MINIMAX PASSBAND GROUP DELAY NONLINEAR FIR FILTER DESIGN WITHOUT IMPOSING DESIRED PHASE RESPONSE

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ABSTRACT

In this paper, a nonlinear phase finite impulse response (FIR) filter is designed without imposing a desired phase response. The maximum passband group delay of the filter is minimized subject to a positivity constraint on the passband group delay response of the filter as well as a specification on the maximum absolute difference between the desired magnitude square response and the designed magnitude square response over both the passband and the stopband. This filter design problem is a nonsmooth functional inequality constrained optimization problem. To tackle this problem, first, the one norm functional inequality constraint of the optimization problem is approximated by a smooth function so that the nonsmooth functional inequality constrained optimization problem is approximated as a nonconvex functional inequality constrained optimization problem. Then, a modified filled function method is applied for finding the global minimum of the nonconvex optimization problem. Computer numerical simulation results show that our designed nonlinear phase peak constrained FIR filter could achieve lower minimum passband group delay than those of existing designs.

1. INTRODUCTION

Nonlinear phase FIR filters are attractive in signal processing applications because they could achieve better frequency selectivities than linear phase filters for the same filter lengths. In addition, bounded input leads to bounded output and the stability of the filter is guaranteed. Consequently, nonlinear phase FIR filters are found in many science and engineering applications [1].

Although many nonlinear phase peak constrained FIR filter designs could be found in literature, most of these designs minimize the maximum absolute differences between the desired magnitude square responses and the designed magnitude square responses [2]. However, these designs have not considered the maximum passband group delays of the filters. To tackle the maximum passband group delays of the filters [3], they require the desired phase responses of the filters. Unlike linear phase filter designs, the desired phase responses of nonlinear phase filters are usually unknown. By imposing certain desired phase responses, the maximum passband group delays of the designed filters are not optimized. Also, the frequency selectivities of the designed filters could be reduced. In this paper, the maximum passband group delay of the filter is minimized subject to a positivity constraint on the passband group delay response of the filter as well as a specification on the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response over both the passband and the stopband of the filter. The one norm functional inequality constraint of the optimization problem is approximated by a smooth function so that the nonsmooth functional inequality constrained optimization problem is approximated as a nonconvex functional inequality constrained optimization problem. Then, a modified filled function method is applied for finding the global minimum of the nonconvex optimization problem. Computer numerical simulation results show that our designed nonlinear phase peak constrained FIR filter could achieve lower maximum passband group delay than those of the existing designs.

The outline of this paper is as follows. The problem formulation and the solution method are presented in Section

2. Computer numerical simulation results are presented in Section 3. Finally, a conclusion is drawn in Section 4.

2. PROBLEM FORMULATION AND SOLUTION METHOD

2.1 Problem formulation

Denote $|H(\omega)|$, $\angle H(\omega)$, $\tau(\omega)$, $D(\omega)$ and $h(n)$ as the magnitude response, the phase response, the group delay response, the desired magnitude response and the impulse response of a nonlinear phase peak constrained FIR filter, respectively. In addition, denote B_p, B_s, N and $(\delta(\omega))^2$ as the passband, the stopband, the length and the specification on the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response of the filter, respectively. The vector of the filter coefficients is given as $\mathbf{x} \equiv [h(0), \quad h(1), \quad \cdots, \quad h(N-1)]^T$, where ^{T} is the transpose operator. Denote the frequency response kernels as

$$\boldsymbol{\iota}_s(\omega) \equiv [0, \quad \sin\omega, \quad \cdots, \quad \sin((N-1)\omega)]^T,$$

$$\boldsymbol{\iota}_c(\omega) \equiv [1, \quad \cos\omega, \quad \cdots, \quad \cos((N-1)\omega)]^T,$$

$$\boldsymbol{\iota}'_s(\omega) \equiv [0, \quad \sin\omega, \quad \cdots, \quad (N-1)\sin((N-1)\omega)]^T$$

and

$$\boldsymbol{\iota}'_c(\omega) \equiv [0, \quad \cos\omega, \quad \cdots, \quad (N-1)\cos((N-1)\omega)]^T.$$

