The role of endogenous capital depreciation rate for business cycle dynamics: lessons from Bulgaria (1999-2018)

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Abstract

We allow for an endogenous depreciation rate of physical capital stock into a real-business-cycle model with a government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2018). We investigate the quantitative importance of the endogenous depreciation rate, and indirectly, the capital utilization mechanism for cyclical fluctuations in Bulgaria. Allowing for endogenous variations in the depreciation rate of capital improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework with constant depreciation and a fixed utilization rate of physical capital, and a fixed depreciation tate *e.g.*, Vasilev (2009).

Keywords: Business cycle fluctuations, capital utilization rate, endogenous depreciation rate, Bulgaria

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1 Introduction and Motivation

As pointed in Vasilev (2018), the average labor productivity in Bulgaria in the period following the introduction of currency board (1997) is highly pro-cyclical. This fact was then rationalized by the fact that the capacity utilization rate was also moving together with output. In other words, at least one of the major inputs of production, labor and capital, is used more intensively during periods of expansions as compared to periods of recessions. However, the standard real-business-cycle (RBC) model cannot account for this stylized fact, as it assumes that the factors of production are fully utilized all the time. In addition, the depreciation rate of capital, which describes the transformation of capital over time, is held fixed. In other words, the standard model assumes that utilizing resources at the highest level is costless, as that increase in wear and tear is not reflected into a higher depreciation.

We speculate that holding the rate of depreciation fixed over the cycle might lead researchers to wrong conclusions. We thus allow the depreciation rate of physical capital to vary. More specifically, we will endogeneize it by linking it to the level of utilization, which will be a decision variable. This modelling approach allows then the depreciation rate to increase during times when capital is utilized at a higher rate, and fall when capital utilization is lower. In other words, the depreciation rate will be also pro-cyclical and respond to the stage of the business cycle. This property of the depreciation rate can potentially provide researchers with a a deeper understanding of the new transmission mechanism for economic fluctuations, working through capital depreciation rate.

We follow several earlier studies, *e.g.* Kydland and Prescott (1988), Greenwood, Hercowitz, and Huffman (1988), and Vasilev (2018), who have incorporated varying depreciation of capital via the endogenously-determined degree of capital utilization in real-business-cycle models.[1] In other words, there is a costs to the capital utilization decision that is at play in the current model, which is a cost in terms of a higher depreciation rate of physical capital stock. We then introduce this extension into a standard real business-cycle model with

[1] Greenwood *et al* (1988) use an investment-specific technological progress instead of the neutral one used in this paper. The other novelty, relative to Vasilev (2018), is the absence of an energy channel in this paper. This allows us to evaluate the isolated effect of capital utilization, and the varying depreciation rate.

a government sector, and investigate whether allowing for cyclically-adjusting depreciation rate helps our augmented real-business-cycle model match the empirical business cycles in Bulgaria in the period after the introduction of the currency board arrangement.[2]

In order to be able to draw plausible quantitative predictions, we calibrate the theoretical economy to approximate Bulgaria in the period 1999-2018. Bulgaria was chosen as a testing ground for the theory, as it is the poorest EU member state, and as a former transition economy, is still developing. Overall, the model with an endogenous depreciation rate performs better than earlier real-business-cycle models vis-a vis data for Bulgaria. Nevertheless, as with the standard RBC model, the model with endogenous depreciation rate of capital falls to generating wage variability of the same magnitude as in data, and the wage rate in the model is very strongly pro-cyclical, while wages are a-cyclical in data.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

2 Model Description

There is a representative households which derives utility out of consumption and leisure. The time available to households can be spent in productive use or as leisure. In addition, the household chooses optimally the rate at which capital stock is being utilized. The government taxes consumption spending and levies a common tax on all income, in order to finance non-productive purchases of government consumption goods, and government transfers. On the production side, there is a representative firm, which hires labor and utilized capital to produce a homogenous final good, which could be used for consumption,

[2] The period of our investigation was chosen due to the fact that the introduction of the hard exchange-rate peg achieved macroeconomic stability in Bulgaria, and thus the time series have good statistical properties.

investment, government purchases, or energy consumption. Depreciation rate is endogenous, and is a function of the endogenous capital utilization rate.

