Particle transport mechanism in liquid-liquid-solid multiphase pipeline flow of high viscosity oil-water-sand

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Abstract

In this study, investigation of sand transport in heavy oil-water multiphase flow is carried out. The study is conducted in three multiphase flow pipeline test facilities with internal diameters (ID) of 1, 1 and 3 inches. The pipeline orientation relative to the horizontal in the facilities are 0˚, +30˚ and 0˚ respectively. Oil viscosity of 3.5 and 10.0 Pa.s with sand volume fraction from 0.010 – 0.100 vol/vol are used in the study. The effect of oil viscosity, upward inclination, sand volume fraction, pipe internal diameter and water cut on sand transport mechanism in pipelines are investigated. In the horizontal test section, flow patterns namely: Dispersed Flow (DF), Plug Flow (PF), Plug Flow with Moving Sand Bed (PFM) and Plug Flow with Stationary Sand Bed (PFS) were identified through flow visualization. In addition to the aforementioned, two flow patterns; the Stratified Wavy Flow with Moving Sand Bed (SWM) and Stratified Wavy Flow with Dunes, (SWD) were observed in the inclined pipeline orientation. Pressure gradient measured, decreased with a decrease in water cut until a minimum value is reached. Beyond the minimum pressure gradient, further reduction in water cut led to an increase in pressure gradient. Sand Minimum Transport Condition (MTC) in the oil-water-sand test were largely the same for the 1-inch 30˚ upward inclined and the 1-inch horizontal test section while that of the 3-inch horizontal test section was considerably higher. An improved MTC predictive correlation is proposed for multiphase heavy oil-water-sand flow. Proposed correlation outperforms existing model when tested on the heavy oil-water-sand dataset.

Highlights

- Experimental investigations of concurrent high viscosity oil, water, and sand (high viscosity oil-water-sand) multiphase flow in pipelines is presented
- Evaluation of existing models’ predictive capability in high viscosity oil-water-sand multiphase flow

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• Investigate the effect of an upward 30 degree inclined pipeline orientation and an increase in pipe internal diameter on sand minimum transport condition (MTC).

• Comparatively evaluate pressure gradient characteristics of water-sand and oil-water-sand multiphase flow

• Propose an improved predictive correlation for MTC of sand in high viscosity oil-water-sand multiphase flow in horizontal pipelines.

1 Introduction

Three-phase oil–water–sand flow in pipelines are prevalent during crude oil production. These flows are encountered when water is used to assist the production and transport of unconventional crude oil resources such as heavy oils and bituminous sand. Numbers released by the British Petroleum’s statistical review of world energy 2019 report (BP, 2019) shows that the world’s primary energy consumption grew by 2.9% from 2018. This is the steepest growth since 2010. Conventional crude oil resources have been significant contributor to global energy consumption. In addition, the report indicates that the year-on-year increase in world energy demand is in its tenth consecutive year. This trend together with advancement in technology, growth in developing economies, and increasing world population, is partly responsible for accelerating depletion of conventional oil reserves. Amidst this depletion, consumption has outpaced production and is fast surpassing the pre-world economic depression figures. One way the oil and gas industry uses to replenish depleting conventional reserves and thus satisfy the world energy demand, is the exploration and production of unconventional resources. Heavy oil is a standout candidate amongst unconventional resources because of reasons including technological advances, availability of large reserves (70% of world oil reserves are constituents of heavy oil) and geopolitics.

Heavy oils are characterised by their high viscosity (>0.1 Pa.s) and low API gravity (<22˚ API), these attributes is due to the presence of asphaltenes which may constitute of up to 50% by weight in some resource. They are generally dense and viscous in nature (similar to molasses) with some impurities. Due to its peculiar characteristics, heavy oil can be difficult to economically produce and transport. It therefore becomes important for the right technology to be developed and deployed for its production and transportation.

Some producers in Canada, Venezuela and the United States use a promising production technique - Cold Heavy Oil Production with Sand (CHOPS) - a technique that encourages the production of heavy crude oil with sand. Co-producing heavy crude oil with sand has several attendant issues such as handling cost for sand (and its disposal), producing wells workover as a result of accumulated sand in the wellbore and downhole pumps, increased wear due to the presence of sand in the produced fluids etc. (Wagg et al. 2009). To curtail these disadvantages which has militated CHOPS implementation and to ensure producers increase its return on investment; production, transportation and
processing facilities with capabilities of handling sand is imperative. The design, operation and control strategies of such facilities must account for sand presence in the produced well fluids. To achieve the aforementioned, an understanding of sand transport mechanism in high viscosity oil-water-sand is essential.

Some three phase studies available in the literature are the works of Karami et al., (2016) also conducted experimental study using 3-phase facility with 6-in.-inner- diameter (ID) facility to investigate characteristics of three-phase stratified wavy flow in horizontal pipelines. The flow characteristic they investigated are wave pattern, liquid holdup, water holdup, pressure gradient, and wetted-wall fraction. Experimental data acquired were compared with predictions obtained from a transient multiphase-simulation software. Results obtained from simulation were in agreement with experiments for measured liquid-holdup and pressure-gradient data. However, the three-phase water-holdup trends were under predicted. Mathew et al., (2001) performed solid transport experiment on the BP Amoco 6-in multiphase flow test facility using several fluids including: water, oil, and carboxymethylcellulose solutions. Viscosity of the fluids used ranged from 0.15 – 0.3 Pa.s. The authors noted in slug flow, water and low-viscosity oil were able to transport the sand uphill, high-viscosity solution on the other hand was unable to transport the solids. The feature was examined in comparison to the model for solids transport in near-horizontal Pipes.

However, most studies in literature on solids transport related to the petroleum industry are conducted using low viscosity liquid and were mostly limited to two-phase liquid-solid flow. Recently, some of such studies include Chen et al. (2020), Archibong-Eso et al. (2020), Fajemidupe et al (2019a), Fajemidupe et al. (2019b), Okeke et al. (2019), Yang et al. (2019) and Leporini et al. (2019). Crowe (2005) characterized the flow of solid particles in single phase liquid flow into: dispersed, scouring, moving dunes and stationary bed.

- **Stationary Bed:** Stationary bed was observed at very low liquid velocity; immobile sand particles form stationary bed with a flat surface at the bottom of the pipe. If the liquid velocity is increased, sand particles attain stable bed height with the particles at the top of the bed moving downstream thus increasing the bed’s length. (Archibong 2015)

- **Moving Dunes:** Sand bed breaks up into moving dunes on increasing the liquid velocity in stationary bed flow patterns, the grains at the dunes surface are rolled along by the liquid from back to front. The grains falls in sheltered regions in front of the dune and the dune passes over the particles until they are once again on the top surface (Yan 2010).
Scouring: Sand particles on top of the dune’s surface in the moving dunes flow pattern, roll along with increased momentum as a result of increase in liquid velocity. Increased momentum ensures that the particles goes further downstream beyond the sheltered region and are swept away as individual scouring grains.

Dispersed: At very high liquid velocity, sand particles pickups its highest momentum and becomes dispersed in the flow in a random and chaotic pattern. The degree of homogeneity of the particles in the liquid is a function of the liquid velocity.

