A model for rising bubbles interacting with crossflowing liquid

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Abstract

The effect of liquid crossflow on the behaviour of submerged bubbles was examined using a theoretical time-varying model based on the equations of motion in two-dimensional space. The results were compared with experimental rise velocities and previously obtained trajectories. Bubble images were recorded using high-speed photography at gas flow rates of 2–25 L/min and crossflow velocities of 0.059–0.334 m/s, respectively. Image processing was then used to measure the bubble rise velocities. Increasing the crossflows strongly limits the bubble rise velocities, which depend on the bubble size, especially at high crossflow velocities. The model's predicted velocities show good agreement with the experimental data, which had uncertainties of around ±10%. The model predicted nearly linear trajectories, which were visually similar to the experimental trajectories. We compared the model's trajectories with swarm centroids and predictions by an empirical correlation developed earlier for the swarm inclination angle. The model's results compared favourably with the experiments, although there were slight overpredictions, especially at high crossflows. This could be corrected in future works by developing more appropriate closure relationships related to crossflows.

Keywords: Bubbly flow, bubble swarms, bubble trajectory, force balance, liquid crossflow

1 Introduction

1.1 Background

Many industrial processes involve gas dispersion by liquid injection from below using submerged orifices. Examples include wastewater treatment, absorption towers, aerated stirred tanks, and bubble columns for chemical and biological processes, such as nitrification and microorganism metabolism. In these applications, the gas–liquid interfacial surface area per unit volume is an important parameter that determines the heat, concentration, and mass-transfer rates. For example, higher surface area-to-volume ratio is associated with higher reaction and mass-transfer rates. Furthermore, smaller bubbles (and hence increased area-to-volume ratio) are created when there is a continuous phase flow across the path of emerging gas bubbles [1]–[5]. Such crossflows can be used to control the bubble ejection frequency and ensure that detached bubbles are swept away from the region near the nozzle, thus reducing the likelihood of coalescence [3, 4]. The crossflow in this study is generated in a boundary layer, different from the uniform crossflow used in Forrester and Reilly [5], Sullivan et al. [6] and Kawase and Ulbrecht [7].

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Forrester and Rielly [5] noted that the drag force created by the flowing liquid and increased boundary layer transport are responsible for the generation of smaller bubbles and their rapid detachment from the orifice. The bubble's rise velocity and trajectory are affected by the momentum of the liquid crossflow. In addition to the buoyancy, virtual mass, surface tension, and Stokes drag forces that the bubbles experience in liquid, there is an additional drag force in the plane of the rising bubbles, which greatly impacts the bubbles' rising profile as there is now a horizontal component.

It is important to predict the extent to which the rise velocities and trajectories are affected by different gas and crossflow conditions, particularly at high gas flow rates (>5 L/min), for which experimental data are scarce. It is difficult to obtain reliable measurements of rise velocities and sizes under high gas fluxes due to bubble overlapping, breakup, and coalescence. Sullivan et al. [6] developed a semi-empirical model based on expansion, vertical displacement, and horizontal displacement of a spherical bubble whose volume was correlated to experimental Reynolds and Froude numbers. Kawase and Ulbrecht [7] developed a model of a crossflowing liquid's effect on a single detaching bubble's diameter based on the force balance. Their approach involved simulating the influence of the continuous phase by virtually inclining the nozzle. Two versions of the model were proposed for low and high dispersed flow rates, and there was less than 10% mean difference from experimental data. However, these studies only investigated the effect of the liquid crossflow on the bubble formation and size distribution in the immediate vicinity of the nozzle. No information was obtained for rise velocities or trajectories.