2.1 Household's problem

There is a representative household, which maximizes its expected utility function, as in Finn (2000):

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \Bigg\{ \ln c_t + \gamma \ln(1-h_t) \Bigg\}, \tag{2.1}$$

where E_0 denotes household's expectations as of period 0, c_t denotes household's private (non-energy) consumption in period t, h_t are hours worked in period t, $0 < \beta < 1$ is the discount factor, $0\gamma > 0$ is the relative weight that the household attaches to leisure.

The household starts with an initial stock of physical capital $k_0 > 0$, and has to decide how much to add to it in the form of new investment, as well as the rate at which the stock of physical capital is being utilized. As a result, every period physical capital depreciates at an endogenous rate, which depends on the level of utilitization rate u_t chosen by the household, so $0 < \delta(u_t) < 1$. Following Vasilev (2018), the functional form for the endogenous depreciation rate is as follows:

$$\delta_t = \delta(u_t) = \omega_0 \frac{u_t^{\omega_1}}{\omega_1} \in (0,1), \tag{2.2}$$

where $\omega_0 > 0$, $\omega_1 > 1$.[3]

The law of motion for physical capital is then

$$k_{t+1} = i_t + (1-\delta_t)k_t, \tag{2.3}$$

and the real interest rate is r_t, hence the before-tax effective (utilized) physical capital income of the household in period t equals $r_t u_t k_t$. In addition to capital income, the household can generate labor income. Hours supplied to the representative firm are rewarded at the

[3]Note that in the case when $\omega_1 = 1$, $\delta_t = \omega_0 = \delta$, the model collapses to the standard setup with a constant depreciation rate.

hourly wage rate of w_t, so pre-tax labor income equals $w_t h_t$. Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit, π_t.

Next, the household's problem can be now simplified to

$$\max_{\{c_t,u_t,h_t,k_{t+1}\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \ln c_t + \gamma \ln(1-h_t) \right\}, \tag{2.4}$$

s.t.

$$(1+\tau^c)c_t + k_{t+1} - (1-\delta_t)k_t = (1-\tau^y)[w_t h_t + r_t u_t k_t] + g_t^t + \pi_t, \tag{2.5}$$

where τ^c is the tax on consumption, τ^y is the proportional income tax rate $(0 < \tau^c, \tau^y < 1)$, levied on both labor and capital income, p_t is the relative (to the aggregate consumption price index) energy price, e_t denotes energy use in period t, and g_t^t denotes government transfers. The household takes the two tax rates $\{\tau^c, \tau^y\}$, government spending categories, $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, profit $\{\pi_t\}_{t=0}^{\infty}$, the realized technology process $\{A_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, and chooses $\{c_t, h_t, u_t, k_{t+1}\}_{t=0}^{\infty}$ to maximize its utility subject to the budget constraint.[4] The constraint optimization problem generates the following optimality conditions:

$$c_t \quad : \quad \frac{1}{c_t} = \lambda_t(1+\tau^c) \tag{2.6}$$

$$h_t \quad : \quad \frac{\gamma}{1-h_t} = \lambda_t(1-\tau^y)w_t \tag{2.7}$$

$$u_t \quad : \quad \delta'(u_t) = \omega_0 u_t^{\omega_1 - 1} = (1-\tau^y)r_t \tag{2.8}$$

$$k_{t+1} \quad : \quad \lambda_t = \beta E_t \lambda_{t+1}[1 + (1-\tau^y)r_{t+1}u_{t+1} - \delta(u_{t+1})] \tag{2.9}$$

$$TVC \quad : \quad \lim_{t\to\infty} \beta^t \lambda_t u_{t+1} k_{t+1} = 0, \tag{2.10}$$

where λ_t is the Lagrangean multiplier attached to household's budget constraint in period t.