Research work on oil-water-solid flow in literature is sparse, a fortiori, viscous oil-water-solid flow. Multiphase flow research community have paid little attention to this study. Heavy oil-gas two-phase flow has gained researchers interest in the past 10-15 years due to interest shown by the oil and gas industry to the exploration and production of heavy oil fields. Some of such studies include Archibong-Eso et al. 2014, 2015, 2017, 2018 and 2019, Baba et al. 2017, 2018, 2019, Al-Safran 2016 and 2018, Zhao 2013 amongst others.

In comparison, high viscosity liquid-liquid-solid three-phase flow has not gained similar interest. Few studies exist in literature for three-phase oil-water-sand flow. Gillies et al. (1995) studied oil-water-sand flow in horizontal pipes. Oil and water with viscosities of 0.066 and 0.01 Pa.s respectively were used as liquid phases. The authors reported a washing out of sand from oil when water was introduced into the oil-sand system in the pipeline. Considerable reduction in pressure gradient was reported. Notably, oil viscosity in their study was low and no information on MTC or flow patterns were presented.

McKibben et al. (2013) conducted a study on water-assisted pipeline transported of heavy oil with sand. Their work was carried out the Saskatchewan Research Council (SRC) under the aegis of a consortium of Canadian heavy-oil producers. To the authors’ knowledge, this study is the only study in literature that was conducted for high viscous oil-water-sand three-phase flows. Oil used in the study to simulate heavy crude oil has viscosity ranging from 0.2 – 31.4 Pa.s. Sand volume fraction of 6 and 12%, with pipeline IDs of 0.05, 0.1 and 0.2 m were used in the study. The authors reported that sand transport in multiphase flow reduced with increase in water cut and reduction in viscosity. They defined water cut as the ratio of the water superficial velocity to the mixture velocity, i.e.: $wc = \frac{V_{sw}}{V_m}$ (generally, superficial velocities are notional velocities of fluid which is calculated as if the given phase or fluid were the only one flowing or present in a given cross sectional area). A sand transport model for the prediction of the minimum velocities required for water-assisted deposit free oil-water-sand flow was developed by comparing the friction velocity $V^*$ that provides a suitable measure of the magnitude of turbulence in a pipeline to the terminal settling velocity, $V_\infty$ (McKibben et al. 2013). McKibben et al.
stated that the correlation provided reasonable estimates of the pressure gradient in high viscous oil-water flow.

The mixture friction factor correlation proposed by McKibben et al. (2013) for mixture is given by:

\[ f_m = 15 \cdot F_r^{-0.5} f_w^{1.3} f_o^{0.32} C_w^{1.2} \]

Froude number, \( F_r \), mixture friction factor for the aqueous phase, \( f_w \) and oil friction factor, \( f_o \) are defined by:

\[ F_r = \frac{V_M}{\sqrt{gD}} > 0.35 \]

\[ f_w = \frac{1}{\sqrt{16 \log \sqrt{5.7/\rho_m V_m^2 D}}} \]

\[ f_o = \frac{16}{Re}; \ Re = \rho_o V_m D/\mu_w \]

The above correlations were only valid for the water-assisted flow region, Froude number, \( F_r > 0.35 \). For \( F_r \leq 0.35 \), flow was considered to be non-water assisted.

To enhance understanding of sand transport mechanism, which is vital in pipeline design, maintenance and in crude oil production & transportation, this study experimentally investigates three-phase high viscous oil-water-sand flow. This study will help inform production and transportation strategy for high viscosity crude oils fields. In addition, it could help improve the maintenance, control and design of pipelines and unit operations equipment used in heavy oil exploration and production.

2 Materials and methods

In this section, description of the fluids (heavy oil and water), solids (sand), measuring devices/equipment are presented. A general description (with aid of schematics) of the high viscosity multiphase flow test facilities used in the study are presented. This test facility have been used by (Aliyu et al., 2017; Archibong-Eso et al., 2018; Baba et al., 2017b, 2017a, 2018).
2.1 Test materials

Sand particles, branded as Congleston HST 95 are used in this study with physical properties stated by manufacturer’s specification thus: density, 2650 kg/m$^3$ and mean particle size of 150 microns. From manufacturer’s specification sheet, the $d_{10}$, $d_{50}$ and $d_{95}$ of the sand particles were 100, 144 and 210 microns respectively as shown in Figure 1. These test particles were chosen because it was similar to properties of sand particles found in most oil and gas production fields. Laboratory tests of particle distribution analysis using sieve techniques was used to validate $d_{10}$, $d_{50}$ and $d_{95}$ of the sand particles stated by manufacturers. This technique involves the weighing of sand from a particular sample and passing them through sieves (screens) of decreasing sizes.

![Particle Sand Distribution for HST 95](image)

**Figure 1: Particle Sand Distribution for HST 95**

The screens are mounted on the sieve shakers. Horizontal and vertical motion is applied to the sieves to enhance particles movement. The quantity of particles remaining in each sieve is weighed, and subtracted from the weight of the empty sieve. Results obtained are used in particle size distribution analysis as shown in Figure 1.

Water and mineral oil were used as the test liquids in this experimental investigation. CYL 680 manufactured by Total UK were used in this study. Physical properties obtained from manufacturer’s specification (and validated by laboratory measurements using Brookfield DV-I™ prime viscometer) are shown in Table 1. Water used was from the public water supply to Cranfield University.

<table>
<thead>
<tr>
<th>Test fluids</th>
<th>Density (kg/m$^3$)</th>
<th>Viscosity (Pa.s)</th>
<th>Interfacial tension (N/m)</th>
<th>API gravity$^1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Water</td>
<td>998</td>
<td>0.001</td>
<td>0.029</td>
<td>--</td>
</tr>
<tr>
<td>CYL680</td>
<td>918</td>
<td>0.90 – 8.00</td>
<td>0.029</td>
<td>22.67</td>
</tr>
</tbody>
</table>

$^1$API gravity, $API = \frac{141.5}{SG} - 131.5$, $SG = \frac{\rho_{\text{oil}}-15°C}{\rho_{\text{water}}-15°C}$, $SG = \text{specific gravity}$
2.2 Experimental test rigs

Two flow loops with different pipe internal diameters were used for horizontal high-viscous oil-water-sand three-phase studies. In addition, a 30° upward inclined flow loop was used to study the effect of upward inclination on the three-phase flow. All the facilities are situated at the Oil and Gas Engineering Centre, Cranfield University, UK. The inclined facility operates in the same fashion and have the same operating equipment as the 1-inch horizontal facility differing only in the main test section inclined orientation. Similarly, the 3-inch facility differ from the 1-inch facility in length and pipe internal diameters. The 1-inch horizontal, 1-inch inclined and 3-inch horizontal facilities have main test sections of 5.5 m, 8.0 m and 17.7 m as can be seen schematically represented in Figure 2 and Figure 3 respectively.