Tsuge et al. [8] also proposed a two-stage model of bubble formation in flowing liquid as an extension to earlier quiescent liquid models. They incorporated pressure fluctuations in the gas chamber, and the calculated bubble volume results show reasonable agreement with experimental results. Ghosh and Ulbrecht [1] solved force balance equations for bubble volume under the influence of crossflowing viscoelastic and pseudo-plastic polymer solutions. Their model was derived in a non-dimensional form using the Weber number and was solved numerically. Marshall et al. [9] used potential flow theory to derive a theoretical model for bubble formation at an orifice exposed to liquid crossflow in a plane wall of a tube. The model provided good predictions for single bubble formation, and they found that bubble growth is primarily controlled by inertial forces, among other forces such as those induced by slip velocity at the interface. A "bubble pressure minimum" criterion was established, with which reasonable prediction of bubble size could be obtained. While all these studies relate to bubble interaction with the crossflowing liquid, they suffer from the deficiency of not obtaining bubble rise velocities or their flow paths but give only volume, size, or shape upon detachment, with each crossflow condition.

Tan et al. [4] used an interface element approach coupled with a mass balance to describe the dynamics of a detaching bubble and predict the bubble shape under liquid crossflow. Low gas velocities were simulated with liquid crossflow velocities of up to 1.2 m/s, and they reported good agreement with literature data. Liu et al. [10] used a one-stage model and solved the force balance equations for the initial bubble size and bubble formation time for different gas and crossflow velocities. The effect of orifice size on the bubble size upon detachment was significant, which was presented as several isosurfaces. There was a maximum error margin of ±21% compared to experimental data from Marshall et al. [9]. These studies have modelled the bubble size, shape, and detachment time, but they do not provide information about the properties after detachment, such as the rise velocities and trajectories.

It has been shown that submerged jet trajectories follow quasilinear trajectories [11], [12] such that an angle θ is sufficient to describe any trajectory. Zhang and Zhu [13] modelled this angle according to the gas and liquid crossflow properties and experimentally studied two-phase submerged jets. Initially, the two-phase jet follows a parabolic trajectory, after which separation occurs. The gas plume then rises along a linear path. They derived correlations for the jet separation height, gas spreading rate, and void fraction. Other approaches to model jet centreline trajectories have mostly been correlational. These include two-stage semi-empirical models called the momentum-dominated near and far field (MDNF and MDFF) models [11], [14]. Following this approach, Zhang [12] used extensive field data to obtain a correlation that modifies the constants of the MDNF and MDFF models, which fit well with the experimental data. We previously reported experimental work on submerged jets with outputs of up to 25 L/min and liquid crossflow velocities of 0.06–0.334 m/s [15]. Since the bubbles follow a linear pathway, we correlated the bubble swarm trajectory using the displacement angle θ as a function of the gas and crossflow velocities expressed in a dimensionless form as Reynolds and capillary numbers. However, as with all correlations, such relationships can only apply within the range of their experimental data, and modelling approaches have to be used, such as momentum balances of the various forces exerted on the rising bubbles.

Based on this background, we present a simple model that describes the rise velocity and trajectory of a single rising bubble under the influence of liquid crossflow. The force balance of a bubble was examined, including the gas momentum, drag, surface tension, viscous, and virtual mass forces. Since a bubble size distribution occurs for each combination of flow conditions, we simulated the rise velocity and trajectory of the largest and smallest bubbles for each condition. The model was solved numerically and predicted differences in rise velocity for the two extremes of bubble size, especially at large gas flows and low crossflow velocities. We validated the rise velocity results with experimental data, and the predicted trajectories showed good agreement. The crossflow ensures that tiny bubbles are carried with the liquid much more easily than the larger ones, resulting in interesting interactions that are not seen in quiescent flows.

1.2 Model description

For a representative bubble within a rising swarm, the 2-D momentum balance equation for its motion can be written as follows:

$$\rho_l V_b\left(\frac{d^2x_b}{dt^2} + \frac{d^2y_b}{dt^2}\right) = F_B - F_g - F_D - F_\sigma - F_\mu - F_{VM} \tag{1}$$

where x_b and y_b are the instantaneous horizontal and vertical displacements of a representative bubble from the nozzle, and F_B, F_g, F_D, F_σ, F_μ, $and\ F_{VM}$ represent the buoyancy, crossflow drag, surface tension, viscous (or Stokes) drag, and virtual (or added) mass forces, respectively; ρ_l is the liquid density and V_b is the bubble volume. Viscous drag is the hindering force of viscosity on an object moving through a viscous fluid, while the virtual mass force is the force associated with an accelerating bubble which must displace some volume of surrounding water equivalent to the volume of the gas bubble since the volumes cannot both occupy the same physical space at the same time. Some of the forces are only valid or dominant in one direction. For example, in the x-direction, viscous drag and surface tension forces are present, but the drag force on the bubble introduced by the crossflowing liquid is dominant by far. If we assume that the crossflowing liquid is time independent, completely horizontal, and has no component in the vertical direction, then $F_{D,y} = 0$. Thus, the momentum balance in Eq. (1) can be rewritten as two separate equations for the horizontal and vertical coordinates:

$$\rho_l V_b \frac{d^2x_b}{dt^2} = F_D - F_{\sigma,y} - F_{\mu,x} - F_{VM,x} \tag{2}$$

$$\rho_l V_b \frac{d^2y_b}{dt^2} = F_B - F_g - F_{\sigma,y} - F_{\mu,y} - F_{VM,y} \tag{3}$$

The drag force on the bubble from the liquid crossflow is:

$$F_D = \frac{1}{2} C_D A_b \rho_l \left(\frac{dx_b}{dt} - u_l(y)\right)\left|\frac{dx_b}{dt} - u_l(y)\right| \tag{4}$$

where $u_l(y)$ is the incoming mean crossflow velocity profile, which is obtained experimentally by time-averaging sufficient instantaneous PIV realisations, and A_b is the bubble cross-sectional area.

If we assume the initial bubble shape upon detaching from the nozzle is spherical, we can assign it the mean-volume equivalent diameter, obtained experimentally. Our observations have shown that the bubbles morph into irregularly shaped ellipsoids with the major axis along the liquid crossflow [16], [17]. The bubble expands slightly while rising. It was deduced that, within the experimental field of view, the diameter along the major and minor axes respectively increase fairly linearly with the x- and y-displacements from the nozzle. Since pressure increases with water depth ($P = h\rho g$), the pressure at the top of the channel is lower than at the bottom, hence causing bubble expansion upon rising.

Furthermore, elongation along the streamwise direction is caused by the crossflowing liquid hence causing the bubbles to be oblate-shaped spheroids (with the leading axis along the direction of the flowing liquid) rather than spheres. These are shown by the plots in Figure 2, where the entire experimental data for the bubble size variation along the respective flow directions were plotted against the vertical or horizontal distance from the nozzle. Data was obtained at five rise or streamwise locations. As can be seen, in both directions straight lines passing through the entire datasets quite sufficiently describe the variation in bubble size for all flow cases. The larger slope of 0.84 indicates faster size increase in the streamwise direction than in the rise direction which has a slope of 0.63, agreeing with observations of bubble shape to be shown later. The equations for these lines are as follows:

$$d_{b,x} = d_i + 0.84x_b \; d_{b,y} = d_i + 0.63y_b \} for\ 0 \leq x_b,\ y_b \leq 0.39\ m \qquad (5)$$

where d_i is volume average bubble dimeter (at the prevailing flow condition) of all the bubbles within the vicinity of the nozzle, where i denotes "initial". This was obtained by image-processing all the bubbles within each 1/5th division of all acquired images. The volume average diameter is now calculated as a volumetric mean of all the bubbles j, i.e. $d_i = \left[\sum\left(n_i d_{b,j}^3\right)/\sum n_j\right]^{\frac{1}{3}}$. The detailed procedure can be found in our previous paper [15].

The drag coefficient C_D in Eq. (4) is calculated using the empirical relationship reported by Kelbaliyev and Ceylan [18] for irregularly shaped bubbles at low Reynolds and Morton numbers. We also use a correlation by Rodrigue [19] that is valid up to $Re = 2000$ ($Re_b = \rho_g u_g d_{b,H}/\mu_g$, where $d_{b,H}$ is the hydraulic diameter of the bubble), which is the boundary between laminar and turbulent flow. There are actually many such correlations in this region for bubbles in liquid, which depend on the shape and the range of liquid properties [20]–[22]. For $Re_b > 2000$ to around 2×10^5, C_D is fairly constant (0.44 – 0.47). We used [18]:

$$C_D = \{ \frac{8}{Re_b}\left[1 + \frac{1}{1-0.5\left(1+250Re^5\right)^{-2}}\right],\ Re_b < 0.5 \ \frac{16}{Re_b}\left[(1 + 32\beta + \frac{1}{2}\sqrt{1 + 128\theta})\right. \qquad (6)$$

$$\text{Where } \beta = 0.018^3 (2/3)^{1/3} Re_b^{8/3} Mo^{1/3}$$

where $Mo = \frac{g\mu_l^4(\rho_l - \rho_g)}{\sigma^4 \rho_l^2}$ is the Morton number, which together with the Reynolds and Eotvos numbers, characterises the shape of bubbles. The Reynolds number is defined based on the bubble hydraulic diameter for an elliptical bubble, the hydraulic diameter is defined as:

$$d_{b,H} = \frac{4d_{b,x}d_{b,y}\left(64-16E^2\right)}{\left(d_{b,x}+d_{b,y}\right)\left(64-3E^4\right)} \tag{7}$$

$$\text{where } E = \frac{d_{b,x}-d_{b,y}}{d_{b,x}+d_{b,y}}$$

Where $d_{b,x}$ and $d_{b,y}$ are minor and major diameters of the bubble respectively; for simplicity in expressing the bubble Reynolds number, we use the bubble hydraulic diameter. Since the incoming gas momentum far exceeds the inherent buoyancy in the liquid, the bubble ejection from the nozzle is momentum dominated and the effect of gravity is on the rising bubbles is negligible, a generalised buoyancy force can be used [23], which is essentially the incoming gas momentum, i.e.:

$$F_B = A_p \rho_g u_g^2 \tag{8}$$

where A_p is the cross-sectional area of the gas inlet nozzle, V_b is the bubble volume, and ρ_g is the gas density. The surface tension and Stokes drag forces are defined as follows:

$$\begin{aligned} -\left(F_{\sigma,x} + F_{\mu,x}\right) &= -\left(\pi\sigma d_{b,x} sin\theta\right) - \left(6\pi\mu_l d_{b,x}\frac{dx_b}{dt}\right) \\ -\left(F_{\sigma,y} + F_{\mu,y}\right) &= -\left(\pi\sigma d_{b,y} cos\theta\right) - \left(6\pi\mu_l d_{b,y}\frac{dy_b}{dt}\right) \end{aligned} \tag{9}$$

Since the angle θ is not known a priori, it is estimated at each time step using the coordinates of the bubble location; i.e. $sin\theta = x_b/\sqrt{x_b^2 + y_b^2}$ and $cos\theta = y_b/\sqrt{x_b^2 + y_b^2}$ (Figure 1). Finally, the virtual mass force is determined. We calculate the virtual mass force as follows:

$$F_{VM,y} = C_{VM}\rho_l V_b \frac{d^2y_b}{dt} \tag{10}$$

where C_{VM} is the virtual mass coefficient, which has a value of 0.5 [24]. $F_{VM,x}$ is calculated analogously. Substituting Eqs. (4)–(10) into Eqs. (2) and (3), we obtain equations that describe the motion of the bubble through the crossflowing liquid:

$$\rho_l V_b \frac{d^2x_b}{dt^2} = \frac{1}{2}C_D A_b \rho_l\left(\frac{dx_b}{dt} - u_l(y)\right)\left|\frac{dx_b}{dt} - u_l(y)\right| - \left(\frac{\pi\sigma}{\rho_l V_b} d_{b,x} sin\theta\right) - \left(\frac{6\pi\mu_l}{\rho_l V_b} d_{b,x} u_y\right) \tag{11}$$

$$\rho_l V_b \frac{d^2y_b}{dt^2} = A_p \rho_g u_g^2 - \left(\pi\sigma d_{b,y} cos\theta\right) - \left(6\pi\mu_l d_{b,y}\frac{dy_b}{dt}\right) - \frac{1}{2}\rho_l V_b \frac{d^2y_b}{dt^2} \tag{12}$$