The interpretation of the first-order conditions above is as follows: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the consumption tax rate. The second equation states that when choosing labor supply optimally, at the margin, each hour spent by the household working for the firm

[4]Note that by choosing k_{t+1} the household is implicitly setting investment i_t optimally. Similarly, by choosing the utilization rate, the household is determining the time-varying depreciation rate.

should balance the benefit from doing so in terms of additional income generates, and the cost measured in terms of lower utility of leisure. The third equation describes the optimal utilization rate, which requires that the change in the depreciation rate, or the marginal cost in terms of an increased depreciation rate resulting from utilizing capital at a higher rate, should equal the after tax return on utilized capital. In other words, the marginal benefit resulting from physical capital services should balance with the user cost of capital. The fourth equation is the so-called "Euler condition," which describes how the household chooses to allocate physical capital over time. The last condition is called the "transversality condition" (TVC): it states that at the end of the horizon, the value of utilized physical capital should be zero.

2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and uses both utilized (effective) physical capital, $u_t k_t$, and labor hours, h_t, to maximize static profit

$$\Pi_t = A_t (u_t k_t)^{\alpha} h_t^{1-\alpha} - r_t u_t k_t - w_t h_t, \tag{2.11}$$

where A_t denotes the level of technology in period t. Since the firm rents the capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, *i.e.*:

$$u_t k_t \quad : \quad \alpha \frac{y_t}{u_t k_t} = r_t, \tag{2.12}$$

$$h_t \quad : \quad (1-\alpha) \frac{y_t}{h_t} = w_t. \tag{2.13}$$

2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on wasteful government purchases, and government transfers. The government budget constraint is as follows:

$$g_t^c + g_t^t = \tau^c c_t + \tau^y [w_t h_t + r_t u_t k_t] \tag{2.14}$$

Tax rates and government consumption-to-output ratio would be chosen to match the average share in data, and government transfers would be determined residually in each period so that the government budget is always balanced.

2.4 Dynamic Competitive Equilibrium (DCE)

For a given process followed by technology $\{A\}_{t=0}^{\infty}$ average tax rates $\{\tau^c, \tau^y\}$, initial capital stock k_0, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t, k_t, u_t, h_t\}_{t=0}^{\infty}$ for the household, a sequence of government purchases and transfers $\{g_t^c, g_t^t\}_{t=0}^{\infty}$, and input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) all markets clear.

3 Data and Model Calibration

To characterize business cycle fluctuations with an endogenous depreciation rate in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2018). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2019), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2019). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2016), the discount factor, $\beta = 0.982$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k/y = 13.964$, in the steady-state Euler equation. The labor share parameter, $1 - \alpha = 0.571$, is obtained as in Vasilev (2017d), and equals the average value of labor income in aggregate output over the period 1999-2016. This value is slightly higher as compared to other studies on developed economies, due to the overaccumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Next, the average income tax rate was set to $\tau^y = 0.1$. This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008. Similarly, the tax rate on consumption is set to its value over the period, $\tau^c = 0.2$.

Next, the relative weight attached to the utility out of leisure in the household's utility function, γ, is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. Next, the steady-state depreciation rate of physical capital in Bulgaria, $\delta = 0.013$, was taken from Vasilev (2016). It was estimated as the average quarterly depreciation rate over the period 1999-2014. In addition, the steady-state capital utilization rate is taken from data, where $\bar{u} = 0.78$. This value, together with the convenient normalization $\omega_0 = \delta$ yields that $\omega_1 = 1.533$.[5]

Finally, the process followed by TFP is estimated from the detrended series by running an AR(1) regression and saving the residuals. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Capital Share	Data average
δ	0.013	Depreciation rate on physical capital	Data average
$\bar{u}$	0.710	Average utilization rate	Data average
ω_0	0.013	Scale parameter, depreciation function	Calibrated
ω_1	1.533	Curvature parameter, depreciation function	Set
τ^y	0.100	Average tax rate on income	Data average
τ^c	0.200	VAT/consumption tax rate	Data average
ρ_a	0.701	AR(1) persistence coefficient, TFP process	Estimated
σ_a	0.044	st. error, TFP process	Estimated

[5]Note that the curvature parameter, ω_1, does not enter the steady state, and only matters for cyclical fluctuations.

