How quantitatively important is public investment for both business cycle fluctuations and output growth in Bulgaria (1999-2018)?*

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Abstract

We introduce government investment into a real-business-cycle setup. We calibrate the model to Bulgarian data for the period 1999-2018. We then proceed to quantitatively evaluate the effect of the public capital accumulation channel as a tool for business cycle propagation, as well the importance of public investment spending on output growth. Government investment shocks, in the absence of other technological disturbances, turn out to be unable to account for observed business cycles in Bulgaria. On the other hand, government investment may be able to increase subsequent output growth, but that effect is estimated to be quite small.

Keywords: business cycles, government investment, Bulgaria

JEL Classification Codes: E24, E32

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1 Introduction and Motivation

The standard real-business-cycle (RBC) model with government sector, e.g. Christiano and Eichenbaum (1992), Braun (1994) and McGrattan (1994), among others, assumes that government purchases are only made on consumption-, or final goods. In reality, however, a non-trivial share of government spending is on investment-, or intermediate goods, such as infrastructure.[1] Indeed, investment decisions in the economy are made not only by households, but also by the government at different level (federal, state and local). The government is an important agent in every economy. Still, very few macroeconomic models consider the role of public capital in a disciplined way.[2] The crucial aspect of the analysis turned out to the the value of the elasticity of output with respect to public capital, which also brings the question of productivity of government spending. Unfortunately, the empirical literature is still ambiguous on the issue.[3]

We take this issue seriously and set up real-business-cycle (RBC) model, where we distinguish between government consumption (12.6 percent of GDP on average over the period 1999-2018) and government investment (2.5 percent of GDP on average over the period 1999-2018). This distinction is important, as in contrast to government consumption, which has mostly a demand effect, public investment in capital has both a demand and a supply effect. More specifically, public capital will enter the model as an important input in the aggregate production function as in Baxter and King (1993). We calibrate the model for Bulgaria in the period 1999-2018, as Bulgaria provides a good testing case for the theory.[4]

[1] In what is to follow, we will use "government" and "public" interchangeably.

[2] The major papers in this literature are Arrow and Kurz (1970), Weizman (1970), Pestieau (1974), Barro (1990), Barro and Sala-i-Martin (1992), Finn (1994, 1998), Glomm and Ravikumar (1994), Cashin (1995), among others.

[3] Aschauer (1989) obtained a mean value of 0.39 for the US, and a range between 0.25 and 0.56. Similarly, Ford and Poret (1991) obtained values between 0.29 and 0.66 for 11 OECD countries. In contrast, Aaron (1990), Tatom (1991), Holz-Eakin (1994), Evans and Karras (1994), Garcia-Mila et al. (1996) obtain elasticity estimates that were not significantly different from zero. Finally, Finn (1994), Cassou and Lansing (1998) obtain elasticity values that are positive but much lower than those in earliesr studies. For a critical review, the reader is referred to Romp and de Haan (2007).

[4] In mid-1997, Bulgaria adopted a currency board regime, which is an extreme form of a fixed exchange rate. This monetary arrangement acted as a "straight jacket" for the then volatile Bulgarian economy, and

This ia because Bulgaria is part of the EU as of 2007, but is still the poorest EU member state. As a former transition economy, this East-European country is still developing, and counts on EU structural and cohesion funds to boost investment and growth in the economy. The increase in government investment began once it declared EU accession as a long-term goal, and thus had to substantively improve its infrastructure to convince the EU that it was worthy of joining the European Union. In that sense, the share of public investment in Bulgaria is much larger than that in the older member states, and some interesting insight can be drawn from the computational fiscal reform exercises performed in this paper.[5] In addition, the study on Bulgaria in this paper would be of interest for other small EU countries like Estonia, Latvia, Lithuania, as well as countries undertaking major infrastructure investment, and/or considering EU accession, such as Serbia, Montenegro, North Macedonia, Kosovo, Albania, Ukraine, Georgia, and others.

Figure 1: Government investment, Bulgaria (1999-2018), in 2015 BGN, mln. (NSI 2020)

brought aggregate economic stability; this explains why the particular period was chosen, and why the model is expressed in terms of real variables.

[5]For a survey of different fiscal (mainly tax-) reforms in Bulgaria and their effect on the business cycle, the reader is referred to Vasilev (2017a), Vasilev (2015b), Di Nola *et al* (2019), as well as the references therein. There are not many studies on Eastern European economies that use dynamic general equilibrium setups with micro-foundations.