To solve these 2nd-order ODEs for the bubble velocity and trajectory, we substitute the velocity in either direction, which is the rate of displacement change. Since the liquid velocity is time independent and only varies along the x-coordinate, we can take local values on the velocity profile along the y-coordinate. Hence, the following substitutions can be made:

$$\frac{dx_b}{dt} = u_x - u_l(y) \Rightarrow \frac{d^2x_b}{dt^2} = \frac{du_x}{dt} \tag{13a}$$

$$\frac{dy_b}{dt} = u_y \Rightarrow \frac{d^2y_b}{dt^2} = \frac{du_y}{dt} \tag{13b}$$

Eqs. (11) and (12) now become:

$$\frac{du_x}{dt} = \frac{2}{3}\left[\frac{1}{2}C_D\frac{A_b}{V_b}\left(u_x - u_l(y)\right)\left|u_x - u_l(y)\right| - \left(\frac{\pi\sigma}{\rho_l V_b}d_{b,x}sin\theta\right) - \left(\frac{6\pi\mu_l}{\rho_l V_b}d_{b,x}u_y\right)\right] \quad (14)$$

$$\frac{du_y}{dt} = \frac{2}{3}\left[\frac{A_p}{V_b}\frac{\rho_g}{\rho_l}u_g^2 - \left(\frac{6\pi\mu_l}{\rho_l V_b}d_{b,y}u_y\right) - \left(\frac{\pi\sigma}{\rho_l V_b}d_{b,y}cos\theta\right)\right] \quad (15)$$

Therefore, Eqs. (13a)–(15) are solved as an initial value problem for the dynamic bubble location and velocities for a specified time interval using the initial conditions:

$$x_b|_{t=0} = x_{b0} = 0.051\ m \text{ (x-coordinate of nozzle location)} \quad (16a)$$

$$y_b|_{t=0} = y_{b0} = 0.000\ m \text{ (y-coordinate of nozzle location)} \quad (16b)$$

$$u_x|_{t=0} = u_{x0} \text{ (taken from specific experimental condition)} \quad (16c)$$

$$u_y|_{t=0} = u_{y0} \text{ (taken from specific experimental condition)} \quad (16d)$$

A solution for the system of equations with these initial conditions is obtained using the non-stiff explicit 4th and 5th-order Runge-Kutta-Felhberg method coded using MATLAB's ode45 function, which was used as the integrator. Using a 1.6 GHz, Core i5 PC with 8 GB RAM, the solution takes around 5 minutes to obtain. However, significant increase in solution times can result if the model is solved for all or a large number of bubbles within the size distribution.

2 Experimental description

To validate the model, experiments were conducted in a low-speed recirculating water channel with a working section of 3000 mm × 600 mm × 700 mm (length × width × height) at the Institute of Fluid Mechanics of Beihang University. Five free-stream velocities in the range of 58.5–334 mm/s, were used, and Figure 3 (a) shows a diagram of the facility for bubble experiments. Prior to the bubble visualisation experiments, the liquid flow velocity was determined using a 2D PIV system with a continuous laser, and the boundary layer was characterised for each flow velocity. The PIV system was previously used by Xu and Wang [25] and Xu et al. [26] and consists of a continuous laser (5 W), a high-speed CMOS camera, a macro lens (Nikon 105 mm f/2.8D), and a personal computer.

A circular brass pipe (which we shall call the "nozzle" throughout this article) was placed vertically in the middle of a flat plate to inject air into the flowing water. The nozzle exit has an outer diameter (D_o) of 8.0 mm and inner diameter (d_{noz}) of 5.0 mm and was 28 mm above the plate. Its centreline was 760 mm away from the leading edge of the plate, as shown in Figure 3 (b). Air was supplied by a commercial air compressor with a capacity to deliver constant flow rates at up to 7.0 bar. A manual valve and a rotameter were used to control the air discharge and measure the flow rate. The air flow rate (Q_g) was set as 2, 5, 10, 15, 20, and 25 L/min, which correspond to air injection velocities (u_g) of 1.7, 4.25, 8.5, 12.75, 17.0, and 21.25 m/s, respectively.