![Schematic of the 1-inch multiphase flow facility](image)

Figure 2: Schematic of the 1-inch multiphase flow facility (Length of main test section are 5.5 m and 8.0 m in the horizontal and inclined orientations respectively).
2.2.1 Slurry System (water-sand)

Water-sand (slurry) system consists of a 0.2 m³ cylindrical plastic vessel with a maximum allowable sand volume fraction of 0.15 volume by volume. An axial flow impeller with twin blades placed about 0.3 m above the vessel's conical base is used to stir the slurry for mixture homogeneity. The conical base opens into a polyvinyl chloride (PVC) pipe that is connected to the suction of the slurry pump, a progressive cavity pump (PCP). The PCP has a maximum flowrate 0 – 2.18 m³/hr and discharge pressure of 10 bara, the PCP pumps the slurry into the test line via a Promag 55S50, DN50 electromagnetic flowmeter with maximum flowrate 2.18 m³/hr. A 4 – 20 mA HART output is used for data acquisition and logging. The 1-inch inclined test section utilizes the same unit operations equipment. For the 3-inch facility, the setup differs in that the cylindrical mixing vessel manufactured from a steel material has a capacity of 2.5 m³ with a three-blade axial impeller used to stir the slurry mixture for homogeneity. The vessel’s conical base opens into the slurry pump; a PCP with discharge pressure of 16 barg, the PCP pumps the slurry into the test line via a Promag 55S800 DN80 electromagnetic flowmeter with maximum flowrate 20 m³/hr.

2.2.2 Oil System

A 0.15-m³ plastic tank capacity insulated on the periphery was used to store oil in the 1-inch horizontal facility. A variable speed PCP with maximum capacity of 0.72 m³/hr is used in pumping oil. Endress+Hauser’s Promass 831 DN 50, a Coriolis flowmeter, with range, 0 – 180 m³/hr is used in oil metering. The flowmeter has three outputs; mass flow rate, density and viscosity. The HART output from the meter is 4 – 20 mA is connected to a data acquisition system for data logging during experimental runs using the LabVIEW.
version 8.6.1 software. The 1-inch inclined facility utilizes the same unit operation equipment. In the 3-inch test facility, mineral oil is stored in a 2 m³ capacity steel tank. A variable speed PCP with maximum capacity 17 m³/hr, is used in pumping oil. A Promass 831 DN 80, Coriolis flowmeter with range 0 – 171 m³/hr, manufactured by Endress+Hauser is used in oil metering.

2.2.3 Multiphase Separation System

Gravity driven separators are used to separate oil-water-sand multiphase mixture at the end of experiment. The separation system consists of a primary and secondary separator. For the 1-inch inclined and horizontal facilities, the primary separator is a rectangular shaped plastic tank with a viewing window that enables visual liquid levels and separation process monitoring. Internally, its design incorporates weir for overflow. The multiphase fluid exits the test section and enters the first partition of the separator where the viewing window is located. Initial separation by gravity takes place in this section. The denser phase settles at the bottom while the less dense phase moves to the second section for further separation. A mixture of high-viscosity oil-water-sand may require a residence time of at least 18 – 24 hours for a near-complete separation to take place. After separation has taken place in the primary separator, oil (with possibly, little mixture of water) is pumped into the secondary separator where further separation takes place. The secondary separator is a modified Intermediate Bulk Container (IBC). On complete separation of the oil-water phase (after a residence time of about 6 hours), single phase oil is recovered and reused while wastewater from the (primary & secondary stage) separators and sand from the first stage separator are disposed of.

2.2.4 Instrumentation and Data Acquisition System

In the 1-inch horizontal and inclined facilities, two GE Druck static pressure transducers, PMP 1400 with pressure range 0 – 4 barg and accuracy 0.04% over the full scale is used to obtain the static pressure in the test sections. Both transducers are placed 2.17 m apart with the first of them positioned 60 pipe diameters from the last injection point, to ensure measurements are taken when flows are fully developed. A differential pressure transducer, Honeywell STD120, with minimum pressure drop measurement of 100 Pa and an accuracy of ±0.05% is used to measure the differential pressure. Temperature of the test fluids on the test section is measured by means of J-type thermal couples with an accuracy of ±0.1°C placed at different locations. Similar arrangement is used in the 3-inch test facility. For temperature regulation in the 1-inch horizontal and inclined test facilities, temperature (and hence viscosity) of mineral oil is regulated by a system consisting of a J-type thermocouple, copper coils (submerged in the oil tanks) and a Thermo Scientific (HAAKE Phoenix II) refrigerated bath circulator. Copper coils in the oil tank are connected to the circulator by running cold or hot glycol in the coils at specific time intervals. The temperature of oil in the tank is thus controlled by heat transfer. The
circulator’s temperature ranges from 0 to +50 °C, with an accuracy of ± 0.01 °C. Similar setup exists for temperature regulation of oil in the 3-inch facility. For repeatability and reproducibility, selected test matrix (chosen in such a way that each test points are represented) were repeated four times. Results showed good agreement for similar flow parameters. This also agreed with uncertainty analysis carried out to estimate errors in measurement. Please refer to Appendix A for information of how the uncertainty was analysed.

Recordings of experimental test runs were done with two Sony Handycam HDR-CX550VE, 12.0 Megapixels, Wide Angle 26.3 MM, Exmor R CMOS Sensor, 1080 50i PAL Full HD, G Lens, focal length 3.8~38.0 mm with sampling frequency of 25 Hz. Both cameras were placed 50 cm from the viewing sections in the side and bottom positions to ensure flow features of sand particulates from those positions are clearly captured. Two lamps powered by four 40 W bulbs each were used to illuminate the test section, the lamp were placed at 45 degrees inclination to the section to minimize reflections.

### 2.3 Test procedure

The procedure used to conduct the test is described. Mineral oil stored in the oil tank is pumped through a recirculation loop between the oil tank and the test facility bypass/injection section to ensure a uniform temperature distribution (and thus, liquid viscosity). Set temperature on the chiller is used to regulate oil viscosity. Temperature at the tank, the viscometer and operating temperature at the single-phase oil line just before the injection point are used to validate the viscosity of the single-phase oil before it mixes with water at the mixing section. In the main test section, transducers are used to log temperature values. Analysis of the data shows that temperature at the main test section are similar to temperature at the tank. The oil tank has submerged temperature controller coils to ensure either heating or cooling of the contents in the tank. On the inception of any experiments, data is first obtained for an empty test section and a single-phase filled facility to ensure noise and zero errors in the devices are eliminated during data analysis.

During experiment, oil is first pumped into the main test line through a pipe that is in-line with the main test section and at flowrates that corresponds to the predetermined oil superficial velocity. The superficial velocity of oil is then kept constant throughout the experimental run time. Slurry is introduced through a flexible pipe connected to the main test section by a Tee-section.

<table>
<thead>
<tr>
<th>Pipe ID (m)</th>
<th>Inclination</th>
<th>Vso (m/s)</th>
<th>Vss (m/s)</th>
<th>Sand concentration (vol/vol)</th>
<th>Viscosity (Pa.s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.0254</td>
<td>0°</td>
<td>0.10 – 0.20</td>
<td>0.1 – 1.4</td>
<td>0.010 – 0.100</td>
<td>3.5, 10.0</td>
</tr>
<tr>
<td>0.0254</td>
<td>30°</td>
<td>0.10 – 0.20</td>
<td>0.1 – 1.0</td>
<td>0.010</td>
<td>3.5</td>
</tr>
</tbody>
</table>
For tests geared towards obtaining MTC, slurry superficial velocity starts from a high superficial velocity and decreases gradually. If the effect of shutdown is to be investigated or pickup velocity of sand particles, slurry velocity begins from its lowest and gradually increases. For each test condition, the flow is allowed to run for 60s from which 30s worth of data is logged at sampling frequency of 250 Hz. In order to ensure results reproducibility, selected test conditions (40% of the test matrix for each experimental run) across the different flow patterns are repeated three times, measurements are obtained during each run and their results are compared to ensure consistency. Video recordings and still photographs are recorded in both the side and bottom view of the test observation section. Table 2 shows the test matrix used for the experimental investigations carried out in this study for the three experimental test facilities.