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 below. The steady-state level of output was normalized to unity (hence the level of technology A differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model matches consumption-to-output and government purchases ratios by construction; The investment ratios are also closely approximated, despite the closed-economy assumption and the absence of foreign trade sector. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where $\bar{r} = (1-\tau^y)r - \delta$ is also relatively well-captured by the model. Lastly, given the absence of debt, and the fact that transfers were chosen residually to balance the government budget constraint, the result along this dimension is understandably not so close to the average ratio in data.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	1.000
c/y	Consumption-to-output ratio	0.624	0.624
i/y	Investment-to-output ratio	0.201	0.175
g^c/y	Energy consumption-to-output ratio	0.151	0.151
wh/y	Labor income-to-output ratio	0.571	0.571
ruk/y	Capital income-to-output ratio	0.429	0.429
h	Share of time spent working	0.333	0.333
$\bar{r}$	After-tax net return on capital	0.014	0.016

5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response functions (IRFs) are presented in Fig. 1 below.

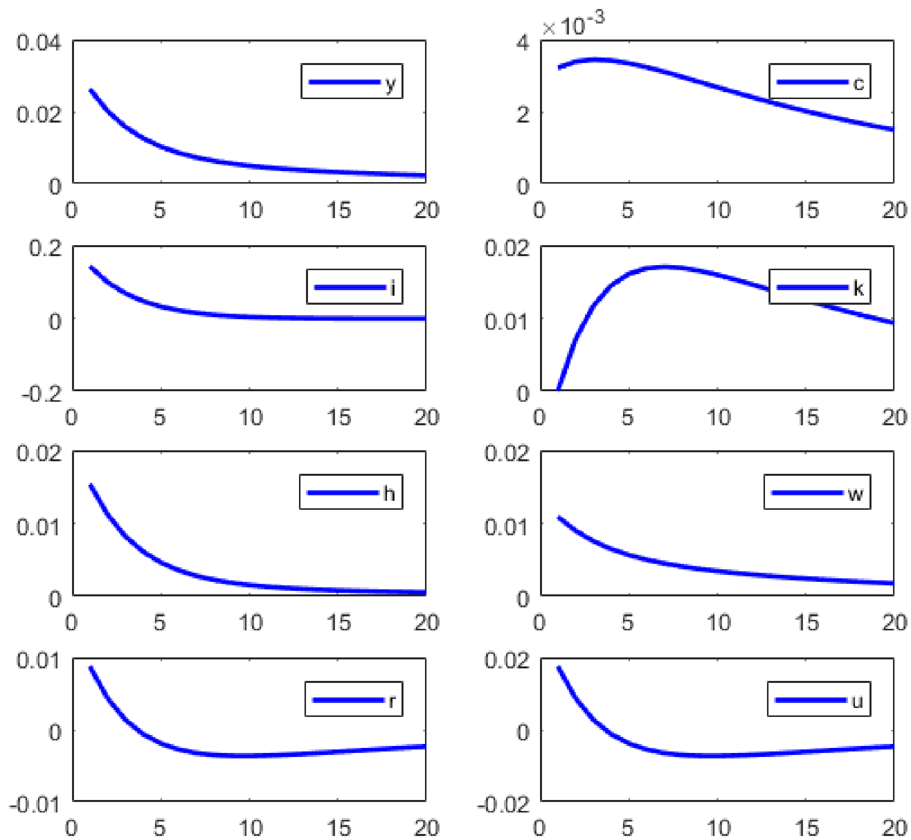

Figure 1: Impulse Responses to a 1% surprise innovation in technology

As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so used of output - consumption, investment, and government consumption all increase contemporaneously. At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and capital. The representative households then respond to the incentives contained in prices and start accumulating capital, and supplies more hours worked. In turn, the increase in capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. Lastly, the utilization rate increases as well, following the increase in the return on capital, but this also increases the endogenous depreciation rate, which in turn decreases the return to capital. In the labor market, the wage rate increases, and the household increases its hours worked. In turn, the increase in total hours further increases output, again indirectly.

