The "Armey Curve" in Bulgaria (2000-18): Theoretical Considerations and Empirical Results

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Abstract

In this paper we provide a theoretical basis for the so-called "Armey curve," the inverted U-shape relationship between the level of government purchases and GDP growth, named after Armey (1995). We use an otherwise standard Keynesian model, augmented with a quadratic relationship between investment and lagged government expenditure, which was documented empirically. This modelling approach is a useful shortcut that aims to capture the common link shared by both variables, namely their dependence on the real interest rate, as suggested also by the extended static IS-LM model. This resulting dynamic relationship is a newly-documented stylized fact, at least in Bulgarian data for the period 2000-2018, and the source in the extended Keynesian model that generates an Armey curve for Bulgaria.

Keywords: Armey curve, GDP growth, government purchases, Bulgaria

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1 Introduction and Motivation

One of the major postulates and policy recommendations of the standard Keynesian theory is that governments could affect economic activity through the use of fiscal policy. In particular, when the economy is in a recession, the government can stimulate aggregate demand by increasing government purchases, by decreasing taxes, or both. Therefore, the theory predicts that a higher level of government purchases can increase the gross domestic product (GDP) of the economy.

Some authors take those recommendations one step further: In addition to the level effect on output, they argue that there is also a systematic growth effect on output. In other words, a higher level of government purchases could affect the growth rate through more public investment in education, healthcare, infrastructure, etc. However, according to Armey (1995), this growth effect is non-linear, and thus not necessarily a positive one. More precisely, Armey (1995) argues that after some level government spending is harmful for economic growth. The existence of a threshold level of government purchases is then a critical issue as it represents an important constraint for policy and public finance consolidation and austerity plans. Such effects deserve a rigorous treatment in order to be understood in depth.

This non-linear relationship between the level of government purchases and GDP growth was a relatively new stylized fact, named "the Armey curve," even though it had been empirically documented earlier in Engen and Skinner (1992), and later in Sheehey (1993). This finding came into stark contrast with a much older empirical relationship, known as the Wagner's law (1883), which postulated that there is only a positive relationship between government spending and economic growth. The finding at that time was due to the fact that the share of government spending in output was very small compared to current, or post-World-War-II levels.

Despite being linked to the Keynesian theory, the Armey curve was never explicitly derived in a formal manner. Most of the studies in the literature, e.g., Sheehey (1993) and later studies[1], are empirical and are all based on *ad hoc* assumptions. Armey (1995) himself argues verbally why the curve is hump-shaped. We aim to bridge that gap by providing a relevant theoretical basis for the Armey curve. The reason is that the Keynesian model is static in nature, while growth is a dynamic concept. Without a dynamic extension of the model, no Armey curve can arise.

We thus start from an otherwise standard Keynesian framework, and extend it another novel stylized fact: the existence of a quadratic relationship between the level of current private investment, and the level of lagged government purchases. The dynamic inter-relationship introduces dynamics in the model in a simple way. We take this underlying dynamic relationship as an empirical regularity, and incorporate it in the model. As a suitable testing case we use Bulgaria over the period 2000-18: a country that is both an EU member state, but also still developing. We can argue that the link captures the common inter-dependence on the main interest rate, which might be rigid due to the fact that the monetary authority, or the central bank, is focused on price stability, and not on full-employment considerations.[2] With this new mechanism in place, the Keynesian framework is now made dynamic, and we can now generate an inverted U-shape relationship between the level of government purchased and output growth. Yet another advantage of the framework is that we can find the approximate threshold-, or congestion level of public spending, i.e., the level of expenditure that maximizes economic growth.

The rest of the paper is organized as follows: In the next section, we present the model setup calibrated to Bulgarian data, and derive the theoretical Armey curve; the optimal-, or the growth-maximizing level of government purchases is solved determined from both the calibrated theoretical model, as well as from data directly, using the empirical Armey curve. The two cases are compared and contrasted. The paper then concludes.