Then, $H(\omega) = \boldsymbol{\iota}_c^T(\omega)\mathbf{x} - j\boldsymbol{\iota}_s^T(\omega)\mathbf{x}$, $\angle H(\omega) = -\tan^{-1}\frac{\boldsymbol{\iota}_s^T(\omega)\mathbf{x}}{\boldsymbol{\iota}_c^T(\omega)\mathbf{x}}$ and $\tau(\omega) = -\frac{d\angle H(\omega)}{d\omega}$. Define

$$\mathbf{Q}_1(\omega) \equiv \boldsymbol{\iota}_c(\omega)\boldsymbol{\iota}'^T_c(\omega) + \boldsymbol{\iota}_s(\omega)\boldsymbol{\iota}'^T_s(\omega)$$

and

$$\mathbf{Q}_2(\omega) \equiv \boldsymbol{\iota}_c(\omega)\boldsymbol{\iota}_c^T(\omega) + \boldsymbol{\iota}_s(\omega)\boldsymbol{\iota}_s^T(\omega).$$

Then, we have $\tau(\omega) = \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}}$. A specification on the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response of the filter is given as $\left||H(\omega)|^2 - (D(\omega))^2\right| \le (\delta(\omega))^2$ $\forall\omega \in B_p \cup B_s$. This is equivalent to $\left|\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x} - (D(\omega))^2\right| \le (\delta(\omega))^2$ $\forall\omega \in B_p \cup B_s$. As the passband group delay response of the filter is required to be positive, we have $-\frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} \le 0$ $\forall\omega \in B_p$. To minimize the maximum passband group delay of the filter subject to the positivity constraint on the passband group delay response of the filter as well as the specification on the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response over both the passband and the stopband of the filter, the filter design problem is formulated as the following optimization problem:

Problem ($\mathbf{P}$)

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) = \max_{\omega \in B_p} \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}},$$

subject to $g_1(\mathbf{x},\omega) = \left|\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x} - (D(\omega))^2\right| - (\delta(\omega))^2 \le 0$ $\forall\omega \in B_p \cup B_s$,

and $g_2(\mathbf{x},\omega) = -\frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} \le 0$ $\forall\omega \in B_p$,

where $f(\mathbf{x})$ is the cost function of the optimization problem, $g_1(\mathbf{x},\omega)$ is the one norm functional inequality constraint and $g_2(\mathbf{x},\omega)$ are the rational functional inequality constraint of the optimization problem.

2.2 Solution method

As Problem ($\mathbf{P}$) is a nonsmooth functional inequality constrained optimization problem, there are oscillations when running conventional optimization algorithms. Hence, it is a challenge to find the global minimum of the optimization problem. To address this difficulty, the one norm functional inequality constraint of the optimization problem is approximated [4] by a smooth function so that the oscillations could be avoided. This is done by defining

$$g_\sigma(\mathbf{x},\omega) \equiv \begin{cases} \left|\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x} - (D(\omega))^2\right| - (\delta(\omega))^2 & \left|\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x} - (D(\omega))^2\right| \ge \frac{\sigma}{2} \\ \frac{\left(\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x} - (D(\omega))^2\right)^2}{\sigma} + \frac{\sigma}{4} - (\delta(\omega))^2 & \textit{otherwise} \end{cases}$$

$\forall\omega \in B_p \cup B_s$. It is worth noting that $g_\sigma(\mathbf{x},\omega) \approx g(\mathbf{x},\omega)$ $\forall\omega \in B_p \cup B_s$ as $\sigma \to 0^+$. Problem ($\mathbf{P}$) could be approximated as the following optimization problem:

Problem ($\mathbf{P}'_\sigma$)

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) = \max_{\omega \in B_p} \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}},$$

subject to $g_\sigma(\mathbf{x},\omega) \le 0$ $\forall\omega \in B_p \cup B_s$,

and $g_2(\mathbf{x},\omega) = -\frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} \le 0$ $\forall\omega \in B_p$.

Problem ($\mathbf{P}'_\sigma$) is a nonconvex functional inequality constrained optimization problem and thus it remains a challenge to find the global minimum of the optimization problem. For this, a modified filled function method [5] is applied. The filled function $H(\mathbf{x})$ is used to escape from the current local minimum and to reach another point in a lower basin of $f(\mathbf{x})$ from the current local minimum.

$$H(\mathbf{x}) \equiv \max_{\omega \in B_p} \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} + \frac{1}{(\mathbf{x} - \mathbf{x}_k^*)^T\mathbf{R}(\mathbf{x} - \mathbf{x}_k^*)},$$

where $\mathbf{R}$ is a positive definite matrix that controls the spread of the hill of $H(\mathbf{x})$ at $\mathbf{x}_k^*$. If $\mathbf{R}$ is a diagonal matrix with all diagonal elements being the same and positive, then large values of these diagonal elements will result in a wide spread of the hill of $H(\mathbf{x})$ at $\mathbf{x}_k^*$ and vice versa. The algorithm for solving Problem ($\mathbf{P}'_\sigma$), incorporating the filled function, is summarized as follows.