Over time, as capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

5.2 Simulation and moment-matching

As in Vasilev (2017b), we will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency. To minimize the sample error, the simulated moments are averaged out over the computer-generated draws. We compare side by side the model with endogenous depreciation rate to the benchmark model with fixed depreciation rate, and constant level of utilization. As in Vasilev (2016, 2017b, 2017c), both models match quite well the absolute volatility of output and investment. By construction, government consumption in the model varies as much as output. However, the model in this paper underestimates the variability in consumption, due to the presence of endogenous capital utilization, which makes investment more volatile, and thus consumption less volatile. Nevertheless, the model is qualitatively consistent with the stylized fact that consumption generally varies less than output, while investment is more volatile than output.

Table 3: Business Cycle Moments

	Data	Model	Benchmark
σ_y	0.05	0.05	0.05
σ_c/σ_y	0.55	0.34	0.82
σ_i/σ_y	1.77	3.61	2.35
σ_g/σ_y	1.21	1.00	1.00
σ_h/σ_y	0.63	0.47	0.28
σ_w/σ_y	0.83	0.55	0.86
$\sigma_{y/h}/\sigma_y$	0.86	0.55	0.86
$corr(c, y)$	0.85	0.90	0.90
$corr(i, y)$	0.61	0.89	0.83
$corr(g, y)$	0.31	1.00	1.00
$corr(h, y)$	0.49	0.98	0.59
$corr(w, y)$	-0.01	0.98	0.96

With respect to the labor market variables, the variability of both employment and wages predicted by the model are much lower the volatility exhibited by their empirical counterparts. Nevertheless, employment variability is much higher in the endogenous depreciation rate setup. Overall, the perfectly-competitive assumption, i.e., that wages equal the marginal product of labor, does not describe very well the dynamics of labor market variables. In addition, as in Vasilev (2017b, 2017c), the model fails in matching unemployment volatility, which in this model varies as much as the employment rate.[6] Next, in terms of contemporaneous correlations, the model systematically over-predicts the pro-cyclicality of the main aggregate variables - consumption, investment, and government consumption. This, however, is a common limitation of this class of models. However, along the labor market dimension, the

[6]The reason behind this mismatch could be driven by several possible explanatory factors: the fact that the model misses the "out-of the-labor-force" segment, as well as the significant emigration to the older EU member states.

contemporaneous correlation of employment with output, and unemployment with output, is relatively well-matched. With respect to wages, the model predicts strong pro-cyclicality, while wages in data are acyclical. This shortcoming is well-known in the literature and an artefact of the wage being equal to the labor productivity in the model.

In the next subsection, as in Vasilev (2015c), we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 on the next page against the averaged simulated AFCs and CCFs. The model compares relatively well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. The persistence of labor market variables are also relatively well-described by the model dynamics. Overall, the model with capital-utilization channel generates too much persistence in output and both employment and unemployment.

Next, as seen from Table 5 below, over the business cycle, in data labor productivity leads employment. The model, however, cannot account for this fact.

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(n_t, n_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(n_t, n_{t-k})$	1.000	0.955	0.902	0.841
	(s.e.)	(0.000)	(0.027)	(0.052)	(0.075)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.956	0.904	0.846
	(s.e.)	(0.000)	(0.026)	(0.051)	(0.074)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.955	0.901	0.838
	(s.e.)	(0.000)	(0.027)	(0.053)	(0.076)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.958	0.909	0.855
	(s.e.)	(0.000)	(0.024)	(0.047)	(0.068)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.955	0.900	0.837
	(s.e.)	(0.000)	(0.028)	(0.053)	(0.077)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.957	0.907	0.850
	(s.e.)	(0.000)	(0.026)	(0.049)	(0.072)

6 Conclusions

We allow for an endogenous depreciation rate of physical capital stock into a real-business-cycle model with a government sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2018). We investigate the quantitative importance of the endogenous depreciation rate, and indirectly, the capital utilization mechanism for cyclical fluctuations in Bulgaria. Allowing for endogenous

