0.62</td>
<td>0.67</td>
</tr>
<tr>
<td>$corr(i, y)$</td>
<td>0.61</td>
<td>0.78</td>
<td>0.77</td>
</tr>
<tr>
<td>$corr(g, y)$</td>
<td>0.31</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td>$corr(h, y)$</td>
<td>0.49</td>
<td>0.77</td>
<td>0.57</td>
</tr>
<tr>
<td>$corr(w, y)$</td>
<td>-0.01</td>
<td>0.66</td>
<td>0.85</td>
</tr>
<tr>
<td>$corr(u, y)$</td>
<td>-0.47</td>
<td>-0.77</td>
<td>-0.57</td>
</tr>
<tr>
<td>$corr(h, y/h)$</td>
<td>-0.14</td>
<td>0.34</td>
<td>0.32</td>
</tr>
</tbody>
</table>

Next, in terms of contemporaneous correlations, the model slightly over-predicts the procyclicality of the main aggregate variables - consumption and government consumption; yet, the MIU model dominates the RBC model. This, however, is a common limitation of this class of models. Still, along the labor market dimension, the contemporaneous correlation of employment with output, and unemployment with output, is relatively well-matched, and much better so in the MIU model. With wages, the model predicts strong cyclicality, while wages in data are acyclical.

In the next subsection, we investigate the dynamic correlation between labor market vari-
ables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 against the simulated AFCs and CCFs. Following Canova (2007), this comparison is used as a goodness-of-fit measure. As seen from Table 4 on the next page, the model compares well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model.

The persistence of labor market variables are also well-described by the MIU model dynamics: the ACFs wages are close to the simulated ones until the third lag. Same holds true for output and investment. The ACF for consumption and employment is well-captured only until the first lag. Overall, the model with one-period nominal wage contracts generates the right persistence in model variables, and is able to respond to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process.

Next, as seen from Table 5 on the next page, over the business cycle, in data labor productivity leads employment. The model with MIU, however, can only partially account for this fact. In this model, as well as in the standard RBC model, a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one. Still, the model with an MIU constraint is a clear improvement over the real setup with perfectly-competitive labor market paradigm used in Vasilev (2009).
Table 4: Autocorrelations for Bulgarian data and the model economy

<table>
<thead>
<tr>
<th>Method</th>
<th>Statistic</th>
<th>k</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>$corr(h_t, h_{t-k})$</td>
<td>0.1000</td>
<td>0.484</td>
<td>0.009</td>
<td>0.352</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>$corr(h_t, h_{t-k})$</td>
<td>0.1000</td>
<td>0.818</td>
<td>0.629</td>
<td>0.441</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.000)</td>
<td>(0.038)</td>
<td>(0.071)</td>
<td>(0.096)</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>$corr(y_t, y_{t-k})$</td>
<td>0.1000</td>
<td>0.810</td>
<td>0.663</td>
<td>0.479</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>$corr(y_t, y_{t-k})$</td>
<td>0.1000</td>
<td>0.818</td>
<td>0.631</td>
<td>0.446</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.000)</td>
<td>(0.034)</td>
<td>(0.061)</td>
<td>(0.083)</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>$corr(a_t, a_{t-k})$</td>
<td>0.1000</td>
<td>0.702</td>
<td>0.449</td>
<td>0.277</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>$corr(a_t, a_{t-k})$</td>
<td>0.1000</td>
<td>0.818</td>
<td>0.632</td>
<td>0.447</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.000)</td>
<td>(0.035)</td>
<td>(0.062)</td>
<td>(0.083)</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>$corr(c_t, c_{t-k})$</td>
<td>0.1000</td>
<td>0.971</td>
<td>0.952</td>
<td>0.913</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>$corr(c_t, c_{t-k})$</td>
<td>0.1000</td>
<td>0.816</td>
<td>0.629</td>
<td>0.445</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.000)</td>
<td>(0.034)</td>
<td>(0.061)</td>
<td>(0.082)</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>$corr(i_t, i_{t-k})$</td>
<td>0.1000</td>
<td>0.810</td>
<td>0.722</td>
<td>0.594</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>$corr(i_t, i_{t-k})$</td>
<td>0.1000</td>
<td>0.817</td>
<td>0.628</td>
<td>0.441</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.000)</td>
<td>(0.038)</td>
<td>(0.069)</td>
<td>(0.093)</td>
<td></td>
</tr>
<tr>
<td>Data</td>
<td>$corr(w_t, w_{t-k})$</td>
<td>0.1000</td>
<td>0.760</td>
<td>0.783</td>
<td>0.554</td>
<td></td>
</tr>
<tr>
<td>Model</td>
<td>$corr(w_t, w_{t-k})$</td>
<td>0.1000</td>
<td>0.816</td>
<td>0.628</td>
<td>0.443</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.000)</td>
<td>(0.033)</td>
<td>(0.061)</td>
<td>(0.084)</td>
<td></td>
</tr>
</tbody>
</table>