As we see from Fig. 1 on the previous page, there are some interesting fluctuations exhibited by public investment series over the period.[6] In terms of co-movement with output, government investment varies more than output ($\sigma_{g^i}/\sigma_y = 1.34$), and is moderately pro-cyclical ($corr(g^i, y) = 0.35$), facts which cannot be explained by the Keynesian literature. We thus proceed to quantitatively evaluate the effect of the public capital accumulation channel as a tool for business cycle propagation, while entirely staying within the neoclassical paradigm. In addition, as a robustness check, we compare and contrast the results to those generated by a model without public capital. The major insight is that the presence of public capital improves the model fit, especially when it comes to labor market dimensions. To the best of our knowledge, this is the first study on the issue for Bulgaria that utilizes modern macroeconomic modelling techniques, and thus of interest to fiscal policy makers in Eastern-European and developing economies.[7]

Last, but not least, we are able to address the relationship between public capital and economic growth, which is a topical question for policy makers.[8] Even though it is generally accepted that improvements in public infrastructure cause a positive effect on economic growth, there is no agreement among economists on the exact quantitative effect: Clarida (1993) and Batina (1998, 1999) found a positive effect of public capital on output from a vector-auto-regressive (VAR) frameworks. In contrast, McMillin and Smith (1994), Otto and Voss (1996) and Voss (2002) find a negative relationship. Unfortunately, for reasonable changes in public investment in Bulgaria, the predicted quantitative effect on growth turn out to be rather small.

The rest of the paper is organized as follows: Section 2 describes the model framework and describes the decentralized competitive equilibrium system, Section 3 discusses the calibration procedure, and Section 4 presents the steady-state model solution. Sections 5 proceeds with the out-of-steady-state dynamics of model variables, and compared the simulated second

[6]Only government investment is plotted, as it constitutes only 1-1.5 percent of GDP.

[7]Due to the short time series, we use a calibrated RBC model to draw some important insights. We also abstract away from monetary aspects and thus differ from Sims and Wolff (2018).

[8]For some of the papers in that literature, the reader is referred to Mera (1973) for Japan, Ratner (1983) on the US, and the classical study by Aschauer (1989) on the US as well.

moments of theoretical variables against their empirical counterparts. Section 6 concludes the paper.

2 Model Description

There is a representative household, which derives utility out of consumption and leisure. The time available to households can be spent in productive use or as leisure. The government taxes consumption spending, and levies a common proportional (”flat”) tax on labor and capital income to finance government purchases, government investment, and lump-sum government transfers. On the production side, there is a representative firm, which hires labor and capital to produce a homogeneous final good, which could be used for consumption, investment, or government consumption and investment.

2.1 Households

There is a representative household, which maximizes its expected utility function

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln c_t + \gamma \ln(1-h_t) \Big\} \tag{2.1}$$

whereE_0 denotes household's expectations as of period 0, c_t denotes household's private consumption in period t, h_t are hours worked in period t, $0 < \beta < 1$ is the discount factor, $0 < \gamma < 1$ is the relative weight that the household attaches to leisure.[9]

The household starts with an initial stock of private physical capital $k_0^p > 0$, and has to decide how much to add to it in the form of new investment. The law of motion for physical capital is

$$k_{t+1}^p = i_t + (1-\delta^p)k_t^p \tag{2.2}$$

and $0 < \delta^p < 1$ is the depreciation rate. Next, the real interest rate is r_t, hence the before-tax capital income of the household in period t equals $r_t k_t^p$. In addition to capital income, the

[9]This utility function is equivalent to a specification with a separable term containing government consumption, e.g. Baxter and King (1993). Since in this paper we focus on the exogenous (observed) policies, and the household takes government spending as given, the presence of such a term is irrelevant. For the sake of brevity, we skip this term in the utility representation above.

household can generate labor income. Hours supplied to the representative firm are rewarded at the hourly wage rate of w_t, so pre-tax labor income equals $w_t h_t$. Lastly, the household owns the firm in the economy and has a legal claim on all the firm's profit, π_t.

Next, the household's problem can be now simplified to

$$\max E_0 \sum_{t=0}^{\infty} \beta^t \Big\{ \ln c_t + \gamma \ln(1-h_t) \Big\} \tag{2.3}$$

s.t.

$$(1+\tau^c)c_t + k^p_{t+1} - (1-\delta^p)k^p_t = (1-\tau^y)[r_t k^p_t + \pi_t + w_t h_t] + g^t_t \tag{2.4}$$

where where τ^c is the tax on consumption, τ^y is the proportional income tax rate ($0 < \tau^c, \tau^y < 1$), and g^t_t denotes government transfers. The household takes the tax rates $\{\tau^c, \tau^y\}_{t=0}^{\infty}$, government spending categories, $\{g^c_t, g^t_t\}_{t=0}^{\infty}$, profit $\{\pi_t\}_{t=0}^{\infty}$, the realized technology process $\{A_t\}_{t=0}^{\infty}$, prices $\{w_t, r_t\}_{t=0}^{\infty}$, and chooses $\{c_t, h_t, k^p_{t+1}\}_{t=0}^{\infty}$ to maximize its utility subject to the budget constraint.[10]