The gas Reynolds number, based on the air injection velocity (u_g) and the nozzle inner diameter (i.e. $Re_g = \rho_l u_g d_{noz}/\mu_g$) was in the range of 574–7179. The bubbles' rise velocity, void fraction, angle of inclination, and size distribution were obtained using an image processing algorithm in Matlab. Xu et al. provide more details about the experimental facility, high-speed camera system, laser sheet, flowmeters, their models and uncertainties, and the image processing method (See Xu et al.). The experimental results of measured bubble rise velocities, rise velocity profile validation, bubble size distribution, and uncertainties have also been extensively reported in Xu et al. [15], the reader should refer to the article for more information. We use the data in that study to validate the model in this study.

3 Results and discussion

3.1 Swarm trajectories

Figure 4 shows a plot of the solution of the x- and y-coordinates of the bubble's displacement, which is superimposed on images from the experiments. The model predictions are represented by continuous blue lines, while the red continuous lines are the mean centroid lines of 2,000 or more processed images obtained from experiments. The blue dashed lines represent the predictions of the empirical relationship developed by Xu et al. [15], which directly correlate swarm inclination angles with dimensionless parameters that characterise the crossflow and gas conditions as follows:

$$\theta = 0.03 \frac{Re_{\infty}^{0.63} We_{\infty}^{0.15}}{Re_{g}^{0.14}} \tag{17}$$

where $Re_{\infty} = \frac{\rho_g u_g d_{ch,H}}{\mu_l}$, $We_{\infty} = \frac{\rho_l U_{\infty}^2 d_{ch,H}}{\sigma}$, $and\ Re_g = \frac{\rho_l u_g d_{noz}}{\mu_g}$ are the freestream Reynolds number, freestream Weber number and gas Reynolds number respectively; $d_{ch,H}$ is the channel height. It is clear from Figure 4 that the model shows general agreement with the experimentally obtained trajectories and those predicted by the correlation. However, these are limited to the current experimental conditions and fluid combination. The paths are all quasilinear, but the predicted trajectories overshoot to different degrees in all cases.

For the low crossflow velocities, the simulated paths produce only slight overshoot compared with the experimental images. However, at high liquid crossflow and especially low gas velocities, the deviation is more notable. A main reason for the discrepancies is that the solution is very sensitive to the drag coefficient, the drag coefficient used here was derived at quiescent flow conditions. To our knowledge, no correlations for drag coefficient in crossflows are available in the literature and will have to be derived in future works. As a result, the utilised drag coefficient is satisfactory at low crossflow conditions and begins to show appreciable deviations as crossflow velocity increases. These deviations are especially pronounced at higher crossflow velocities, which promote bubble breakup and result in more bubbles. Consequently, a more detailed model that incorporates bubble interaction is required, but this requires simulating the paths and velocities of individual bubbles. Such an approach would be data-driven with size distributions obtained from experiments, which would be input at each time step of the solution, but this would profoundly increase computational requirements. Nevertheless, Figure 4 shows that the simple model in this study provides quite satisfactory predictions of the trajectories and is more generally applicable than pure correlational analysis since it is derived from the fundamental equations of motion.

3.2 Rise velocities

Figure 5 shows the model results for the development of the bubble rise velocities along the height of the channel in comparison with the experimental data. Equal axes are used for all the cases to show the relative magnitudes of the rise velocity in each case. Five experimental data points were obtained for each case: at the bottom of the FOV immediately after ejection from the nozzle; then at 95 mm, 174, 252 mm from the nozzle; and towards the top of the FOV 330 mm from the nozzle. The error bars on the experimental data points indicate the estimated uncertainties obtained as the mean of the standard deviations between three measurements, which are generally around 10%. However, in conditions of high gas flow rates and high crossflow velocities, the errors are as high as 20% because of bubble overlapping caused by the large number of bubbles produced. Nevertheless, within the 10% error limit mentioned, there is good agreement between the experimental data and numerical results of the model.

The model results for the unsteady bubble rise velocities through the crossflowing liquid for $Q_g =$ 10–25 L/min are shown in Figure 6. We did not include the others because of space since the figure is a close up of Figure 7. It is immediately clear that higher liquid crossflow velocity delays the attainment of terminal velocity. Furthermore, there is continuous increase in rise velocities at all crossflows from 10 to 25 L/min. For the high gas flow rates shown, a progressive increase in rise velocities occurs over time at each crossflow prior to achieving terminal velocity. Other investigators have reported similar behaviour [24]. This rise is much less pronounced at 10 L/min than at 25 L/min. At 10 L/min, there is almost a balance between the liquid crossflow and the forces normal to it, thus promoting bubble buoyancy.

