

# Recent Works on Optimization for Image and Video Signal Processing

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# Outline

- ❖ Motion Estimations
- ❖ Digital Forensics
- ❖ Conclusions
- ❖ References
- ❖ Questions and Answers

# Motion Estimations

- ❖ Format of digital videos
  - ⌘ Consists of frames of digital images.
- ❖ Why motion estimations?
  - ⌘ There are lots of redundancy among consecutive frames of video sequences, such as background and moving objects.
  - ⌘ By estimating the motions of moving objects in the current frame, an estimated digital image in the next frame can be constructed. The exact digital image in the next frame is characterized by the residue error and the motion vectors.
  - ⌘ By coding the residue error and the motion vectors, very high compression ratio could be achieved for most of digital videos.

# Motion Estimations

- ❖ Existing techniques for motion estimations
  - ⌘ Full integer pixel search
    - ❖ Estimate the motion vectors at all integer pixel locations.
  - ⌘ Half pixel search
    - ❖ Estimate the motion vectors at all integer and half pixel locations.
  - ⌘ Quarter pixel search
    - ❖ Estimate the motion vectors at all integer, half and quarter pixel locations.

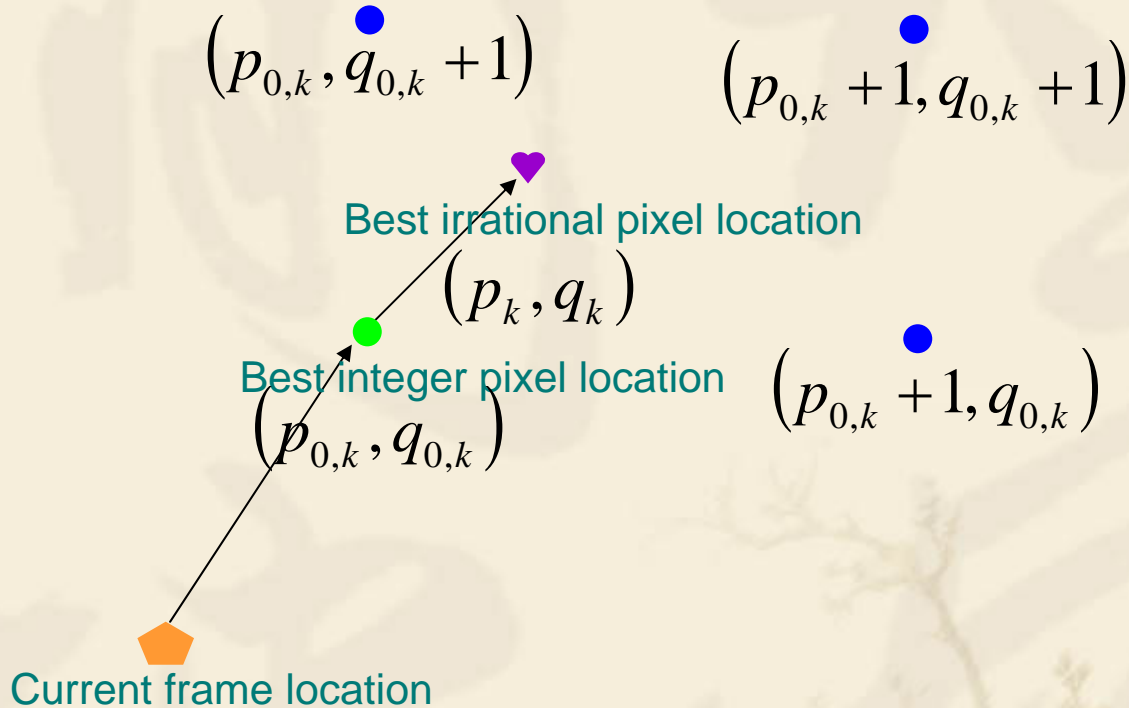


# Motion Estimations

- ❖ Drawbacks of existing motion estimation techniques
  - ⌘ Computational efforts of existing algorithms are very heavy.
  - ⌘ Existing pixel search algorithms could only achieve motion vectors with rational pixel precisions.
- ❖ Problem statement
  - ⌘ Derive optimal motion vectors with irrational pixel precisions in a single step.