The first-order optimality conditions as as follows:

$$c_t \;:\; \frac{1}{c_t} = \lambda_t(1+\tau^c) \tag{2.5}$$

$$h_t \;:\; \frac{\gamma}{1-h_t} = \lambda_t(1-\tau^y)w_t \tag{2.6}$$

$$k_{t+1} \;:\; \lambda_t = \beta E_t \lambda_{t+1}\Big[1 + [1-\tau^y]r_{t+1} - \delta^p\Big] \tag{2.7}$$

$$TVC \;:\; \lim_{t\to\infty} \beta^t \lambda_t k^p_{t+1} = 0 \tag{2.8}$$

where λ_t is the Lagrangean multiplier attached to household's budget constraint in period t. The interpretation of the first-order conditions above is as follows: the first one states that for each household, the marginal utility of consumption equals the marginal utility of wealth, corrected for the consumption tax rate. The second equation states that when choosing labor supply optimally, at the margin, each hour spent by the household working for the firm should balance the benefit from doing so in terms of additional income generates, and the cost measured in terms of lower utility of leisure. The third equation is the so-called

[10]Note that by choosing k_{t+1} the household is implicitly setting investment i_t optimally.

”Euler condition,” which describes how the household chooses to allocate physical capital over time. The last condition is called the ”transversality condition” (TVC): it states that at the end of the horizon, the value of physical capital should be zero.

2.2 Firm problem

There is a representative firm in the economy, which produces a homogeneous product. The price of output is normalized to unity. The production technology is Cobb-Douglas and, as in Bater and King (1993), Finn (1994, 1998), Cassou and Lansing (1998), and Sims and Wolff (2018), uses both private and public physical capital, k_t^p and k_t^g, and labor hours, h_t, to maximize static profit

$$\Pi_t = A_t(k_t^p)^\alpha h_t^{1-\alpha}(k_t^g)^\sigma - r_t k_t^p - w_t h_t, \tag{2.9}$$

where A_t denotes the level of technology in period t, and $\sigma > 0$ captures the increasing returns to scale generated by the presence of public capital.[11] Since the firm rents the capital from households, the problem of the firm is a sequence of static profit maximizing problems. In equilibrium, there are no profits, and each input is priced according to its marginal product, *i.e.*:

$$k_t^p \quad : \quad \alpha \frac{y_t}{k_t^p} = r_t, \tag{2.10}$$

$$h_t \quad : \quad (1-\alpha)\frac{y_t}{h_t} = w_t. \tag{2.11}$$

In equilibrium, given that the inputs of production are paid their marginal products, $\pi_t = 0$, $\forall t$.

2.3 Government

In the model setup, the government is levying taxes on labor and capital income, as well as consumption, in order to finance spending on government purchases, government investment, and government transfers.[12] The government period budget constraint is as follows:

$$g_t^c + g_t^i + g_t^t = \tau^c c_t + \tau^y[w_t h_t + r_t k_t + \pi_t] \tag{2.12}$$

[11]Note that the firm takes the amount of public capital as given.

[12]Given the low foreign debt in Bulgaria (21 percent of GDP on average over the period), we abstract away from debt considerations.

subject to the law of motion for public capital:[13]

$$k_{t+1}^g = (1-\delta^g)k_t^g + g_t^i \tag{2.13}$$

and an initial $k_0^g > 0$. Government investment will be assumed to follow a simple rule:

$$g_t^i = B_t \theta y_t, \tag{2.14}$$

where B_t denotes the efficiency of government investment in period t, and $0 < \theta < 1$ captures the average government investment-to-output ratio.

Consumption tax rate, income tax rate, and government investment-to-output and consumption-to-output ratios would be chosen to match the average share in data. Finally, government transfers would be determined residually in each period so that the government budget is always balanced.

2.4 Exogenous stochastic processes

The exogenous processes for total factor productivity, A_t, and government investment productivity, B_t, will follow AR(1) processes in natural logarithms:

$$\ln A_{t+1} = (1-\rho_a)\ln A + \rho_a \ln A_t + \epsilon_{t+1}^a \tag{2.15}$$

$$\ln B_{t+1} = (1-\rho_b)\ln B + \rho_b \ln B_t + \epsilon_{t+1}^b, \tag{2.16}$$

where A, B are the steady-state values of the two processes, $0 < \rho_a, \rho_\theta < 1$ are the respective persistence parameters, and the productivity innovations and changes to institutional quality are drawn from the following distributions: $\epsilon_t^a \sim i.i.dN(0,\sigma_a^2)$ and $\epsilon_t^b \sim i.i.dN(0,\sigma_b^2)$, respectively.