Figure 7 shows the model results of rise velocity and the trends are generally different at lower inlet gas velocities (10 L/min and below) than above 10 L/min. There is a progressive decrease in the rise velocity over time before some form of steady state (terminal velocity) is reached, which is delayed at low crossflow velocity. At $Q_g =$ 2 L/min and $U_\infty =$ 0.059 m/s, terminal velocity is not attained within the height of the channel, but is gradually reached at more than 2 s to less than 1 s as U_∞ moves from 0.115 to 0.334 m/s. This trend is more or less the same at $Q_g =$ 5 L/min, but the rates of terminal velocity attainment are different. This behaviour at low gas flow rates could be explained by the gas phase momentum being lower than that of the crossflow, which results in decreasing velocity. On the other hand, for all cases, as the gas flow rate increases to about $Q_g =$ 10 L/min, the dynamic behaviour of the trends changes from decreasing to increasing before reaching terminal velocity. A similar trend was obtained by Sathe et al. [24] in their simulation for bubbly jets at high gas flows in quiescent liquid. Back to Figure 6, at 10 L/min , the initial increase is mostly followed by a drop, which we presume to be caused by viscous drag overcoming the initial gas momentum, which results in deceleration. Beyond $Q_g =$ 10 L/min, the gas momentum is dominant before terminal velocity is achieved.

At $Q_g =$ 25 L/min, the inverse effect of increased crossflow velocity on the bubbles' terminal velocity is only slight. Thus, we can conclude that the time to reach terminal velocity is not affected by the liquid crossflow at high gas flows. Essentially, there is a complex interaction between the bubble momentum force, crossflow drag, and viscous drag, as outlined in Eq. **(1)**. The dynamic response of this interaction is highly dependent on the magnitude of each force.

Figure 8 shows the predicted rise velocities as two isosurfaces, which give an overall view of the effects of the main parameters: the crossflow velocity, inlet gas flow rate, and bubble size. The rise velocities are proportional to the superficial gas velocity at the inlet at all crossflow conditions. This is consistent with the experimental studies of Xu et al. [15], Wang and Socolofsky [27] and Zhang and Zhu [17]. However, increasing the liquid crossflow velocity severely limits the bubble rise velocities. At low superficial gas velocities, increasing the crossflow velocity results in rapid damping of the rise velocities before levelling off at high crossflows. For the intermediate gas flow rates, an increase in the crossflow velocity first results in a gradual decrease before a steeper decrease and then finally levelling off at high crossflows. At the highest gas flows, the crossflow velocity generally has a more gradual effect of damping the rise velocities beyond $U_\infty = 0.15$ m/s. This suggests that the higher momentum of the rising bubbles begins to counterbalance the effect of the crossflowing drag since only a quarter reduction in the rise velocity is lost, as opposed to half at $Q_g = 5$ L/min. This behaviour was observed experimentally in a previous study [15]. Wang and Socolofsky [27] also observed a rapid decrease of the rise velocities at low gas flow rates with increasing liquid crossflows.

Each isosurface in Figure 8 represents velocities for the maximum and minimum bubble sizes within the bubble size distribution for each set of conditions. The larger bubbles consistently have higher rise velocities than the smaller bubbles despite the larger drag resulting from their size. This is consistent with observations where smaller bubbles trail larger ones. The difference in rise velocities ranges from 1 to 10% and is more apparent at high crossflow velocities and low gas flow rates. This combination of conditions results in less size variability, and we report in Table 1, the minimum and maximum bubble diameters for each condition. Xu et al. [15] provides more discussion about the effect of the crossflow velocity on the size distribution.