3 Results and Discussion

3.1 Flow Characterization

Results and discussion for the flow patterns observed in test facilities with 1-inch, 1-inch inclined and 3-inch pipe internal diameters are presented here. The section describes the flow patterns with the aid of images and presents a flow pattern map to highlight the flow condition in which each flow pattern was observed. The transitions from one pattern to the other are highlighted in the map. Pipe diameter and inclination effects are also discussed.

3.1.1 Flow Pattern Maps

Representative still images obtained from high definition cameras are presented in Figure 4. Images shown follows flow patterns observed with a gradual reduction in slurry velocity from bottom to top. At the highest superficial slurry (water-sand) velocity and a fixed oil superficial velocity, sand particles in the multiphase flow attains its highest momentum. This momentum causes the sand particles in the pipelines to be dispersed in the flow in a random and chaotic pattern. The degree of homogeneity of the particles in the liquid is a function of the liquid velocity. This flow pattern observed is termed the Dispersed Flow (DF) pattern is observed (Figure 4d). DF is characterised by oil dispersion in a continuous water phase (Figure 4d1 and 4d2). Fouling in the form of oil streaks on the pipe wall (as a result of the high viscosity of oil) is observed in this regime with sand particles homogeneously distributed.

When superficial slurry velocity is decreased, the very high momentum observed in the DF is reduced, this allows some dispersed oil to form globules in water. These globules (oil plugs) are observed to be flowing in a continuous water. Oil streaks on the pipe walls, first noticed in the DF regime is observed to grow thicker due to a reduction in water cut and sand concentration in the flow (decreased flow momentum ensures more oil is now
able to foul the pipe walls). Figure 4c2 shows a relatively lower homogenous distribution of sand particles relative to the DF. This is also attributable to the decrease in turbulent energies that enhanced sand particles dispersion. Further reduction in the slurry superficial velocity oil-coated sand particles are walls of the pipe. Figure 4c1 shows that oil plugs were predominantly near the top section of the pipe. This flow pattern was termed Plug Flow (PF). Additional reduction of the slurry superficial velocity resulted in a flow pattern similar to PF. However, a moving/sliding sand bed was observed at the bottom of the pipe. The oil streaks on the walls of the pipe and the oil plugs increased in size relative to the PF. Flow patterns with these characteristic features were termed Plug Flow with Moving Sand Bed (PFM). The onset of sand deposition at the bottom of the pipeline. This flow pattern was in the PFM. The mean mixture velocity of the flow at which the sand particles were observed to be sliding on the bottom (as shown in Figure 4b2) of the pipeline is the MTC. Further reduction in slurry superficial velocity resulted in a denser sand bed, elongated and thicker oil plugs. This is attributable to a reduction in flow turbulence which ensures more sand particles settle to the bottom of the pipe and the oil plugs agglomerate into thicker sizes as shown in Figure 4a2. In addition, gravity forces in flow are dominant at relative lower flow velocities leading to the settling of sand particles. Decreasing sand superficial velocity further, resulted in more sand particles settling at the sand bed as gravity forces dominance increases. Figure 4a2 shows that oil wetted sand particles were observed to agglomerate in patches within the sandbed. Some patches were observed to be moving at slow speed relative to the mean speed of the sand bed. This flow pattern was termed the Plug Flow with Static Sand Bed (PFS).

Flow pattern maps were constructed with the superficial oil and slurry velocities on the ordinate and abscissa respectively, as shown in Figure 5. From the plots shown, it is clearly seen that the transition from one flow pattern to the other depends on the oil superficial velocities, the sand concentration and the slurry velocity.
Flow patterns earlier described transits from one type to the other based on this flow conditions. As an example, for a fixed superficial oil velocity of 0.11 m/s at nominal oil viscosity of 10.0 Pa.s and sand concentration of 10%, DF flow pattern is observed at the highest slurry superficial velocity of 0.90 m/s. A reduction in this superficial velocity to about 0.7 m/s sees the flow transit to PF. Subsequent reduction to lower slurry velocities sees transition through the MTC to the PFM and the PFS flow patterns. For the 30° inclined test section, a flow pattern map was similarly constructed and is shown in Figure 5 (d). The different flow conditions at which each flow pattern exists are clearly shown in the map.
Figure 5: Flow pattern maps for oil-water-sand flow for sand concentrations of (a) 1% (b) 5% (c) 10% v/v at oil viscosity 10.0 Pa.s and (d) 1% v/v sand concentration at oil viscosity of 3.5 Pa.s.

Flow patterns observed in the 1-inch 30° upward inclined test section are shown in Figure 6 with increasing slurry superficial velocity from top bottom. For a constant superficial oil velocity and at the highest superficial slurry velocity tested in this investigation, *Dispersed Flow (DF)* is observed. Oil streaks and thus fouling on the pipe wall, hitherto observed in the horizontal test section, were considerably thinner and sometimes, non-existent in the inclined section. This may be attributable to the increased effect of gravity which enhances the draining speed of oil from the pipe walls. *Plug Flow (PF)* was observed on reducing the slurry velocity. Further reduction resulted in the *Plug Flow with Moving Sand Bed (PFM)* similar to the earlier description in the horizontal test section. At superficial velocities of slurry below *PFM*, a flow pattern in which oil, water and sand flowed in different strata was observed. The densest phase (sand) flowed at the bottom while the least dense phase (oil) flowed at the top. The interface between oil and water was wavy and irregular. The sand bed in this regime was transported along the pipeline at low velocities, it also thicker compared to *PFM*. This flow pattern was termed *Stratified Wavy Flow (SWM)*.
A further decrease led to observations of flow pattern similar to the SWM, however, the moving sand bed were transformed to dunes, particles at the upper layer of the dunes were observed to be saltating into the sheds in front of its parent dune. The particles in the shed stayed in place until the slower moving dune travels the distance and reabsorbs the particle into its "cell". The wavy interface of the oil was observed to make infrequent contact with the sand particles at the dunes’ topmost layer. This flow pattern was called Stratified Wavy Flow with Dunes, SWD. It is worth noting that SWM and SWD were not observed in the horizontal flow patterns at similar test conditions, their observation here can be explained by the increased effect of gravity in the flow due to test section inclination which favours flow stratification in high viscous oil-water flow.