2.5 Dynamic Competitive Equilibrium (DCE)

For the given processes followed by the technology variables $\{A_t, B_t\}_{t=0}^\infty$, the two tax rates $\{\tau^c, \tau^y\}_{t=0}^\infty$, and initial capital stocks $\{k_0^p, k_0^g\}$, the decentralized dynamic competitive equilibrium is a list of sequences $\{c_t, i_t, k_t^p, k_t^g, h_t\}_{t=0}^\infty$ for the household, a sequence of government

[13] Note that public capital is a non-market type of physical capital.

purchases, investment and transfers $\{g_t^c, g_t^i, g_t^t\}_{t=0}^{\infty}$, output series $\{y_t\}_{t=0}^{\infty}$ and input prices $\{w_t, r_t\}_{t=0}^{\infty}$ such that (i) the household maximizes its utility function subject to its budget constraint; (ii) the representative firm maximizes profit; (iii) government budget is balanced in each period; (iv) government capital evolves according to its law of motion, and (v) all markets clear: $c_t + i_t + g_t^c + g_t^i = y_t$.

3 Data and Model Calibration

To characterize business cycle fluctuations in Bulgaria, we will focus on the period following the introduction of the currency board (1999-2018). Quarterly data on output, consumption and investment was collected from National Statistical Institute (2020), while the real interest rate is taken from Bulgarian National Bank Statistical Database (2020). The calibration strategy described in this section follows a long-established tradition in modern macroeconomics: first, as in Vasilev (2016), the discount factor, $\beta = 0.982$, is set to match the steady-state capital-to-output ratio in Bulgaria, $k^p/y = 8.964$, in the steady-state Euler equation. The labor share parameter, $1 - \alpha = 0.571$, is obtained as in Vasilev (2017d), and equals the average value of labor income in aggregate output over the period 1999-2018. This value is slightly higher as compared to other studies on developed economies, due to the over-accumulation of physical capital, which was part of the ideology of the totalitarian regime, which was in place until 1989. Following Baxter and King (1993), we set $\sigma = 0.025$ to correspond to the average government investment-to-output ratio.

Next, the average labor and capital income tax rate was set to $\tau^y = 0.1$. This is the average effective tax rate on income between 1999-2007, when Bulgaria used progressive income taxation, and equal to the proportional income tax rate introduced as of 2008. Similarly, the average tax rate on consumption is set to its value over the period, $\tau^c = 0.2$. Further, the relative weight attached to the utility out of leisure in the household's utility function, γ, is calibrated to match that in steady-state consumers would supply one-third of their time endowment to working. This is in line with the estimates for Bulgaria (Vasilev 2017a) as well over the period studied. Next, the depreciation rate of private physical capital in Bulgaria, $\delta^p = 0.013$, was taken from Vasilev (2016). It was estimated as the average quarterly depreciation rate over the period 1999-2014. Due to the lack of data, the depreciation rate of physical capital was set to $\delta^g = 0.005$. Given an average government-investment share of $\theta = 0.025$, this produces a steady-state public capital-to-output ratio of $k^g/y = 5$. The scale parameter B, capturing the steady-state of government investment efficiency, will be set to unity.

Finally, the process followed by the TFP process is estimated from the detrended series by running an AR(1) regression and saving the residuals. Due to the lack of data, the process followed by government investment technology progress will feature the same parameters. Table 1 below summarizes the values of all model parameters used in the paper.

Table 1: Model Parameters

Parameter	Value	Description	Method
β	0.982	Discount factor	Calibrated
α	0.429	Pvt capital Share	Data average
σ	0.025	Public capital Share	Data average
γ	0.873	Relative weight attached to leisure	Calibrated
θ	0.025	Government inv-to-output ratio	Data average
δ^p	0.013	Depreciation rate on pvt physical capital	Data average
δ^g	0.005	Depreciation rate on govt physical capital	Data average
τ^y	0.100	Average tax rate on labour income	Data average
τ^c	0.200	VAT/consumption tax rate	Data average
B	1.000	Steady-state value of govt inv. eff. process	Set
ρ_a	0.701	AR(1) persistence coefficient, TFP process	Estimated
σ_a	0.044	st. error, TFP process	Estimated
ρ_b	0.701	AR(1) persistence coefficient, govt inv. process	Set
σ_b	0.044	st. error, govt inv. process	Set

4 Steady-State

Once the values of model parameters were obtained, the steady-state equilibrium system solved, the "big ratios" can be compared to their averages in Bulgarian data. The results are reported in Table 2 below. The steady-state level of output was normalized to unity (hence the level of technology A differs from one, which is usually the normalization done in other studies), which greatly simplified the computations. Next, the model matches consumption-to-output and government purchases ratios by construction; The investment ratios are also closely approximated, despite the closed-economy assumption and the absence of foreign trade sector. The shares of income are also identical to those in data, which is an artifact of the assumptions imposed on functional form of the aggregate production function. The after-tax return, where $\bar{r} = (1-\tau^y)r - \delta^p$ is also relatively well-captured by the model. Lastly, given the absence of debt, and the fact that transfers were chosen residually to balance the government budget constraint, the result along this dimension is understandably not so close to the average ratio in data.