6 Conclusions

We augment an otherwise standard business cycle model with a richer government sector, and add money-in-utility (MIU) considerations. In particular, real money balances enter in a non-separable way with consumption and leisure. This specification is then calibrated to Bulgarian data after the introduction of the currency board (1999-2020), gives a role to money in accentuating economic fluctuations. In particular, the modified MIU approach allows for a second asset, which is a substitute for capital. This novel mechanism allows the framework to reproduce - better than the RBC framework - the observed variability and
Table 5: Dynamic correlations for Bulgarian data and the model economy

<table>
<thead>
<tr>
<th>Method</th>
<th>Statistic</th>
<th>-3</th>
<th>-2</th>
<th>-1</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>corr($h_t, (y/h)_{t-k}$)</td>
<td>-0.342</td>
<td>-0.363</td>
<td>-0.187</td>
<td>-0.144</td>
<td>0.475</td>
<td>0.470</td>
<td>0.346</td>
</tr>
<tr>
<td>Model</td>
<td>corr($h_t, (y/h)_{t-k}$)</td>
<td>-0.005</td>
<td>-0.003</td>
<td>0.014</td>
<td>0.316</td>
<td>0.009</td>
<td>-0.009</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.738)</td>
<td>(0.652)</td>
<td>(0.541)</td>
<td>(0.788)</td>
<td>(0.528)</td>
<td>(0.641)</td>
<td>(0.732)</td>
</tr>
<tr>
<td>Data</td>
<td>corr($h_t, w_{t-k}$)</td>
<td>0.355</td>
<td>0.452</td>
<td>0.447</td>
<td>0.328</td>
<td>-0.040</td>
<td>-0.390</td>
<td>-0.57</td>
</tr>
<tr>
<td>Model</td>
<td>corr($h_t, w_{t-k}$)</td>
<td>-0.005</td>
<td>-0.003</td>
<td>0.014</td>
<td>0.316</td>
<td>0.009</td>
<td>-0.009</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>(s.e.)</td>
<td>(0.738)</td>
<td>(0.652)</td>
<td>(0.541)</td>
<td>(0.788)</td>
<td>(0.528)</td>
<td>(0.641)</td>
<td>(0.732)</td>
</tr>
</tbody>
</table>

correlations among model variables, and those characterizing the labor market in particular.

This is good news for monetary models, as money (and long-term money non-neutrality) is shown to be an important phenomenon, and provides a quantitatively relevant model ingredient to explain aggregate economic activity. Still, the limitations of this research need to be pointed out: as explained earlier, the MIU approach is a short-cut, and it can be further micro-founded using a cash-in-advance-, or a shopping-time role for money. This, however, is left for future research.

References


Sofia University St. Kliment Ohridski, Faculty of Economics and Business Administration, Sofia, Bulgaria.