[1]The interested reader is referred to Afonso and Furceri (2010), Arpaia and Turrini (2007), Dar and Amirkhalkhali (2002), Folster and Henrekson (2001), Gwartney *et al.* (1998), Lin (1994), Sattar (1993), Engen and Skinner (1992), and the references therein.

[2]Another possible explanation could be the dynamic negative effects of high debt levels. For now, we leave this interesting interest rate channel for future study.

2 Model Setup

This section describes a two-period open economy standard Keynesian model with the simple extension, as outlined in the introduction. We need at least 2 periods in the timing of the model in order for the setup to have a dynamic dimension. After all, the Armey curve features economic growth. The results obtained using a 2-period model can be then easily extended to any number of periods.

We begin with the national income accounting identity

$$Y_t = C_t + I_t + G_t + X_t - M_t, \tag{1}$$

where Y_t denotes GDP in period t, C_t is private final consumption spending, I_t denotes investment, G_t are government purchases, X_t are exports, and M_t are imports. In other words, output equals the sum of its uses.

Next, aggregate consumption behavior is assumed to be characterized by a standard Keynesian consumption function:

$$C_t = \bar{C} + b(Y_t - T_t), \tag{2}$$

where $\bar{C} > 0$ denotes the autonomous consumption spending, $0 < b < 1$ is the marginal propensity to consume out of disposable (after-tax) income, and T_t denote lump-sum taxes in period t. Such a relationship has been also documented for Bulgaria in Vasilev (2015b).

Next, in an open economy context, imports are proportional to disposable income, with the degree of proportionality $0 < m < 1$, also referred to as the marginal propensity to imports:

$$M_t = m(Y_t - T_t) \tag{3}$$

In contrast, since exports depend on foreign countries' demand,which is taken to be exogenous in the model, we will set $X_t = \bar{X}$, and keep it unchanged.

For the sake of realism, we set total tax revenue to be proportional to income, or:

$$T_t = tY_t, \tag{4}$$

where $0 < t < 1$ is the average (effective) tax rate. As in Vasilev (2015a), that corresponds to a proportional tax system, where all forms of income - labor, capital, and profit (corporate) income are all taxed at the same rate of 10 percent.[3]

Lastly, the novelty in this paper is that the model will try to capture the (partial) crowding out effect of government purchases, and the fact that more public spending discourages private saving, and leaves less resources for private investment tomorrow.[4] The intertemporal price of those resources is the real interest rate, hence there is a direct link between the two variables, which can be represented after some simple algebra as:

$$I_t = f(G_{t-1}), \quad \text{where} \quad f'(.) > 0, f''(.) < 0. \tag{5}$$

The assumptions imposed on this function are easily verified using data on Bulgaria over the period 2000-18, where all data is from NSI (2019). As documented in Fig. 1 below (where G_{lag} denotes G_{t-1}), a non-linear relationship was established when a quadratic regression specification was fitted through the scatterplot. In addition, the formal regression estimation output is presented in Fig. 2 on the next page. R^2 is 48 percent, which means that the model explains half of the variation in investment, and all variables are statistically significant.

Now that the assumed relationship between investment and lagged government purchases has been empirically verified, we proceed and introduce it into the model framework. In addition, we also need to fix initial investment. Therefore, in the first period, we will assume that private investment is pre-determined, and set to some exogenous level, or $I_1 = \bar{I}$. Therefore, in period 1,

$$Y_1 = \bar{C} + b(Y_1 - tY_1) + \bar{I} + G_1 + \bar{X} - m((Y_1 - tY_1). \tag{6}$$

Similarly, in period 2,

$$Y_2 = \bar{C} + b(Y_2 - tY_2) + f(G_1) + G_2 + \bar{X} - m(Y_2 - tY_2). \tag{7}$$

[3]Given that total revenues are now endogenous, the government will use spending as an instrument to achieve a balanced budget, which is what is observed in Bulgarian data over most of the period 2000-18. With an Armey curve a higher spending feeds into higher future output, and thus higher future tax revenue.