Table 2: Data Averages and Long-run Solution

Variable	Description	Data	Model
y	Steady-state output	N/A	1.000
c/y	Consumption-to-output ratio	0.648	0.674
i/y	Investment-to-output ratio	0.201	0.175
k^p/y	Pvt capital-to-output ratio	8.960	8.960
k^g/y	Govt capital-to-output ratio	5.000	5.000
g^c/y	Government consumption-to-output ratio	0.126	0.126
g^i/y	Government inv-to-output ratio	0.025	0.025
wh/y	Labor income-to-output ratio	0.571	0.571
rk^p/y	Capital income-to-output ratio	0.429	0.429
h	Share of time spent working	0.333	0.333
$\bar{r}$	After-tax net return on capital	0.014	0.016

5 Out of steady-state model dynamics

Since the model does not have an analytical solution for the equilibrium behavior of variables outside their steady-state values, we need to solve the model numerically. This is done by log-linearizing the original equilibrium (non-linear) system of equations around the steady-state. This transformation produces a first-order system of stochastic difference equations. First, we study the dynamic behavior of model variables to an isolated shock to the total factor productivity process, and then we fully simulate the model to compare how the second moments of the model perform when compared against their empirical counterparts.

5.1 Impulse Response Analysis

This subsection documents the impulse responses of model variables to a 1% surprise innovation to technology. The impulse response functions (IRFs) are presented in Fig. 2 and Fig. 3 for the TFP and government investment shock, respectively. As a result of the one-time unexpected positive shock to total factor productivity, output increases upon impact. This expands the availability of resources in the economy, so used of output - consumption, private investment, government investment, and government consumption also increase contemporaneously.

At the same time, the increase in productivity increases the after-tax return on the two factors of production, labor and private physical capital, that are under the control of the household. The representative household then respond to the incentives contained in prices and start accumulating private physical capital, and supplies more hours worked. In turn, the increase in private physical capital input feeds back in output through the production function and that further adds to the positive effect of the technology shock. In the labor market, the wage rate increases, and the household increases its hours worked. In turn, the increase in total hours further increases output, again indirectly.

With public physical capital in the model, there is first a positive effect on government investment, which is proportional to output. In turn, there is a feedback effect from physical investment on next-period output through public capital, as the latter is an input in the aggregate production function. However, this indirect effect is rather small, due to the very mild increasing returns to scale. Similarly, the positive effect on both the transitional path and the steady-state of public physical capital is also quite small.

Over time, as private physical capital is being accumulated, its after-tax marginal product starts to decrease, which lowers the households' incentives to save. As a result, the private physical capital stock eventually returns to its steady-state, and exhibits a hump-shaped dynamics over its transition path. The rest of the model variables return to their old steady-states in a monotone fashion as the effect of the one-time surprise innovation in technology dies out.