Algorithm

Step 1: Initialize a minimum improvement factor ε, an accepted error ε', an initial search point $\tilde{\mathbf{x}}_1$, a positive definite matrix $\mathbf{R}$ and an iteration index $k = 1$.

Step 2: Find a local minimum of the following optimization Problem ($\mathbf{P}_f$) via the integration approach [6] based on the initial search point $\tilde{\mathbf{x}}_k$.

Problem ($\mathbf{P}_f$)

$$\min_{\mathbf{x}} \quad f(\mathbf{x}) = \max_{\omega\in B_p} \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}},$$

subject to $g_\sigma(\mathbf{x},\omega)\le 0 \quad \forall\omega\in B_p\cup B_s$,

$$g_2(\mathbf{x},\omega) = -\frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} \le 0 \quad \forall\omega\in B_p,$$

and

$$g_3(\mathbf{x}) \equiv \max_{\omega\in B_p} \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} - (1-\varepsilon)\max_{\omega\in B_p}\frac{(\tilde{\mathbf{x}}_k)^T\mathbf{Q}_1(\omega)\tilde{\mathbf{x}}_k}{(\tilde{\mathbf{x}}_k)^T\mathbf{Q}_2(\omega)\tilde{\mathbf{x}}_k} \le 0,$$

where $g_3(\mathbf{x})\le 0$ is a discrete constraint we imposed. Denote the obtained local minimum as $\mathbf{x}_k^*$.

Step 3: To escape from the current local minimum and to reach another point in a lower basin of $f(\mathbf{x})$ from $\mathbf{x}_k^*$, we find a local minimum of the following optimization Problem ($\mathbf{P}_H$) via the integration approach [6] based on the initial search point $\mathbf{x}_k^*$.

Problem ($\mathbf{P}_H$)

$$\min_{\mathbf{x}} \quad H(\mathbf{x}) = \max_{\omega\in B_p} \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} + \frac{1}{(\mathbf{x}-\mathbf{x}_k^*)^T\mathbf{R}(\mathbf{x}-\mathbf{x}_k^*)},$$

subject to $g_\sigma(\mathbf{x},\omega)\le 0 \quad \forall\omega\in B_p\cup B_s$,

$$g_2(\mathbf{x},\omega) = -\frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} \le 0 \quad \forall\omega\in B_p,$$

and

$$g_4(\mathbf{x}) \equiv \max_{\omega\in B_p} \frac{\mathbf{x}^T\mathbf{Q}_1(\omega)\mathbf{x}}{\mathbf{x}^T\mathbf{Q}_2(\omega)\mathbf{x}} - (1-\varepsilon)\max_{\omega\in B_p}\frac{(\mathbf{x}_k^*)^T\mathbf{Q}_1(\omega)\mathbf{x}_k^*}{(\mathbf{x}_k^*)^T\mathbf{Q}_2(\omega)\mathbf{x}_k^*} \le 0,$$

where $g_4(\mathbf{x})\le 0$ is a discrete constraint we imposed. Denote the obtained local minimum as $\tilde{\mathbf{x}}_{k+1}$. Set $k=k+1$.

Step 4: Iterate Step 2 and Step 3 until

$$\left|\max_{\omega\in B_p}\frac{(\mathbf{x}_k^*)^T\mathbf{Q}_1(\omega)\mathbf{x}_k^*}{(\mathbf{x}_k^*)^T\mathbf{Q}_2(\omega)\mathbf{x}_k^*} - \max_{\omega\in B_p}\frac{(\mathbf{x}_{k-1}^*)^T\mathbf{Q}_1(\omega)\mathbf{x}_{k-1}^*}{(\mathbf{x}_{k-1}^*)^T\mathbf{Q}_2(\omega)\mathbf{x}_{k-1}^*}\right| \le \varepsilon'.$$

Take the final vector of $\mathbf{x}_k^*$ as the global minimum of the original optimization problem.