# Motion Estimations

## ❖ Nonlinear Block Matched Motion Models



∞ Note that

$$(1 - p_k)(1 - q_k) + p_k(1 - q_k) + (1 - p_k)q_k + p_k q_k = 1$$

# Motion Estimations

## ❖ Nonlinear Block Matched Motion Models

### ∞ Left lower direction

$$\tilde{B}_{k,p_k,q_k}^{LL}(x,y) \equiv (1-p_k)(1-q_k)B_k(x+p_{0,k},y+q_{0,k}) + (1-p_k)q_kB_k(x+p_{0,k}-1,y+q_{0,k}) \\ + p_k(1-q_k)B_k(x+p_{0,k},y+q_{0,k}+1) + p_kq_kB_k(x+p_{0,k}-1,y+q_{0,k}+1)$$

### ∞ Upper lower direction

$$\tilde{B}_{k,p_k,q_k}^{UL}(x,y) \equiv (1-p_k)(1-q_k)B_k(x+p_{0,k},y+q_{0,k}) + (1-p_k)q_kB_k(x+p_{0,k}+1,y+q_{0,k}) \\ + p_k(1-q_k)B_k(x+p_{0,k},y+q_{0,k}+1) + p_kq_kB_k(x+p_{0,k}+1,y+q_{0,k}+1)$$

### ∞ Right lower direction

$$\tilde{B}_{k,p_k,q_k}^{LR}(x,y) \equiv (1-p_k)(1-q_k)B_k(x+p_{0,k},y+q_{0,k}) + (1-p_k)q_kB_k(x+p_{0,k}-1,y+q_{0,k}) \\ + p_k(1-q_k)B_k(x+p_{0,k},y+q_{0,k}-1) + p_kq_kB_k(x+p_{0,k}-1,y+q_{0,k}-1)$$

### ∞ Right upper direction

$$\tilde{B}_{k,p_k,q_k}^{UR}(x,y) \equiv (1-p_k)(1-q_k)B_k(x+p_{0,k},y+q_{0,k}) + (1-p_k)q_kB_k(x+p_{0,k}+1,y+q_{0,k}) \\ + p_k(1-q_k)B_k(x+p_{0,k},y+q_{0,k}-1) + p_kq_kB_k(x+p_{0,k}+1,y+q_{0,k}-1)$$



# Motion Estimations

## ❖ Objectives

- ∞ Minimize the mean square error between the translated block in the current frame and the reference block in the next frame.

$$MSE_k(p_k, q_k) \equiv \min \left\{ \begin{array}{l} \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left| \tilde{B}_{k,|p_k|,|q_k|}^{UL}(x, y) - B_{k+1}(x, y) \right|^2, \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left| \tilde{B}_{k,|p_k|,|q_k|}^{UR}(x, y) - B_{k+1}(x, y) \right|^2, \\ \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left| \tilde{B}_{k,|p_k|,|q_k|}^{LL}(x, y) - B_{k+1}(x, y) \right|^2, \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left| \tilde{B}_{k,|p_k|,|q_k|}^{LR}(x, y) - B_{k+1}(x, y) \right|^2 \end{array} \right\}$$