3.3 Streamwise bubble velocities

Bubble velocities in the liquid streamwise direction follow increasing trends over time as both the gas flow rate and crossflow velocity increase, as shown in Figure 9. Unlike the rise velocities, however, steady state is almost never reached in the case of the streamwise velocity within the same time interval. The freestream velocity does not seem to limit the streamwise bubble velocities, which can be attributed to the added effect of the horizontal component of bubble momentum. It would take longer to reach steady-state velocity, which can be estimated for each case as the vector sum of the initial streamwise bubble velocity and the crossflow velocity. The crossflow velocity is not uniform but has a profile from the bottom of the channel, and its effect is integrated at each vertical height. It gives an idea of how the steady-state streamwise bubble velocity is more difficult to attain than in the case of the rise velocity. The curves show that bubbles are more accelerated with increasing gas and crossflow velocities, and the former has a more dominant effect. Just like the rise trajectories, the bubble predicted streamwise velocities generate noticeable divergence from the experimental results at high crossflows. Since this is in the streamwise direction, it is expected since the drag coefficient relationship as stated earlier was derived for quiescent flows. At U_{∞} = 0.0585 m/s for example, which is the closest to quiescent, agreement between the model and experimental results, within the error limits, is remarkable. We begin to see increasing difference between them as crossflow velocity is increased for all gas flow rates. This buttresses the need for further work in order to improve drag coefficient relationships under the influence of high liquid crossflows.

4 Concluding remarks

In this paper, we described a theoretical model of the effect of liquid crossflow on submerged bubble swarms. The model is based on a force balance of the dominant forces exerted on rising bubbles, which include the inlet momentum, buoyancy, and viscous and crossflow drag. The resulting equations of motion were two-dimensional, unsteady, and numerically solved as an initial value problem to obtain bubble rise velocities and trajectories. For experimental validation, we used previously reported data obtained by recording high-speed images of the flow, and image analysis was used to measure the bubble rise velocities, size distribution, and trajectories for each case. The model's initial velocities and bubble size were obtained from the nozzle's exit conditions determined from the image analysis. We then used a linear approximation of bubble expansion along the channel's height at each time step. The experimental data showed that increasing the liquid crossflow velocity resulted in bubble breakup and strongly limited the bubble rise velocities.

We showed that the model agrees with the experimental rise velocities. For the trajectories, the model's trace of the bubble paths closely followed the experimentally obtained bubble centroids and the predictions of an empirical correlation relating swarm inclination and dimensionless flow parameters. However, the model consistently produced slight overpredictions in each set of conditions. We suggest that the predictions could be improved by experimentally determining more appropriate relationships of the drag coefficient determined for crossflows and embedding them within the model. Furthermore, instead of approximating bubble expansion using a linear fit to the data, equations of state could be used. This seems reasonable for small bubbles that are more evenly distributed at low gas flow rates, but deviations are inevitable at high gas rates and high crossflows, where bubble sizes are more widely distributed and bubble breakup or coalescence occur. The entire bubble size distribution (or representative parts of it) can also be used rather than an average bubble size for each case. However, this will require more computational resources and time, as well as a more intricate model.

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Competing interests

Non declared.

Nomenclature

Roman		
A	[m^2]	Area
Ca	[-]	Capillary number
d	[m]	Diameter
Fr	[-]	Froude number
Mo	[-]	Morton number
Q	[L/min or m^3/s]	Volumetric flow rate
Re	[-]	Reynolds number
t	[s]	Time
u	[m/s]	Velocity specified by a subscript
V	[m^3]	Bubble volume specified by subscript b
We	[-]	Weber number
x	[m]	Horizontal spatial coordinate
y	[m]	Vertical spatial coordinate
Greek		
γ	[-]	Velocity profile shape parameter
μ	[kg/s-m]	Dynamic viscosity
ρ	[kg/m^3]	Density
σ	[N/m]	Liquid surface tension

θ	[°]	Swarm angle of inclination or trajectory (for linear and quasi linear swarms)
τ	[s]	Cross-correlation time lag
Subscripts		
b	Bubble	
ch	Channel	
g	Gas phase	
H	Hydraulic, used to specify that bubble diameter is a hydraulic diameter	
l	Liquid phase	
noz	Nozzle	
s	Slip, used for slip velocity	
sg	Superficial gas	
∞	Freestream	

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