The working principle of the algorithm has been discussed in [5]. In this paper, an analytical bound on the computational complexity of the algorithm is derived. Suppose that the algorithm takes K iterations before the termination. As the constraint $g_3(\mathbf{x})$ is imposed on Problem ($\mathbf{P}_f$), a new local minimum of Problem ($\mathbf{P}_f$), which is $\mathbf{x}_k^*$, will not be located at $\tilde{\mathbf{x}}_k$, that is $\mathbf{x}_k^*\ne\tilde{\mathbf{x}}_k$, and the cost value evaluated at the new local minimum will be lower than or equal to $1-\varepsilon$ multiplied to the cost value evaluated at $\tilde{\mathbf{x}}_k$, that is $f(\mathbf{x}_k^*)\le(1-\varepsilon)f(\tilde{\mathbf{x}}_k)$ for $k\le K$. Similarly, as the constraint $g_4(\mathbf{x})$ is imposed on Problem ($\mathbf{P}_H$), a new local minimum of Problem ($\mathbf{P}_H$), which is $\tilde{\mathbf{x}}_{k+1}$, will not be located at $\mathbf{x}_k^*$, that is $\tilde{\mathbf{x}}_{k+1}\ne\mathbf{x}_k^*$, and the cost value evaluated at the new local minimum will be lower than or equal to $1-\varepsilon$ multiplied to the cost value evaluated at $\mathbf{x}_k^*$, that is $f(\tilde{\mathbf{x}}_{k+1})\le(1-\varepsilon)f(\mathbf{x}_k^*)$ for $k\le K$. Hence, we have $f(\tilde{\mathbf{x}}_{k+1})\le(1-\varepsilon)f(\mathbf{x}_k^*)\le(1-\varepsilon)^2 f(\tilde{\mathbf{x}}_k)$ for $k\le K$. This further implies that $\tilde{\mathbf{x}}_k$ for $k\le K$ will not be stuck at local minima of $f(\mathbf{x})$ because $0<1-\varepsilon<1$. Also, we have $f(\tilde{\mathbf{x}}_k)\le(1-\varepsilon)^{2(k-1)}f(\tilde{\mathbf{x}}_1)$ for $k\le K$. Let the global minimum of the optimization problem be $\mathbf{x}^\bullet$, then we have $f(\mathbf{x}^\bullet)\le(1-\varepsilon)^{2(K-1)}f(\tilde{\mathbf{x}}_1)$ for $k\le K$. This implies that $K\le 1-\frac{\log f(\tilde{\mathbf{x}}_1)-\log f(\mathbf{x}^\bullet)}{2\log(1-\varepsilon)}$ and the algorithm always converges. Let $\lceil z\rceil$ be the nearest integer of z such that $\lceil z\rceil\ge z$. Then, the computational complexity of the algorithm is bounded by that required for finding $2\left\lceil 1-\frac{\log f(\tilde{\mathbf{x}}_1)-\log f(\mathbf{x}^\bullet)}{2\log(1-\varepsilon)}\right\rceil$ local minima of the optimization problem.

3. COMPUTER NUMERICAL SIMULATION RESULTS

Since desired phase responses of nonlinear phase peak constrained FIR filters are imposed in existing designs, it is very difficult to have a fair comparison. We intend to compare our works to that presented in [3] because the works presented in [3] are the most related works to our works found in literature.

Both the length and the desired magnitude response of the filter are chosen the same as that in [3] in order to have a fair comparison, that is $N=30$ and

$$D(\omega)=\begin{cases}1 & |\omega|\le 0.12\pi\\ 0 & |\omega|\ge 0.24\pi\end{cases}.$$

As the optimization problem is to minimize the maximum passband group delay of the filter subject to the positivity constraint on the passband group delay response of the filter, the specification on the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response over the passband of the filter is set exactly the same as that over the stopband of the filter. Since there are tradeoffs among the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response, the length, the bandwidth and the center frequency of the filter, $\delta(\omega)$ is set to -34.5 dB for $|\omega|\le 0.12\pi$ and $|\omega|\ge 0.24\pi$. In order to have a good approximation between the nonsmooth functional inequality constrained optimization problem and the corresponding nonconvex functional inequality constrained optimization problem, σ should be small. Here, $\sigma=10^{-6}$ is chosen. Also, to obtain a high accuracy of the obtained global minimum without the termination of our algorithm, both ε and ε' should be small. Here, $\varepsilon=\varepsilon'=10^{-6}$ is chosen. The initial condition $\tilde{\mathbf{x}}_1$ of the global optimization algorithm is obtained based on the method discussed in [2]. Since local minima of nonlinear phase peak constrained FIR filters are usually located very close together, the spreads of the hills of $H(\mathbf{x})$ at $\mathbf{x}_k^*$ should be small and $\mathbf{R}$ is chosen as the diagonal matrix with all diagonal elements equal to 10^{-3}.