# Motion Estimations

## ❖ Solution Methods

$$\begin{aligned}
 MSE_k^{UL}(p_k, q_k) &\equiv \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left( (1-p_k)(1-q_k)B_k(x+p_{0,k}, y+q_{0,k}) + (1-p_k)q_k B_k(x+p_{0,k}+1, y+q_{0,k}) \right. \\
 &\quad \left. + p_k(1-q_k)B_k(x+p_{0,k}, y+q_{0,k}+1) + p_k q_k B_k(x+p_{0,k}+1, y+q_{0,k}+1) - B_{k+1}(x, y) \right)^2 \\
 \\
 \frac{\partial MSE_k^{UL}(p_k, q_k)}{\partial p_k} &= \frac{2}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left( \begin{aligned}
 & \left( p_k q_k (B_k(x+p_{0,k}, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}+1, y+q_{0,k}) + B_k(x+p_{0,k}+1, y+q_{0,k}+1)) \right. \\
 & + p_k (B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}, y+q_{0,k})) \\
 & + q_k (B_k(x+p_{0,k}+1, y+q_{0,k}) - B_k(x+p_{0,k}+1, y+q_{0,k}+1)) \\
 & \left. + B_k(x+p_{0,k}, y+q_{0,k}) - B_{k+1}(x, y) \right) \\
 & \left( q_k (B_k(x+p_{0,k}, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}+1, y+q_{0,k}) + B_k(x+p_{0,k}+1, y+q_{0,k}+1)) \right. \\
 & \left. + B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}, y+q_{0,k}) \right)
 \end{aligned} \right) \\
 \\
 &= p_k q_k^2 \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \frac{2}{N^2} (B_k(x+p_{0,k}, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}+1, y+q_{0,k}) + B_k(x+p_{0,k}+1, y+q_{0,k}+1))^2 \\
 &+ p_k q_k \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \frac{4}{N^2} (B_k(x+p_{0,k}, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}+1, y+q_{0,k}) + B_k(x+p_{0,k}+1, y+q_{0,k}+1)) \\
 &\quad (B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}, y+q_{0,k})) \\
 &+ p_k \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \frac{2}{N^2} (B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}, y+q_{0,k}))^2 \\
 &+ q_k^2 \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \frac{2}{N^2} (B_k(x+p_{0,k}+1, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k})) \\
 &\quad (B_k(x+p_{0,k}, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}+1, y+q_{0,k}) + B_k(x+p_{0,k}+1, y+q_{0,k}+1)) \\
 &+ q_k \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \frac{2}{N^2} \left( \begin{aligned}
 & (B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}, y+q_{0,k})) (B_k(x+p_{0,k}+1, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k})) \\
 & + (B_k(x+p_{0,k}, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}+1, y+q_{0,k}) + B_k(x+p_{0,k}+1, y+q_{0,k}+1)) \\
 & (B_k(x+p_{0,k}, y+q_{0,k}) - B_{k+1}(x, y))
 \end{aligned} \right) \\
 &+ \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \frac{2}{N^2} (B_k(x+p_{0,k}, y+q_{0,k}) - B_{k+1}(x, y)) (B_k(x+p_{0,k}, y+q_{0,k}+1) - B_k(x+p_{0,k}, y+q_{0,k}))
 \end{aligned}$$

# Motion Estimations

## ❖ Solution Methods

$$\frac{\partial MSE_k^{UL}(p_k, q_k)}{\partial p_k} = p_k (c_{k,pq^2} q_k^2 + c_{k,pq} q_k + c_{k,p}) + c_{k,q^2} q_k^2 + c_{k,q} q_k + c_k$$

$$\frac{\partial MSE_k^{UL}(p_k, q_k)}{\partial q_k} = q_k (z_{k,qp^2} p_k^2 + z_{k,qp} p_k + z_{k,q}) + z_{k,p^2} p_k^2 + z_{k,p} p_k + z_k$$

$$\left. \frac{\partial MSE_k^{UL}(p_k, q_k)}{\partial p_k} \right|_{(p_k, q_k) = (p_k^{UL*}, q_k^{UL*})} = 0 \quad \text{and} \quad \left. \frac{\partial MSE_k^{UL}(p_k, q_k)}{\partial q_k} \right|_{(p_k, q_k) = (p_k^{UL*}, q_k^{UL*})} = 0$$

$$p_k^{UL*} = - \frac{c_{k,q^2} q_k^{UL*2} + c_{k,q} q_k^{UL*} + c_k}{c_{k,pq^2} q_k^{UL*2} + c_{k,pq} q_k^{UL*} + c_{k,p}}$$