3.2 Minimum Transport Condition (MTC)

The Minimum Transport Condition (MTC) is defined as: “the slurry superficial velocity in which water wetted sand, deposits and forms a sliding/moving sand bed on the bottom of a horizontal or slightly inclined pipeline”. MTC of sand in oil-water multiphase flow test was determined through visual observations, video recordings and pressure gradient logs. Results obtained for MTC is discussed in the following subsections.
3.2.1 Effect of Oil Superficial Velocity on MTC

MTC results in the 1 inch horizontal test facility for nominal oil viscosity, 10.0 Pa.s and sand concentrations of 1, 5 & 10% volume/volume is presented in Figure 7. For sand concentration of 1% vol./vol. and nominal oil viscosity of 10 Pa.s, an increase in oil superficial velocity from 0.10 m/s to 0.14 m/s results in MTC reduction from 0.41 m/s to 0.26 m/s.

![Figure 7: MTC as a function of oil superficial velocity, (a) sand concentration and (b) water cut at oil viscosity of 10.0 Pa.s](image)

Further increase in oil superficial velocity from 0.14 m/s to 0.20 m/s marginally decreased the MTC to 0.25 m/s. The same trend is observed for sand volume fractions of 5 and 10%. This decrease in MTC with increase in oil superficial velocity indicates that an increased in oil content in the pipeline have the effect of increasing sand transportation capabilities of sand. Overall, an increased mixture velocity (sum of slurry and oil superficial velocities) will ensure increased turbulence thus ensuring sand particles remain suspended in flow at relatively lower MTC.

3.2.2 Effect of Sand Concentration on MTC

Figure 7 a shows that an increase in sand concentration have the effect of increasing the MTC for all the oil superficial velocities investigated. Gravity forces that acts to force sand particles to settle on the pipe’s bottom is a function of gravitational acceleration and mass of the sand. Increasing the sand concentration will increase the mass of sand in the pipeline and hence, gravity forces. This implies that the gravity forces dominate over a relatively larger mixture velocity thus requiring a relatively larger superficial velocity for the onset of sand particles settling in the pipe’s bottom. This characteristic behaviour, therefore, results in a higher MTC value. Higher lift forces are also required to keep sand particles in suspension. This trend of MTC increase with an increase in sand concentration was also observed for the low viscosity water-sand flow investigated by Archibong-Eso et al. (2019), Fajemidupe et al. (2019), Archibong-Eso (2015), and Yan (2010).
3.2.3 Effect of Water Cut on MTC

Figure 7b shows water cut \( \frac{V_{ss}}{(V_{ss} + V_{so})} \) as a function of both the oil superficial velocity and the sand concentration. It is pertinent to note that in this study, a modified water cut equation was used by substituting the water superficial velocity with slurry superficial velocity. For the three-sand concentrations investigated in this study, water cut values at the MTC reduced with increase in oil superficial velocity. This follows a similar explanation made earlier; increased oil superficial velocity increases the turbulence in flow hence allowing suspension of particles at lower water cut. Practically, this implies that for fields with high oil content, the water cut required to transport sand particles will be lower compared to a similar field with low oil content.

3.2.4 Viscosity effects on MTC

Experimental investigations conducted with oil viscosity of 3.5 Pa.s and 10.0 Pa.s in the 1-inch horizontal flow loop are used to discuss the effect of viscosity on MTC for sand. Results shown in Table 3

Table 3 indicates that the MTC for three-phase oil-water-sand flow increased with increase in MTC for both sand concentrations used in this section of the study.

<table>
<thead>
<tr>
<th>Vso (m/s)</th>
<th>Oil Viscosity (Pa.s)</th>
<th>Sand Concentration (%)</th>
<th>MTC (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>3.50</td>
<td>1.00</td>
<td>0.32</td>
</tr>
<tr>
<td>0.10</td>
<td>3.50</td>
<td>5.00</td>
<td>0.36</td>
</tr>
<tr>
<td>0.10</td>
<td>10.00</td>
<td>1.00</td>
<td>0.41</td>
</tr>
<tr>
<td>0.10</td>
<td>10.00</td>
<td>5.00</td>
<td>0.60</td>
</tr>
<tr>
<td>0.10</td>
<td>10.00</td>
<td>10.00</td>
<td>0.66</td>
</tr>
</tbody>
</table>

While an increase of about 28% was observed at 1% sand concentration when oil viscosity increased from 3.5 Pa.s to 10.0 Pa.s, a similar increase in oil viscosity at 5% sand concentration yielded a 67% increase in MTC value. Kinetic turbulence is essential in ensuring sand particles are suspended during the three-phase flow, an increase in oil viscosity reduces the turbulence thus ensuring that particles require a much higher flow velocity to produce significant inertia forces capable of overcoming the viscous forces.

3.2.5 Pipe diameter effects on MTC

The 3-inch horizontal test facility was used to investigate the effect of scaling up the internal pipe diameter on MTC. Sand concentration studied was 1 and 5% at oil viscosity of 3.5 Pa.s and oil superficial velocity of 0.10 – 0.20 m/s. Table 4 presents the result obtained from both horizontal facilities. Result indicates that MTC increased with increase in pipe diameter at the same oil superficial velocity. Comparatively, MTC increased from 0.57 m/s at sand concentration of 1% and oil superficial velocity of 0.10 m/s in the 1-inch
pipe to about 0.72 m/s in the 3-inch pipe at similar flow conditions. Similar trend is observed for the 5% sand concentration. This may be attributed to the decrease in the flow turbulence for similar flow conditions. As pipe diameter increases and thus the effective flow area, Reynolds Number, a major pointer to the turbulence in flow (which acts to keep sand particle suspended rather than depositing on the pipe’s bottom) decreases hence requiring higher velocities to keep the same sand concentration suspended in flow.

Table 4: Oil-Water-Sand MTC in 1 and 3-inch ID test facilities

<table>
<thead>
<tr>
<th>Oil Superficial Velocity (m/s)</th>
<th>Sand concentration (%)</th>
<th>Pipe Diameter (m)</th>
<th>Oil Viscosity (Pa.s)</th>
<th>MTC (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.10</td>
<td>1.00</td>
<td>0.0254</td>
<td>3.50</td>
<td>0.32</td>
</tr>
<tr>
<td>0.10</td>
<td>1.00</td>
<td>0.0764</td>
<td>3.50</td>
<td>0.54</td>
</tr>
<tr>
<td>0.19</td>
<td>1.00</td>
<td>0.0254</td>
<td>3.50</td>
<td>0.25</td>
</tr>
<tr>
<td>0.20</td>
<td>1.00</td>
<td>0.0764</td>
<td>3.50</td>
<td>0.76</td>
</tr>
<tr>
<td>0.10</td>
<td>5.00</td>
<td>0.0254</td>
<td>3.50</td>
<td>0.36</td>
</tr>
<tr>
<td>0.11</td>
<td>5.00</td>
<td>0.0764</td>
<td>3.50</td>
<td>0.88</td>
</tr>
</tbody>
</table>

3.2.6 Inclination Effects on MTC

Results obtained from the 1-inch 30° inclined test section were used to compare with the 1-inch horizontal test section for the oil-water-test as shown in Table 5. Results indicate that within the test conditions of the experiments, MTC is only slightly affected by the inclination angle. MTC at 30° test section was similar to that of the horizontal with only slight changes observed. Generally, it was observed that a 30° upward inclination had negligible effect on MTC when compared to the horizontal test for oil-water-sand flows. Results presented are for nominal oil viscosity of 3.5 Pa.s.