Based on the parameters chosen above, it only takes three iterations for the algorithm to terminate. Hence, our proposed method is very efficient. Figures 1 and 2 plot the maximum passband group delay as well as the square root of the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response over the passband and the stopband of the filter designed via our proposed approach. It can be seen from the figures that they are 6.8778, -56.9425 dB and -34.6062 dB, respectively, in which the required constraints are all satisfied. Compared to the results obtained in [3], those values are 12.43898, -26.7653 dB and -44.7822 dB, respectively. Although the performance on the square root of the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response over the stopband of our designed filter is slightly worse than that of [3], both the maximum passband group delay and the square root of the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response over the passband of our designed filter are significantly better than that of [3]. This is because our proposed algorithm could find the global minimum of the nonconvex optimization problem, in which the method discussed in [3] does not optimize the maximum passband group delay of the filter.

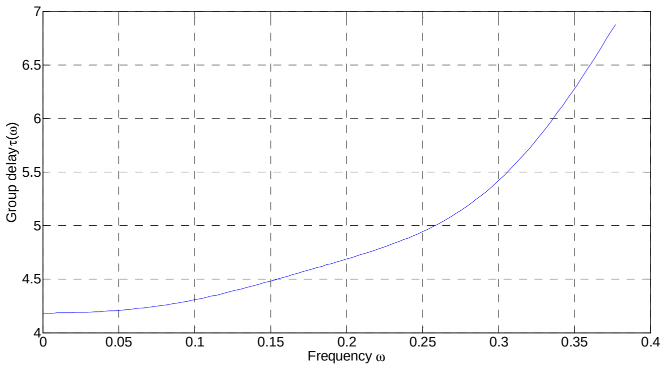
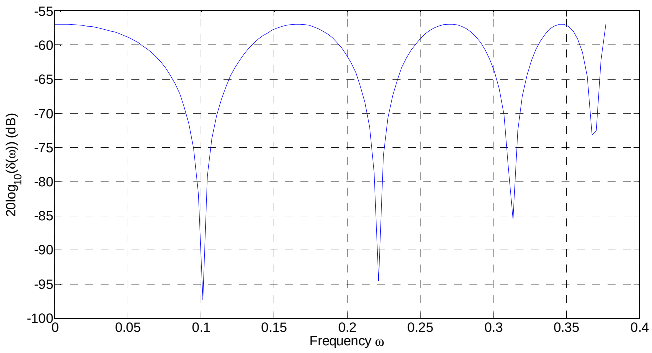


Figure 1. The passband group delay response of our designed nonlinear phase peak constrained FIR filter.



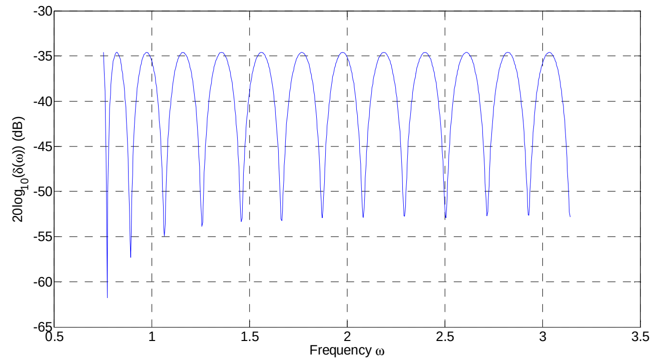


Figure 2. (a) The square root of the maximum absolute difference between the designed magnitude square response and the desirable magnitude square response over the passband of our designed nonlinear phase peak constrained FIR filter; (b) that over the stopband.

4. CONCLUSION

This paper formulates a minimax passband group delay nonlinear phase peak constrained FIR filter design problem as a nonsmooth functional inequality constrained optimization problem. The one norm of the functional inequality constraint of the optimization problem is first approximated by a smooth function so that the nonsmooth functional inequality constrained optimization problem is approximated as a nonconvex functional inequality constrained optimization problem. Then, a modified filled function method is applied for finding the global minimum of the nonconvex optimization problem. Computer numerical simulation results show that our proposed method could efficiently and effectively design a minimax passband group delay nonlinear phase peak constrained FIR filter without imposing a desirable phase response.

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