$$q_k^{UL*} \left( z_{k,qp^2} \left( - \frac{c_{k,q^2} q_k^{UL*2} + c_{k,q} q_k^{UL*} + c_k}{c_{k,pq^2} q_k^{UL*2} + c_{k,pq} q_k^{UL*} + c_{k,p}} \right)^2 + z_{k,qp} \left( - \frac{c_{k,q^2} q_k^{UL*2} + c_{k,q} q_k^{UL*} + c_k}{c_{k,pq^2} q_k^{UL*2} + c_{k,pq} q_k^{UL*} + c_{k,p}} \right) + z_{k,q} \right) + z_{k,p^2} \left( - \frac{c_{k,q^2} q_k^{UL*2} + c_{k,q} q_k^{UL*} + c_k}{c_{k,pq^2} q_k^{UL*2} + c_{k,pq} q_k^{UL*} + c_{k,p}} \right)^2 + z_{k,p} \left( - \frac{c_{k,q^2} q_k^{UL*2} + c_{k,q} q_k^{UL*} + c_k}{c_{k,pq^2} q_k^{UL*2} + c_{k,pq} q_k^{UL*} + c_{k,p}} \right) + z_k = 0$$

# Motion Estimations

## ❖ Solution Methods

$$MSE_k^{UL}(0, q_k) = \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left( \begin{array}{l} (1-p_k)(1-q_k)B_k(x+p_{0,k}, y+q_{0,k}) \\ + (1-p_k)q_k B_k(x+p_{0,k}+1, y+q_{0,k}) \\ + p_k(1-q_k)B_k(x+p_{0,k}, y+q_{0,k}+1) \\ + p_k q_k B_k(x+p_{0,k}+1, y+q_{0,k}+1) \\ - B_{k+1}(x, y) \end{array} \right)^2 \Big|_{p_k=0}$$

$$= \frac{1}{N^2} \sum_{x=0}^{N-1} \sum_{y=0}^{N-1} \left( q_k (B_k(x+p_{0,k}+1, y+q_{0,k}) - B_k(x+p_{0,k}, y+q_{0,k})) + B_k(x+p_{0,k}, y+q_{0,k}) - B_{k+1}(x, y) \right)^2$$

$$\frac{\partial MSE_k^{UL}(0, q_k)}{\partial q_k} = \tilde{c}_{k,0,q} q_k + \tilde{c}_{k,0}$$

$$\frac{\partial MSE_k^{UL}(0, q_k)}{\partial q_k} \Big|_{q_k = \tilde{q}_k^{0,UL}} = 0$$

$$\tilde{q}_k^{0,UL} = -\frac{\tilde{c}_{k,0}}{\tilde{c}_{k,0,q}}$$

$$\tilde{F}_{k,0,q}^{UL} \equiv \left\{ \left( 0, -\frac{\tilde{c}_{k,0}}{\tilde{c}_{k,0,q}} \right) \right\}$$