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(n_t, (y/n)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(n_t, (y/n)_{t-k})$	0.101	0.134	0.179	0.925	0.335	0.233	0.156
	(s.e.)	(0.342)	(0.301)	(0.254)	(0.058)	(0.240)	(0.277)	(0.314)
Data	$corr(n_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(n_t, w_{t-k})$	0.101	0.134	0.179	0.925	0.335	0.233	0.156
	(s.e.)	(0.342)	(0.301)	(0.254)	(0.058)	(0.240)	(0.277)	(0.314)

variations in the depreciation rate of capital improves the model performance against data, and in addition this extended setup dominates the standard RBC model framework with constant depreciation and a fixed utilization rate of physical capital, and a fixed depreciation tate *e.g.*, Vasilev (2009).

References

Bulgarian National Bank. 2019. Bulgarian National Bank Statistics. Available on-line at www.bnb.bg. Accessed on July 21, 2019.

Greenwood, Jeremy, Zvi Hercowitz, and Gregg Huffman. 1988. "Investment, Capacity Utilization, and the Real Business Cycle," *American Economic Review* 78: 402-17.

Hodrick, Robert and Edward Prescott. 1980. "Post-war US business cycles: An empirical investigation." *Unpublished manuscript* (Carnegie-Mellon University, Pittsburgh, PA).

Kydland, Fynn and Edward Prescott. 1988. "The Workweek of Capital and Its Cyclical Implications," *Journal of Monetary Economics* 21: 343 -60.

National Statistical Institute. 2019. Aggregate Statistical Indicators. Available on-line at www.nsi.bg. Accessed on July 21, 2019.

Vasilev, Aleksandar. 2018. ”The Role of Energy in a Real Business Cycle Model with an Endogenous Capital Utilization Rate in a Government Sector: Lessons from Bulgaria (1999–2016),” *Central European Economic Journal* 5(1): 130-141.

Vasilev, Aleksandar. 2017a. ”Business Cycle Accounting: Bulgaria after the introduction of the currency board arrangement (1999-2014), *European Journal of Comparative Economics*, 14(2): 197-219.

Vasilev, Aleksandar. 2017b. ”A Real-Business-Cycle model with efficiency wages and a government sector: the case of Bulgaria,” *Central European Journal of Economics and Econometrics*, 9(4): 359-377.

Vasilev, Aleksandar. 2017c. ”A Real-Business-Cycle model with reciprocity in labor relations and fiscal policy: the case of Bulgaria,” *Bulgarian Economic Papers* BEP 03-2017, Center for Economic Theories and Policies, Sofia University St. Kliment Ohridski, Faculty of Economics and Business Administration, Sofia, Bulgaria.

Vasilev, Aleksandar. 2017d. ”VAT Evasion in Bulgaria: A General-Equilibrium Approach,” *Review of Economics and Institutions*, 8(2): 2-17.

Vasilev, Aleksandar. 2016. ”Search and matching frictions and business cycle fluctuations in Bulgaria,” *Bulgarian Economic Papers* BEP 03-2016, Center for Economic Theories and Policies, Sofia University St. Kliment Ohridski, Faculty of Economics and Business Administration, Sofia, Bulgaria.

Vasilev, Aleksandar. 2015a. ”Welfare effects of at income tax reform: the case of Bulgaria,” *Eastern European Economics* 53(2): 205-220.

Vasilev, Aleksandar. 2015b. ”Welfare gains from the adoption of proportional taxation in a general-equilibrium model with a grey economy: the case of Bulgaria's 2008 flat tax reform," *Economic Change and Restructuring*, 48(2): 169-185.

Vasilev, Aleksandar. 2015c. "Macroeconomic Effects of Public-Sector Unions," *LABOUR* 29(2): 101-126.

Vasilev, Aleksandar. 2009. "Business cycles in Bulgaria and the Baltic countries: an RBC approach," *International Journal of Computational Economics and Econometrics*, 1(2): 148-170.