[4]As mentioned earlier, the lagged effect could be driven by some stickiness exhibited in the behavior of the interest rate.

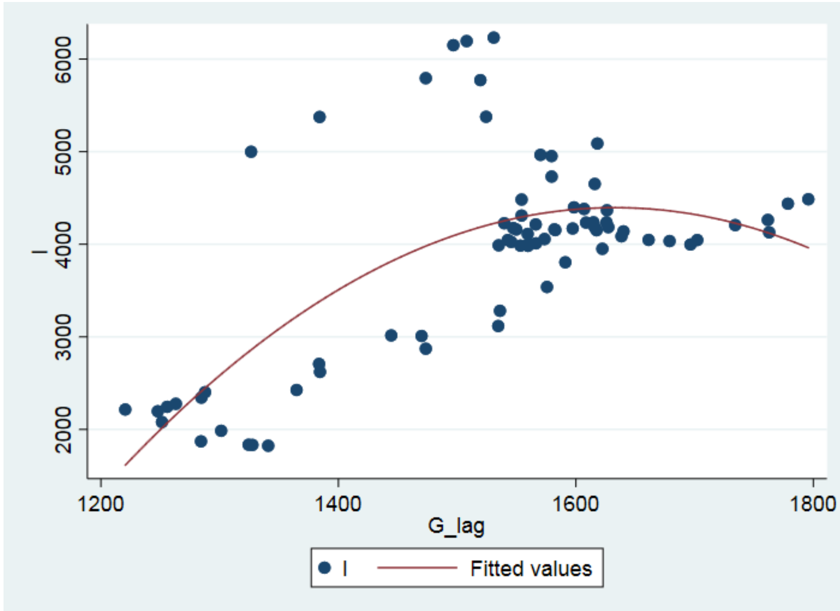

Figure 1: The facts

Source	SS	df	MS			
				Number of obs	=	74
				F(2, 71)	=	33.33
Model	41454837.5	2	20727418.7	Prob > F	=	0.0000
Residual	44151447.4	71	621851.372	R-squared	=	0.4842
				Adj R-squared	=	0.4697
Total	85606284.9	73	1172688.83	Root MSE	=	788.58

I	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
G_lag	53.39736	12.18918	4.38	0.000	29.09281	77.7019
G_lagsq	-.0163497	.0040864	-4.00	0.000	-.0244977	-.0082018
_cons	-39202.68	9035.525	-4.34	0.000	-57219	-21186.35

Figure 2: Regression Output: Investment Function

Differencing output produces the following expression:

$$\Delta Y = Y_2 - Y_1 = b(1-t)\Delta Y + f(G_1) - \bar{I} + G_2 - G_1 - m(1-t)\Delta Y, \tag{8}$$

or

$$\Delta Y = \frac{1}{1-(b-m)(1-t)}[f(G_1) - \bar{I} + G_2 - G_1] \tag{9}$$

Divide now by Y to transform the expression into output growth rate to obtain

$$\frac{\Delta Y}{Y} = \frac{1}{Y}\frac{1}{1-(b-m)(1-t)}[f(G_1) - \bar{I} + G_2 - G_1]. \quad (10)$$

Thus, the effect of government purchases on economic growth equals

$$\partial\left(\frac{\Delta Y}{Y}\right)/\partial G_1 = \frac{1}{Y}\frac{1}{1-(b-m)(1-t)}[f'(G_1) - 1]. \quad (11)$$

In other words, depending on the level of government purchases G_1, the effect of government spending on output growth can be either positive, zero, or negative. For low levels of spending, $f'(.) > 1$, i.e., the demand effect is very large (like it was during the Great Depression), and the effect is positive. In contrast, for large levels of spending, the effect is negative $f'(.) < 1$. There could be also some value for intermediate G_1 for which $f'(G_1) = 1$, so there is zero effect on growth.