# Motion Estimations

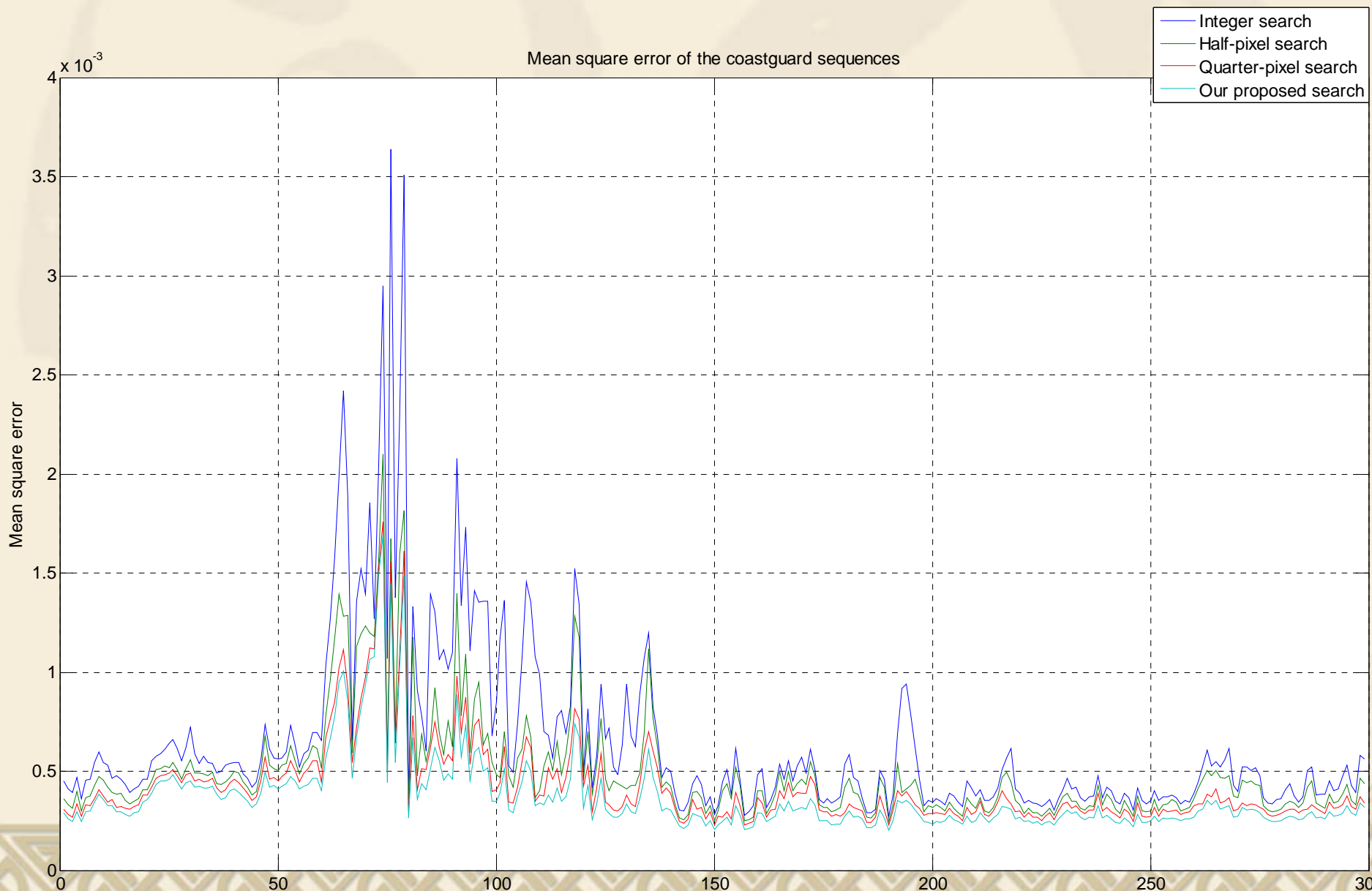
## ❖ Solution Methods

$$F_k^{UL} \equiv \tilde{F}_{k,0,q}^{UL} \cup \tilde{F}_{k,1,q}^{UL} \cup \tilde{F}_{k,0,p}^{UL} \cup \tilde{F}_{k,1,p}^{UL} \cup \{(0,0), (p_k^{UL*}, q_k^{UL*})\}$$
$$(p_k^*, q_k^*) \equiv \arg \left\{ \begin{array}{l} \arg \left\{ \min_{(p_k, q_k) \in F_k^{UL}} MSE_k^{UL}(p_k, q_k) \right\}, \arg \left\{ \min_{(p_k, q_k) \in F_k^{UR}} MSE_k^{UR}(p_k, q_k) \right\}, \\ \arg \left\{ \min_{(p_k, q_k) \in F_k^{LL}} MSE_k^{LL}(p_k, q_k) \right\}, \arg \left\{ \min_{(p_k, q_k) \in F_k^{LR}} MSE_k^{LR}(p_k, q_k) \right\} \end{array} \right\}$$