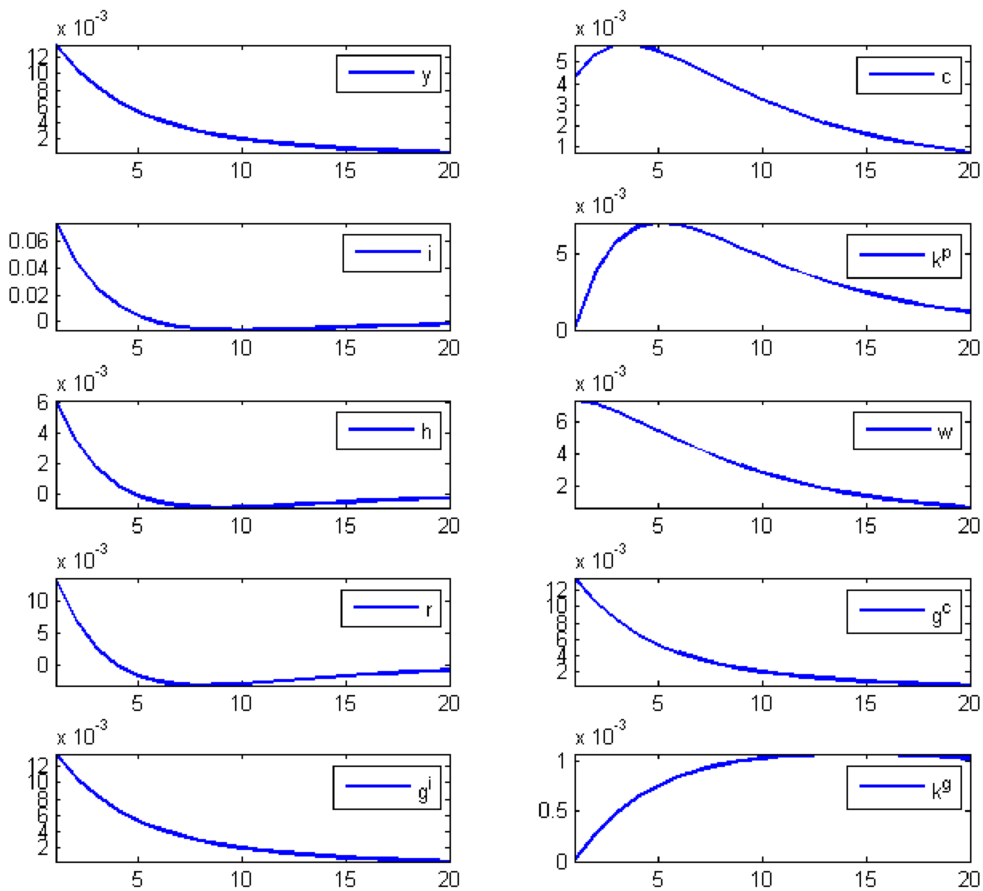

Figure 2: Impulse Responses to a 1% surprise innovation in technology

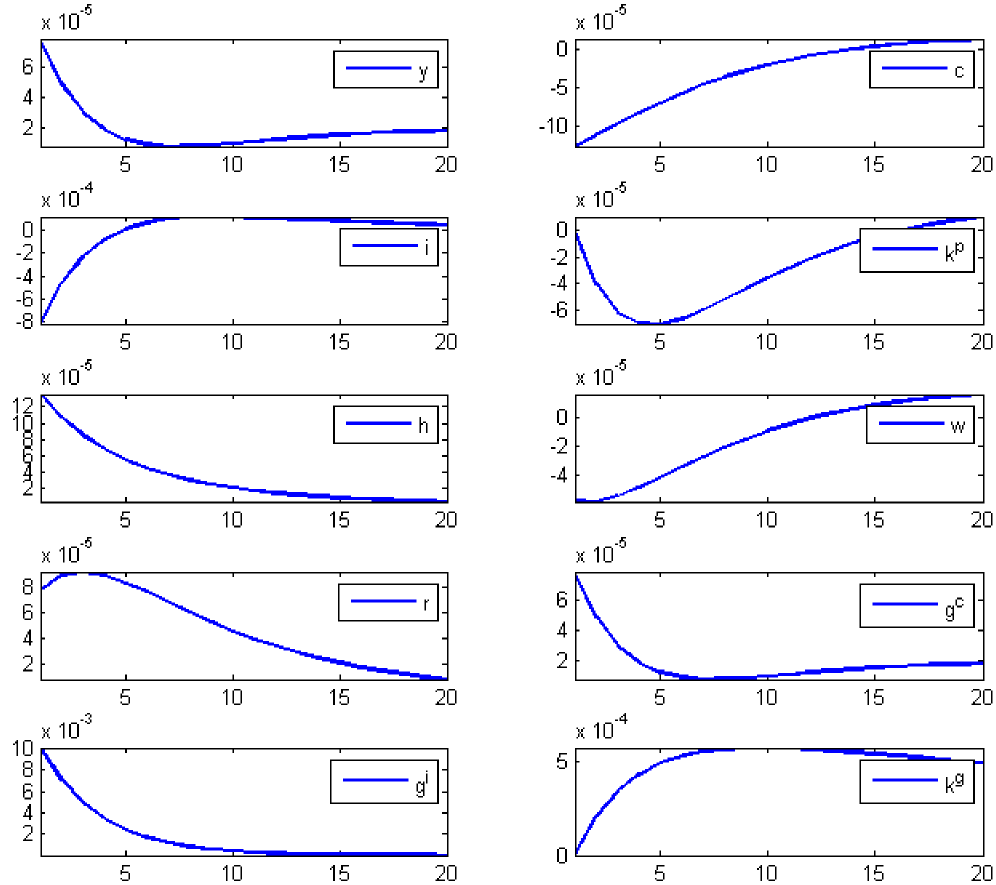

Figure 3: Impulse Responses to a 1% surprise innovation in government investment efficiency

In contrast, the effect of shock to government investment efficiency is quite short-lived, and rather small in magnitude as shown in Fig. 3 on the next page. Given the mild increasing returns to scale, driven by the presence of public capital in the aggregate production function, we do not see a multiplier effect. Just the opposite, there is a complete crowding out effect on private consumption and private investment upon impact of the shock, as the household switches towards accumulating more public capital. This negative effect makes households fell poorer, and they start working more, which in turn decreases wages. As the effect of the efficiency of government investment shock dies out, the household turns back to private capital, and investment, physical capital and labor return to their old steady-states.