We now use the mean level of government spending over the period 2000-18, $G_{avg} = 1526.466$ (in BGN mln.), in Bulgaria, as well as the estimated functional form for $f(.)$ in order to make some computational experiments. In particular

$$f'(G_{avg}) - 1 = 53.397 - 0.016 * 2G_{avg} - 1 = 3.551 > 0. \quad (12)$$

or the effect on growth at the average level of public expenditure on growth has been positive. This result shows that spending can be increased further in order to speed up economic growth. In particular, we can obtain the model-predicted threshold level of government purchases, denoted by $\hat{G}$, that maximizes economic growth by setting

$$f'(\hat{G}) = 1 \quad \text{i.e.,} \quad \hat{G} = 1637.4.$$

In Bulgarian data, we observe such values (in BGN mln.) and above from 2016 onwards, which is an indication that the economy is now operating beyond the peak of the Armey curve, and the government needs to lower the level of government spending.

Alternatively, we can estimate the Armey curve empirically, and obtain the growth-maximizing level of government spending by running the following regression:

$$Growth_{GDP_t} = \gamma_0 + \gamma_1 G_t + \gamma_2 G_t^2 + \nu_t \quad (13)$$

The results are presented in the Fig. 3 below, where g_Y denotes output growth rate, G is the level of government purchases, and Gsq is the square of government purchases. According to the OLS estimates produced, the growth of the economy is maximized at $G^* = 1572.43$ (in BGN mln.), which is lower than the level predicted by the calibrated Keynesian model above. This value corresponds to the level observed in 2012. The qualitative conclusion - that the economy is now operating beyond the peak of the Armey curve, and the government needs to lower the level of government spending - continues to hold. In addition, given the low R^2, any inference based on this regression is to be taken with some caution.

Source	SS	df	MS			
				Number of obs	=	74
				F(2, 71)	=	1.39
Model	.000269636	2	.000134818	Prob > F	=	0.2551
Residual	.006873452	71	.000096809	R-squared	=	0.0377
				Adj R-squared	=	0.0106
Total	.007143087	73	.000097851	Root MSE	=	.00984

g_Y	Coef.	Std. Err.	t	P>|t|	[95% Conf.	Interval]
G	.0002247	.0001645	1.37	0.176	-.0001033	.0005528
Gsq	-7.29e-08	5.54e-08	-1.32	0.193	-1.83e-07	3.76e-08
_cons	-.180432	.1213136	-1.49	0.141	-.4223245	.0614605

Figure 3: Armey curve regression

This result can be also seen from the fitted Armey curve presented in Fig. 4 on the next page. Thus, the presence of a peaking relationship between the level of government spending and economic growth has been established both theoretically and empirically in Bulgaria over the period 2000-18.

3 Conclusions

In this paper we provide a theoretical basis for the so-called "Armey curve," the inverted U-shaped relationship between the level of government purchases and GDP growth, named after Armey (1995). We use an otherwise standard Keynesian model, extended with a quadratic relationship between investment and government expenditure, which is a new documented