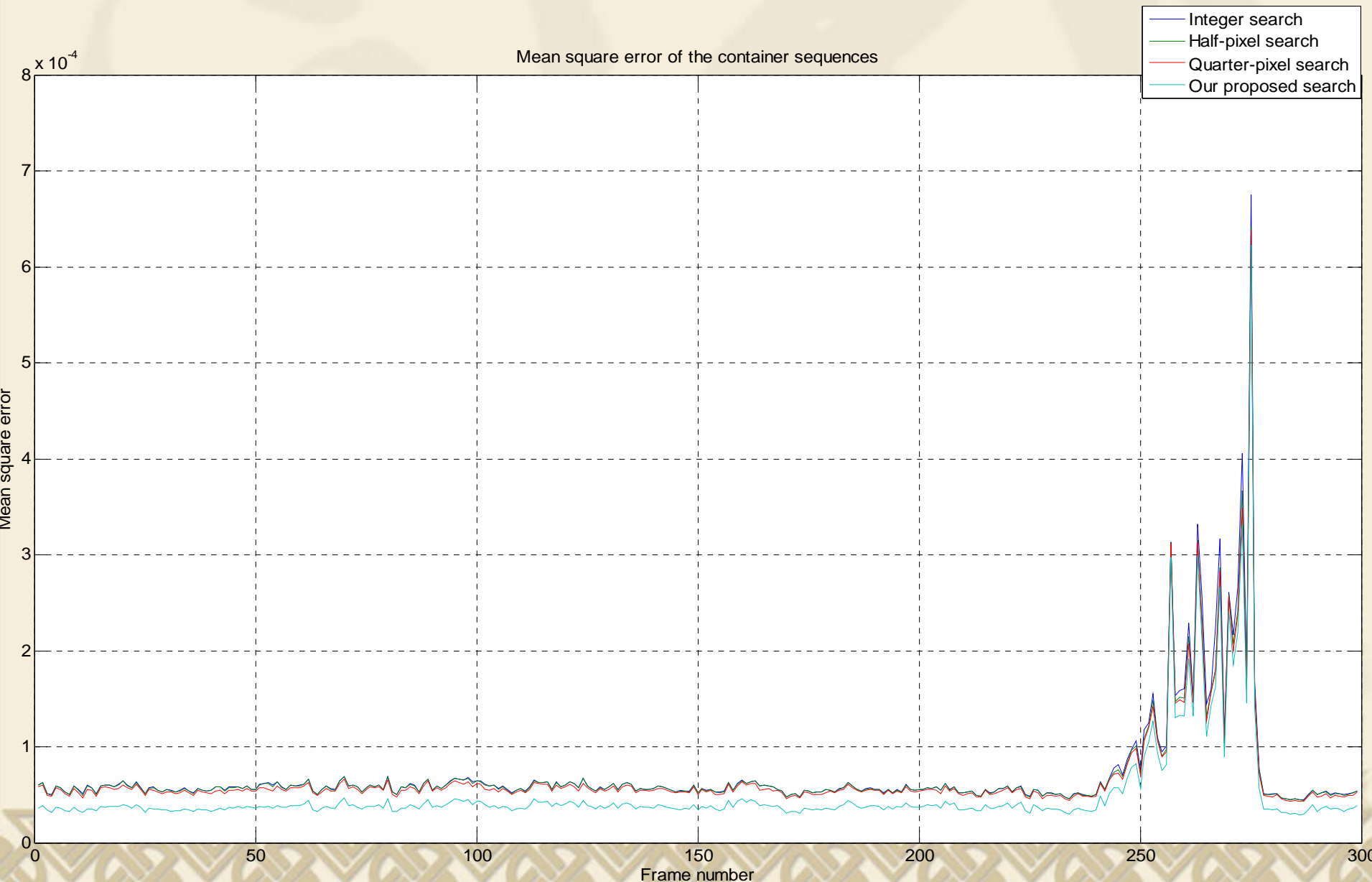
# Motion Estimations

## ❖ Computer Numerical Simulation Results (8×8)