Note that there is a long-run effect on the level of output, but it is quite small. If Bulgaria doubles the government investment share to a 5% to output (possibly through the use of EU funds), which is an extremely unlikely scenario, that would increase output by only 0.186% per annum.[14]

5.2 Simulation and moment-matching

As in Vasilev (2017b), we will now simulate the model 10,000 times for the length of the data horizon. Both empirical and model simulated data is detrended using the Hodrick-Prescott (1980) filter. Table 3 on the next page summarizes the second moments of data (relative volatilities to output, and contemporaneous correlations with output) versus the same moments computed from the model-simulated data at quarterly frequency. The "Model" is the case with the public capital, while the "Benchmark RBC" is the standard setup without government capital. In addition, to minimize the sample error, the simulated moments are averaged out over the computer-generated draws.

The model with government investment efficiency shocks are not able to generate business cycles of the magnitude of the observed fluctuations in Bulgaria. Its isolated effect is also very small when both shocks are at work. However, the model with TFP shock, constant government investment efficiency, and government capital accumulation seems to slightly dominate the standard RBC model without public capital considerations, especially in terms of some correlations, and the hours-wages correlation in particular. Still, in all of the models, the predicted consumption and investment volatilies are too high.[15] Overall, the models are qualitatively consistent with the stylized fact that consumption generally varies

[14]More specifically, the effect is 0.77% in the first year, 0.49% for the second year, 0.2% for year 3, then drops to 0.1%, before rebounding to 0.186% figure.

[15]This is a typical result in the literature. To address it, some authors have incorporated frictions, such as habits in consumption, and/or capital/investment adjustment costs. In this paper we do not incorporate such mechanisms, as we want to keep the model parsimonious, and focus on the transmission mechanism of public investment shocks.

less than output, while investment is more volatile than output.[16] The model with public capital produces more volatile consumption and investment series, but the quantitative effect is relatively small. In addition, the predicted volatility of public investment is relatively close to the one observed in data.

Table 3: Business Cycle Moments

	Data	Benchmark RBC (no public inv)	Model with gov't inv. (both shocks)	TFP shock only	Gov't inv. eff. shock only
σ_c/σ_y	0.55	0.82	0.90	0.90	2.82
σ_i/σ_y	1.77	2.35	2.53	2.55	5.78
σ_{g^c}/σ_y	1.21	1.00	1.00	1.00	1.00
σ_{g^i}/σ_y	1.34	-	1.14	1.00	10.7
σ_h/σ_y	0.63	0.28	0.19	0.20	2.08
σ_w/σ_y	0.83	0.86	0.93	0.93	1.87
$corr(c, y)$	0.85	0.90	0.95	0.95	-0.18
$corr(i, y)$	0.61	0.83	0.73	0.72	0.11
$corr(g^c, y)$	0.31	1.00	1.00	1.00	1.00
$corr(g^i, y)$	0.35	-	0.83	1.00	0.39
$corr(h, y)$	0.49	0.59	0.45	0.44	0.45
$corr(w, y)$	-0.01	0.96	0.98	0.97	-0.03
$corr(w, h)$	0.32	0.36	0.28	0.26	-0.85

With respect to the labor market variables, the variability of hours predicted by the model with public capital is lower than both the standard model without government capital and data, but the variability of wages in both model is very close to that in data (again a bit higher in the model with public capital). This is yet another confirmation that the perfectly-competitive assumption, e.g. Vasilev (2009), as well as the benchmark calibration here, does not describe very well the dynamics of labor market variables. Next, in terms of contemporaneous correlations, the models systematically over-predicts the pro-cyclicality of the main

[16]This differs from other emerging economies where consumption varies more than output, e.g. Ghate (2016). In this sense, Bulgarian economy behaves like a developed one, with a strong consumption smoothing mechanism/consideration in place.

aggregate variables - consumption and private and public investment. This, however, is a common limitation of this class of models. Again, the model with public capital produces a closer figure for the private investment correlation (at the cost of predicting a larger figure for the consumption correlation). Next, along the labor market dimension, the contemporaneous correlation of hours with output is a bit low, but closer to data, as compared to the model without government capital. With respect to wages, the models predict strong cyclicality, while wages in data are acyclical. This shortcoming is well-known in the literature and an artifact of the wage being equal to the labor productivity in both setups.

In the next subsection, as in Vasilev (2016), we investigate the dynamic correlation between labor market variables at different leads and lags, thus evaluating how well the model matches the phase dynamics among variables. In addition, the autocorrelation functions (ACFs) of empirical data, obtained from an unrestricted VAR(1) are put under scrutiny and compared and contrasted to the simulated counterparts generated from the model.