Table 5: MTC for oil-water-sand test in horizontal and inclined test facilities (Nominal oil viscosity, 3.5 Pa.s)

<table>
<thead>
<tr>
<th>Sand concentration (%)</th>
<th>Pipe Diameter (m)</th>
<th>Inclination (°)</th>
<th>Oil Superficial Velocity (m/s)</th>
<th>MTC (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>0.0254</td>
<td>0</td>
<td>0.10</td>
<td>0.42</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0254</td>
<td>30</td>
<td>0.10</td>
<td>0.41</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0254</td>
<td>0</td>
<td>0.14</td>
<td>0.44</td>
</tr>
<tr>
<td>1.00</td>
<td>0.0254</td>
<td>30</td>
<td>0.15</td>
<td>0.46</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0254</td>
<td>0</td>
<td>0.20</td>
<td>0.54</td>
</tr>
<tr>
<td>5.00</td>
<td>0.0254</td>
<td>30</td>
<td>0.20</td>
<td>0.55</td>
</tr>
</tbody>
</table>
Figure 8: Pressure gradient as a function of water cut (sand concentration, 1% v/v) – 30° inclined pipeline.

3.3 Pressure Gradient Analysis

3.3.1 Effect of Water Cut on Pressure Gradient

Figure 8 shows plots of pressure gradient as a function of water cut for representative flow conditions in the study. It is observed that initial increase in water cut decreases the pressure gradient due to the reduction of the effective viscosity in the flow until a minimum is reached. Further increase is observed to result in a corresponding increase in pressure gradient; this is because the increased water cut has relatively little effect on viscosity reduction compared to the increased flow velocity, which aids pressure increase. Additionally, some of the dispersed oil and oil plugs are carried back to the walls of the pipe resulting in increased shear. This behaviour is particularly similar to the water-oil flow.

3.3.2 Effect of Oil Input Content on Pressure Gradient

The effect of increasing oil input content (oil superficial velocity) on pressure gradient is shown in Figure 9 below where pressure gradient as a function oil superficial velocity of 0.10, 0.14 and 0.20 m/s are presented. The plot shows a general increase in pressure gradient with increase in oil superficial velocities. This is attributed to the increase oil content in the line. An increase in oil content increases the shear on the pipe wall and thus the pressure gradient as earlier explained in the oil-water-sand 1-inch horizontal test. Notably, below a slurry velocity of about 0.2 m/s (where the flow is largely unassisted by water), the effect of increasing the oil content in the pipe is not distinct. Further investigations may be necessary to determine the flow characteristic behaviour.
Figure 9: Pressure gradient as a function of oil and sand superficial velocities at sand concentration of (a) 1% (b) 5% and (c) 10% v/v and oil viscosity of 10 Pa.s.

A further explanation on the effect of oil input on pressure gradient can be made using the superficial oil velocity. For all the conditions depicted in Figure 9, the pressure gradient progressively declines with a reduction in slurry velocity. However, the magnitude of the pressure gradients increases with increasing oil viscosity due to the combined effect with sand concentration. For example, at Vso of 0.1 m/s, the pressure gradient is seen to increase from about 2 kPa/m to just below 3 kPa/m. Furthermore, it can be seen in Figure 9c that the pressure gradient experiences a sharp increase as Vso goes from 0.14 to 0.2 m/s. From the Hagen-Poiseuille pressure gradient model, it is observed that pressure gradient is an inverse function of pipe flow area (diameter) thus explaining the observed phenomenon. Several authors in literature have stated that MTC occurs at the minimum pressure gradient in liquid-solid studies, this was not necessarily the case in the present study. Howbeit, the minimum pressure gradient reduction occurred close to the MTC and the disparity in conclusion may be a result of the differences in MTC definition, uncertainty in measurements and/or the subjectivity involved in MTC definition.

In comparison to the pressure gradient data of Mckibben et al. (2013) in their 4-inch pipe, the range of pressure gradients we obtained in the 3-inch pipe were similar in magnitude as theirs. However, the trend with mixture velocity ($V_m$) showed that the current data had
a gentler rising slope (especially at the 1% sand concentration case) as compared to Mckibben’s trend which had a linear and quasi-quadratic trend for their various water cut and single phase conditions. These were given in Figure 4 of their article. The oils used in their experiments had a range from 0.2–3.7 Pa.s and then a big jump to 31.4 Pa.s. As such, their friction factor correlation’s applicable range is affected by the missing viscosities between 3.7 and 31.4 Pa.s. It is reiterated here that the current data were obtained with oils of 3.5 – 10 Pa.s which may considered a more stable range. Secondly, it is noted that our flow regimes were varied (oil plugs to dispersed oil with sand beds) as against their co-annular regime. It is hence imperative to adjust the friction factor correlation to be more applicable to the conditions of the current experiments. We report the correlation procedure in subsequent sections.

3.3.3 Effect of Sand Concentration on Pressure Gradient

Table 6 below shows the pressure gradient at MTC for different sand concentration at the same superficial oil velocity. Results shown indicates that the pressure gradient increases with increase in sand concentration, this is expected as the increased sand concentration in the flow line decreases the effective flow area of the mixture thereby increasing the pressure gradient.

<table>
<thead>
<tr>
<th>Sand concentration v/v (%)</th>
<th>Vso (m/s)</th>
<th>Oil Viscosity (Pa.s)</th>
<th>MTC (m/s)</th>
<th>Pressure Gradient (kPa/m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.10</td>
<td>10.0</td>
<td>0.51</td>
<td>1.90</td>
</tr>
<tr>
<td>5</td>
<td>0.10</td>
<td>10.0</td>
<td>0.60</td>
<td>3.02</td>
</tr>
<tr>
<td>10</td>
<td>0.10</td>
<td>10.0</td>
<td>0.66</td>
<td>3.18</td>
</tr>
</tbody>
</table>

Figure 10: Pressure gradient as a function of slurry velocity (or water superficial velocity) in a variety of multiphase flow
3.3.4 Water-Sand, Oil-Water & Oil-Water-Sand Pressure Gradient

Comparison of pressure gradient obtained in water-sand, oil-water and oil-water-sand flow against slurry velocity (or water superficial velocity, as in the case of oil-water flow) is presented in Figure 10. Comparatively, at similar flow conditions, oil-water pressure gradient is lower than the pressure gradient logged for oil-water-sand; the presence of sand in the latter may be responsible for this observation. Sand in flow acts to reduce the effective flow area of the pipe thus increasing the pressure gradient. A further comparison with the water-sand test shows clearly the influence of oil and sand presence in flow. In comparison between the water-sand, oil-water and oil-water-sand flow, it is observed that the water-sand flow pressure gradient was the lowest by several order of magnitudes due to the absence of oil in flow. The presence of oil in flow results in increase shear and fouling on the pipe wall, hence increasing the pressure gradient.