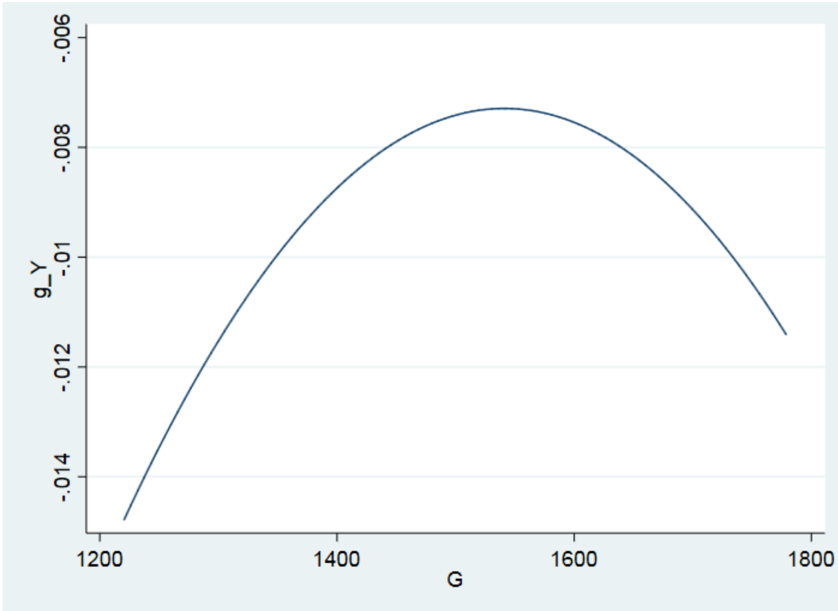

Figure 4: Empirical Armey curve in Bulgaria, 2000-18

stylized fact in Bulgarian data for the period 2000-2018. The link is through the dependence of both on the interest rate. The model is able to generate a realistic Armey curve for Bulgaria through this new transmission channel alone.

As a future extension, we may consider next a dynamic IS-LM model, in order to provide more detail on the interest rate link outlined above. The ambition is eventually to construct a micro-founded New Keynesian general equilibrium model with physical capital, maybe along the lines of Barro (1990) and Easterly and Rebelo (1993), and augmented with sticky prices, in order to understand better the quantitative effect of this new propagation mechanism.

References

Afonso, A. and Furceri, D. (2010) "Government size, composition, volatility and economic growth," *European Journal of Political Economy*, 26(4), p. 517-32.

Armey, R. (1995) *The freedom revolution.* Washington, DC: Rogney Publishing Co.

Arpaia, A. and Turrini, A. (2007) "Government expenditure and economic growth in the EU: long-run tendencies and short-run adjustment," SSRN paper 2004461.

Barro, R.J. (1990) "Government spending in a simple model of economic growth," *Journal of Political Economy* 98, p. 103-125.

Dar, A., and Amirkhalkhali, S. (2002) "Government size, factor accumulation, and economic growth: evidence from OECD countries," *Journal of Policy Modeling* 24, p. 679-692.

Easterly, W. and Rebelo, S. (1993) "Fiscal policy and economic growth," *Journal of Monetary Economics* 32(3), p. 417-458.

Engen, E.M. and Skinner, J.S. (1992) "Fiscal policy and economic growth," *NBER Working Paper.*

Folster, S. and Henrekson, M. (2001) "Growth effects of government expenditure and taxation in rich countries," *European Economic Review* 45(8), p. 1501-1520.

Gwartney, J., Holcombe, R., and Lawson, R. (1998) "Scope of Government and the Wealth of Nations," *The Cato Journal* 18, p. 163.

Lin, S.A. (1994) "Government spending and economic growth," *Applied Economics* 26(1), p. 83-94.

National Statistical Institute (2019) *Aggregate Statistical Indicators.* Available on-line at *www.nsi.bg.* Accessed on Feb. 28, 2019.

Sheehey, E.J. (1993) "The Effect of government size on economic growth," *Eastern Economic Journal*, p.321-328.

Sattar, Z. (1993) "Public expenditure and economic performance: A comparison of developed and low-income developing countries," *Journal of International Development* 5(1), p. 27-49.

Vasilev, A. (2015a) "Welfare effects of at income tax reform: the case of Bulgaria," *Eastern European Economics* 53(2): 205-220.

Vasilev, A. (2015b) "Modeling Real Private Consumption Expenditure in Bulgaria after the Currency Board Implementation (1997-2005)," *Zagreb International Review of Economics and Business*, vol. 18(1), pages 81-89.

Wagner, A. (1883) *Finanzwissenschaft*, 2nd and 3rd., Leipzig. Partly reprinted in RA Musgrave and AT Peacock, Eds. (1958). *Classics in the Theory of Public Finance.*