5.3 Auto- and cross-correlation

This subsection discusses the auto-(ACFs) and cross-correlation functions (CCFs) of the major model variables. The coefficients empirical ACFs and CCFs at different leads and lags are presented in Table 4 on the next page against the averaged simulated AFCs and CCFs.[17] For the sake of brevity, we only present the result for the case when both shocks are present in the model setup. Overall, the model compares relatively well vis-a-vis data. Empirical ACFs for output and investment are slightly outside the confidence band predicted by the model, while the ACFs for total factor productivity and household consumption are well-approximated by the model. The persistence of labor market variables are also relatively well-described by the model dynamics. Overall, the model with public capital generates too much persistence in output and both hours and unemployment (defined in the model as $u_t = 1 - h_t$), and is subject to the criticism in Nelson and Plosser (1992), Cogley and Nason (1995) and Rotemberg and Woodford (1996b), who argue that the RBC class of models do not have a strong internal propagation mechanism besides the strong persistence in the TFP process. In those models, e.g. Vasilev (2009), and in the current one, labor market is

[17]Following Canova (2007), this is used as a goodness-of-fit measure.

modelled in the Walrasian market-clearing spirit, and output and unemployment persistence is low.

Table 4: Autocorrelations for Bulgarian data and the model economy

		k			
Method	Statistic	0	1	2	3
Data	$corr(u_t, u_{t-k})$	1.000	0.765	0.552	0.553
Model	$corr(u_t, u_{t-k})$	1.000	0.949	0.884	0.809
	(s.e.)	(0.000)	(0.030)	(0.057)	(0.081)
Data	$corr(h_t, h_{t-k})$	1.000	0.484	0.009	0.352
Model	$corr(h_t, h_{t-k})$	1.000	0.949	0.884	0.809
	(s.e.)	(0.000)	(0.030)	(0.057)	(0.081)
Data	$corr(y_t, y_{t-k})$	1.000	0.810	0.663	0.479
Model	$corr(y_t, y_{t-k})$	1.000	0.956	0.904	0.844
	(s.e.)	(0.000)	(0.026)	(0.050)	(0.072)
Data	$corr(a_t, a_{t-k})$	1.000	0.702	0.449	0.277
Model	$corr(a_t, a_{t-k})$	1.000	0.955	0.900	0.836
	(s.e.)	(0.000)	(0.027)	(0.051)	(0.074)
Data	$corr(c_t, c_{t-k})$	1.000	0.971	0.952	0.913
Model	$corr(c_t, c_{t-k})$	1.000	0.958	0.908	0.850
	(s.e.)	(0.000)	(0.024)	(0.047)	(0.068)
Data	$corr(i_t, i_{t-k})$	1.000	0.810	0.722	0.594
Model	$corr(i_t, i_{t-k})$	1.000	0.950	0.888	0.815
	(s.e.)	(0.000)	(0.029)	(0.055)	(0.081)
Data	$corr(w_t, w_{t-k})$	1.000	0.760	0.783	0.554
Model	$corr(w_t, w_{t-k})$	1.000	0.957	0.907	0.848
	(s.e.)	(0.000)	(0.025)	(0.048)	(0.070)

Next, as seen from Table 5 below, over the business cycle, in data labor productivity leads hours. The model, however, cannot account for this fact. As in the standard RBC model a technology shock can be regarded as a factor shifting the labor demand curve, while holding the labor supply curve constant. Therefore, the effect between employment and labor productivity is only a contemporaneous one.

Table 5: Dynamic correlations for Bulgarian data and the model economy

		k						
Method	Statistic	-3	-2	-1	0	1	2	3
Data	$corr(h_t, (y/h)_{t-k})$	-0.342	-0.363	-0.187	-0.144	0.475	0.470	0.346
Model	$corr(h_t, (y/h)_{t-k})$	0.013	0.016	0.019	0.281	0.006	-0.047	-0.079
	(s.e.)	(0.335)	(0.290)	(0.237)	(0.303)	(0.223)	(0.254)	(0.284)
Data	$corr(h_t, w_{t-k})$	0.355	0.452	0.447	0.328	-0.040	-0.390	-0.57
Model	$corr(h_t, w_{t-k})$	0.014	0.020	0.024	0.362	0.030	-0.029	-0.069
	(s.e.)	(0.328)	(0.287)	(0.237)	(0.289)	(0.212)	(0.246)	(0.280)

6 Conclusions

We introduce government investment into a real-business-cycle setup augmented with a public sector. We calibrate the model to Bulgarian data for the period following the introduction of the currency board arrangement (1999-2018). We then proceed to quantitatively evaluate the effect of the public capital accumulation channel as a tool for business cycle propagation, as well the importance of public investment spending on output growth. Government investment shocks, in the absence of other technological disturbances, turn out to be unable to account for observed business cycles in Bulgaria. On the other hand, government investment may be able to increase subsequent output growth, but that effect is estimated to be quite small.

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