3.4 Correlation for MTC in oil-water-sand pipe flow

Based on the experimental data collected that have been collected, it is imperative to correlate the onset of sand transport in three-phase oil–water–sand flow. Since it was shown that MTC varies as a function of the inlet oil and water conditions as well as the sand slurry flow conditions, these quantities will be used for correlation in dimensionless form. The correlation method is based on the method of McKibben et al. and can be considered an improvement of their model. A similar method have been adopted by(Aliyu, 2015; Aliyu et al., 2015) The MTC which gives the onset of particle deposition in the pipeline, is defined as the condition whereby the friction velocity is greater than the particles’ terminal settling velocity i.e. when $V^* > V_\infty$. The settling velocity is defined as

$$V_\infty = \sqrt{\frac{1.33 g D (\rho_s - \rho_f) / C_D \rho_f}{C}};$$

where $\rho_f$ and $C_D$ are the solid carrier fluid (water) density and the drag coefficient of the solids respectively. Other parameters are as previously defined. In fact, Mckibben et al. found that with oils of up to 31.4 Pa.s, the relationship between the friction velocity and the particle terminal settling velocity at MTC is as follows.

$$V^* = 9V_\infty$$  \hspace{1cm} (1)

meaning that the minimum transport condition or velocity is:

$$V_{MTC} = \frac{9V_\infty}{\sqrt{f_m/2}}$$  \hspace{1cm} (2)

since the friction velocity and the terminal settling velocity are related by:

$$V^* = V_{MTC} \frac{f_m}{\sqrt{2}}$$  \hspace{1cm} (3)

where $f_m$ is the three-phase mixture friction factor which McKibben et al. (2013) found the following relation to fit their experimental data:
\[ f_m = 15 \cdot Fr^{-0.5} f_w^{1.3} f_o^{0.32} C_w^{-1.2} \] (4)

where \( Fr \), is the mixture Froude number, \( f_w \) is the mixture friction factor for the water phase, \( C_w \) is the water cut (water fraction), and \( f_o \) is the oil friction factor. They are defined as follows:

\[ Fr = \frac{V_M}{\sqrt{gD}} > 0.35; \quad f_w = \frac{1}{\sqrt{16 \log \sqrt{5.7/\rho_m V_M^2 D}}}; \]

\[ f_o = \frac{16}{Re}; \quad Re = \frac{\rho_o V_m D}{\mu_w} \] (5)

For the current study, as the viscosity range is lower than that used by McKibben and co-workers, we therefore correlated the mixture friction factor using the measurements obtained the current study in addition to those of McKibben’s using non-linear least squares fitting. For simplicity, the same dimensionless parameters used by McKibben et al. (2013) were selected since they carried out an extensive dimensional analysis. We therefore express \( f_m \) as a power law function of \( Fr \), \( f_w \), \( f_o \), and \( C_w \) as follows:

\[ f_m = A \cdot Fr^b f_w^c f_o^d C_w^e \] (6)

where the coefficient \( A \) and indices \( b-e \) are regression constants to be determined by fitting Equation 6 to the horizontal experimental data for the two pipe sizes in this study, using the least squares method expressed as a minimisation problem as follows:

\[ \min \sum_{i=1}^{N} [(f_m)_{exp,i} - (A \cdot Fr^b f_w^c f_o^d C_w^e)_{pred,i}]^2 \] (7)

where \( N \) is the number of experimental data points, the subscripts “pred” and “exp” denote predicted and experimental respectively. Equation 7 was solved iteratively to determine the values of \( A-e \) using the Generalised Reduced Gradient method. Initial values of \( A-e \) were chosen and constrained to be realistic. Hence, the new three-phase friction factor correlation, which covers a wider range of experimental conditions and oil viscosity, is given as:

\[ f_m = 20 \cdot Fr^{-0.5} f_w^{1.2} f_o^{0.4} C_w^{-0.32} \] (8)

It will be shown in Figure 11 that the MTC predicted using the new correlation is a marked improvement over that using Equation (4). In addition to the new mixture friction factor correlation (Equation 6), calculation methods for other key quantities in the MTC correlation were employed that consider the mechanisms at play. For example, for calculating the terminal velocity, the Burddyck model was used. It was developed for the transition zone (0.1 < \( d_p < 0.1 \ mm \)) and clearly defines the characteristic drag coefficient of the particles:
\[ V_\infty = \frac{8.8}{d_p} \left[ \sqrt{1 + 95 \frac{S_s - S_f}{S_f}} d_p^3 - 1 \right] \] (9)

where \(d_p, S_s\) and \(S_f\) are defined as the particle diameter, specific density of solids relative to water, and the specific density of carrier fluids relative to water. Secondly, an effective diameter, \(D_H\) is used for implementation to ensure the flow area takes the presence of sand particles into account. This is achieved by estimating a sand holdup, \(H_s\) and hence the effective diameter for the flow area:

\[ H_s = \frac{C_v V_{ss}}{V_m} \] (10)

\[ D_H = D(1 - H_s) \] (11)

where \(C_v\) and \(D\) are the sand concentration and the pipe internal diameter respectively.

To account for the sand concentration, the gravity effect is associated with the sand concentration thus: \(g(s - 1)(1 - C_v)\). This is consistent with the fact that the gravity force in flow is associated to the weight of sand and thus its concentration in the flow line. It is worth noting that at lower sand concentration this may be negligible. However, as sand concentration increases it becomes increasingly important and will lead to errors in the model if not considered. Therefore, a modified Froude number is used:

\[ F_r = \frac{V_m}{\sqrt{g D_H (s - 1)(1 - C_v)}} \] (12)

Figure 11 shows that the McKibben et al. (2013)'s model over estimates the MTC by up to 50%, this error in prediction may be attributed to the fact that the model was developed on the basis of significantly different flow conditions (e.g. pipeline used in their study had internal diameters of 4 and 6-in) and fluids of a higher viscosity range than those used in the current experiments. On the other hand, the MTC prediction based on the modified friction factor which was derived by combining the current experimental data and those of McKibben et al.'s shows markedly improved predictions which are predominantly within a ±20% error band.
Figure 11: Comparison of McKibben et al. (2013) Model with the proposed modified model

Table 7 shows the statistical performance evaluation of the new model and that of McKibben’s. The evaluation indicates that the proposed model outperforms McKibben’s MTC model for all the statistical errors investigated. The statistical parameters $E_1$–$E_4$ are defined for each data point $i$ as follows. $E_1$ is the average relative error:

$$E_1 = \frac{1}{N} \sum_{i=1}^{N} (y_{predicted} - y_{measured})_i$$

(13)

The absolute of average relative error is given as:

$$E_2 = \frac{1}{N} \sum_{i=1}^{N} |(y_{predicted} - y_{measured})_i|$$

(14)

While standard deviation about the relative error is given by:

$$E_3 = \sqrt{\frac{\sum_{i=1}^{N} (y_i - Y_1)^2}{N - 1}}$$

(15)

The average actual error

$$E_4 = \sum_{i=1}^{N} (y_{predicted} - y_{measured})_i^2$$

(16)

The mean error, $E_1$, the mean absolute error, $E_2$, standard deviation, $E_3$ of the errors and the sum square of errors, $E_4$ of the proposed model were $-0.03$, $0.17$, $0.19$ and $0.42$ respectively relative to McKibben’s model of $0.63$, $0.63$, $0.24$ and $5.08$ respectively. Caution is advised in using the correlation outside the range of experimental data used in its development. Nevertheless, the correlation provides a useful tool in obtaining
improved predictions of sand MTC in three-phase oil–water–sand flows involving highly-viscous oils.

Table 7: Statistical evaluation of proposed model with the McKibben et al. (2013) model

<table>
<thead>
<tr>
<th>Correlation</th>
<th>E1</th>
<th>E2</th>
<th>E3</th>
<th>E4</th>
</tr>
</thead>
<tbody>
<tr>
<td>McKibben et al. (2013)</td>
<td>0.63</td>
<td>0.63</td>
<td>0.24</td>
<td>5.08</td>
</tr>
<tr>
<td>Proposed</td>
<td>-0.03</td>
<td>0.17</td>
<td>0.19</td>
<td>0.42</td>
</tr>
</tbody>
</table>

4 Conclusions

In this study, the transport of high viscous oil-water-sand has been experimentally investigated in the 1 & 3-inch horizontal and 1-inch 30° upward inclined test facilities. The study involved flow pattern identification, pressure gradient analysis and MTC prediction.

The main findings in the study are as follows:

In the horizontal multiphase oil-water-sand test, new flow patterns namely: Dispersed Flow (DF), Plug Flow (PF), Plug Flow with Moving Sand Bed (PFM) and Plug Flow with Stationary Sand Bed (PFS) were reported in the horizontal pipe.

For the inclined flow, similar flow patterns were observed with two additional flow patterns; the Stratified Wavy Flow with Moving Sand Bed (SWM) and Stratified Wavy Flow with Dunes, (SWD).

Pressure gradient were generally observed to decrease initially with decrease in superficial slurry velocity (and hence water cut) before reaching a minimum. Further reduction in the slurry velocity beyond this minimum results in an increase in pressure gradient. Pressure gradient in the oil-water-sand test was much higher than the water-sand test and higher when compared to the oil-water test for similar flow conditions.

The MTC in oil-water-sand test were observed to be largely the same for the 1-inch 30° upward inclined and the 1-inch horizontal test section while that of the 3-inch horizontal test section was considerably higher. A new empirical model has been proposed and shown to outperform other sand–oil–water models including that of McKibben et al. (2013).

Acknowledgement

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References


A.1 Estimating Uncertainties

In this research, work, experimental uncertainties were determined using a stepwise procedure similar to (Archibong 2015) and it is explained below:

1. If an experimentally measured output, \( y \) is a function of inputs \( x_1, x_2, x_3 \ldots x_n \) then:
   \[
   y = f(x_1, x_2, x_n \ldots x_n)
   \]  
\( (A-1) \)

2. Estimate uncertainty of \( x_i \)
   \( u(x_i) \) — uncertainty in absolute terms
   \( u(x_i) \) — uncertainty in fractional terms

3. Compute sensitivity of \( y \) to changes in \( x_i \), i.e. partial differentiation for each input.
   The absolute and fractional inputs are given by:
   \[
   c_i = \frac{\partial y}{\partial x_i}
   \]
   \( (A-2) \)
   \[
   c_i^* = \frac{\partial y}{\partial x_i} \cdot \frac{x_i}{y}
   \]
   \( (A-3) \)

4. A combination of the uncertainties for a set of uncorrelated inputs is obtained by summation of uncertainties for each input. The absolute and fractional terms are expressed as:
   \[
   u_c(y) = \sqrt{\sum_{i=1}^{n} (c_i \cdot u(x_i))^2}
   \]
   \( (A-4) \)
Uncertainties in the superficial liquid and gas liquid velocities, liquid holdup, pressure gradient and liquid viscosity is described in the following sections together with sample calculation.

A.1.1 Uncertainty in superficial liquid velocity

The liquid superficial velocity, $V_{SL}$ is a function of the mass flow rate, $\dot{M}_L$, liquid density, $\rho_L$ and flow area, $A$. For the purpose of this evaluation, the pipe diameter and hence the flow area will be considered constant. The Coriolis mass flow meter is used to obtain the mass flow rate and liquid density, from the manufacturer’s guide, the maximum error in measurement are $\pm0.5\%$ and $\pm0.5\,\text{kg/m}^3$ for mass flow rate and liquid density respectively. In evaluating the uncertainty, we follow the steps:

1. Determine standard uncertainty of each of the functions based on their respective measurement equipment and confidence level. Assuming a 95.4% confidence level;

$$V_{SL} = \frac{\dot{M}_L}{\rho_L A} = \frac{4\dot{M}_L}{\rho_L \pi D^2}$$  \hspace{1cm} (A-1.1)

2. Partial derivatives of the inputs, $Q_L$ and $\rho_L$ is given by:

$$\frac{\partial V_{SL}}{\partial \dot{M}_L} = \frac{4}{\rho_L \pi D^2}$$  \hspace{1cm} (A-1.2)

$$\frac{\partial V_{SL}}{\partial \rho_L} = -\frac{4\dot{M}_L}{\pi (\rho_L D)^2}$$  \hspace{1cm} (A-1.3)

3. Combined uncertainty in measurement of the superficial liquid velocity is thus:

$$u_c^*(y) = \sqrt{\sum_{i=1}^{n} (c_i^* \cdot u^*(x_i))^2}$$  \hspace{1cm} (A-5)
\[
    u_c(V_{SL}) = \sqrt{\sum_{i=1}^{n} (c_i^+ \cdot u^+(V_{SL}))^2}
\]

\[
    = \sqrt{\left( u_c^+(\dot{M}_L) \cdot \frac{\partial V_{SL}}{\partial \dot{M}_L} \right)^2 + \left( u_c^+(\rho_L) \cdot \frac{-4\dot{M}_L}{\pi(\rho_L D)^2} \right)^2}
\]

(A-1.4)

**Case Study**

In a test case for the 0.0254 m pipe ID test facility, \( V_{SL} = 0.10 \text{ m/s}, \dot{M}_L = 180.5 \text{ kg/h}, \rho_L = 905.8 \text{ kg/m}^3 \). As earlier stated, pipe diameter is considered a constant while the uncertainty for \( \rho_L \) and \( \dot{M}_L \) are as given in the manufacturer’s manual. Substituting in the combined uncertainty equation at 95.4 % confidence level, the superficial liquid velocity for this condition is \( \pm 0.54 \% \). Similar analysis for the electromagnetic flowmeter used for metering slurry showed similar results.

Generally the uncertainty in measurement of the \( V_{SLS} \) (superficial velocity of water, slurry and oil) showed an uncertainty of \( \pm 0.55, 0.50 \text{ and } 0.50 \% \) respectively. The uncertainty in measuring the sand loadings was negligible (\( \pm 10 g \)) when compared to the sand loading (loadings > 1kg).

**Uncertainty in pressure gradient**

Pressure gradient in this work was obtained directly from measurements by the differential and point pressure transducers. Based upon this premise, the uncertainty in the measurements of pressure gradient is sourced directly from the stated uncertainties in the manufacturer’s guide. For measurements using the single points and the differential pressure transducers, the uncertainties were given as \( \pm 2 \text{ and } \pm 0.04\% \) for the range of \( 0 \text{ to } 6 \text{ barg and } -200 \text{ to } +200 \text{ mbar} \) respectively.

**Uncertainty in liquid viscosity**
The uncertainty in liquid viscosity measurement is obtained from the viscometer supplied by Brookfield. The accuracy of the viscometer is given as ±1% of the full range with a repeatability of ±0.2% of the full range.