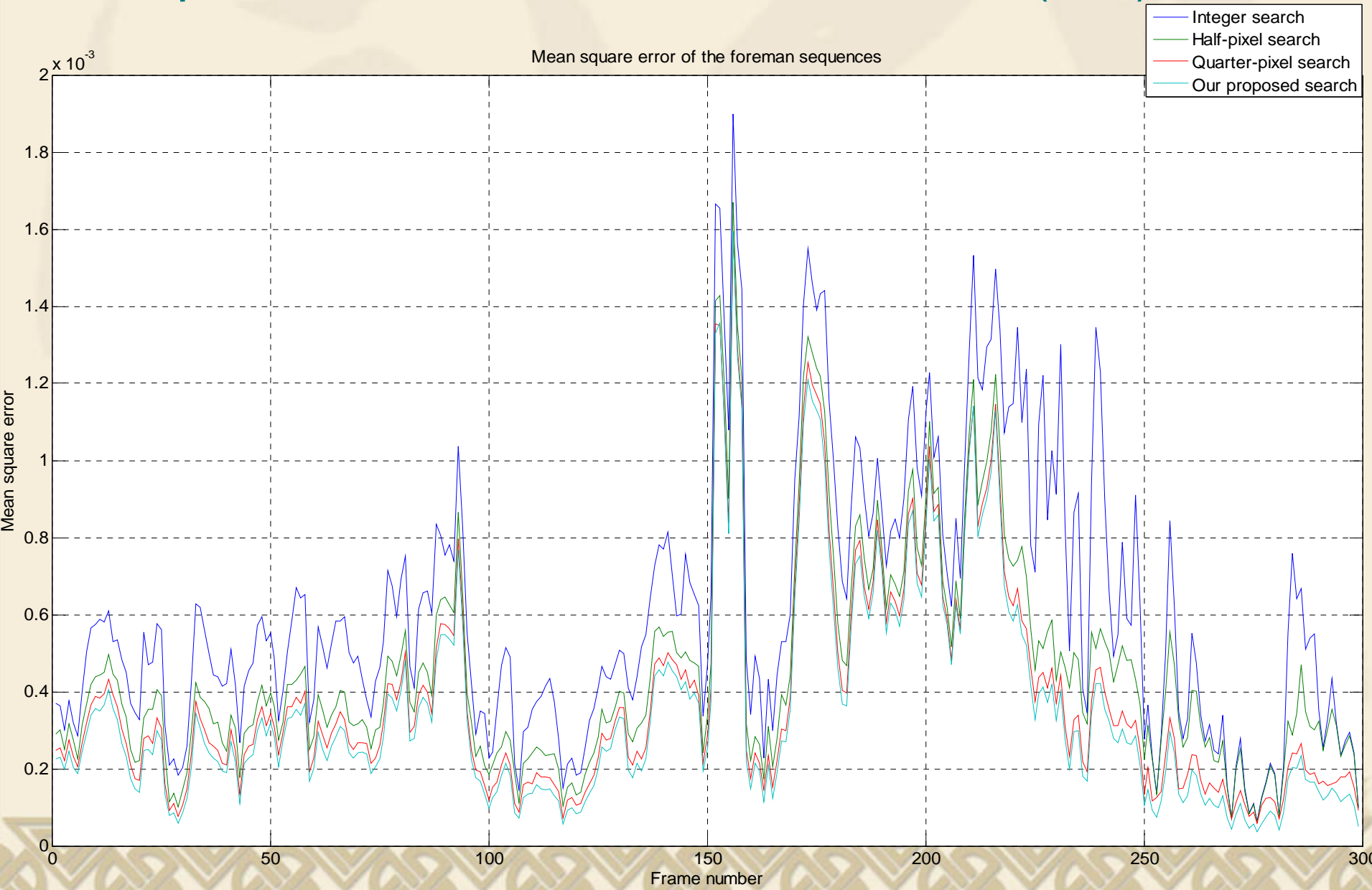
# Motion Estimations

## ❖ Computer Numerical Simulation Results (8×8)



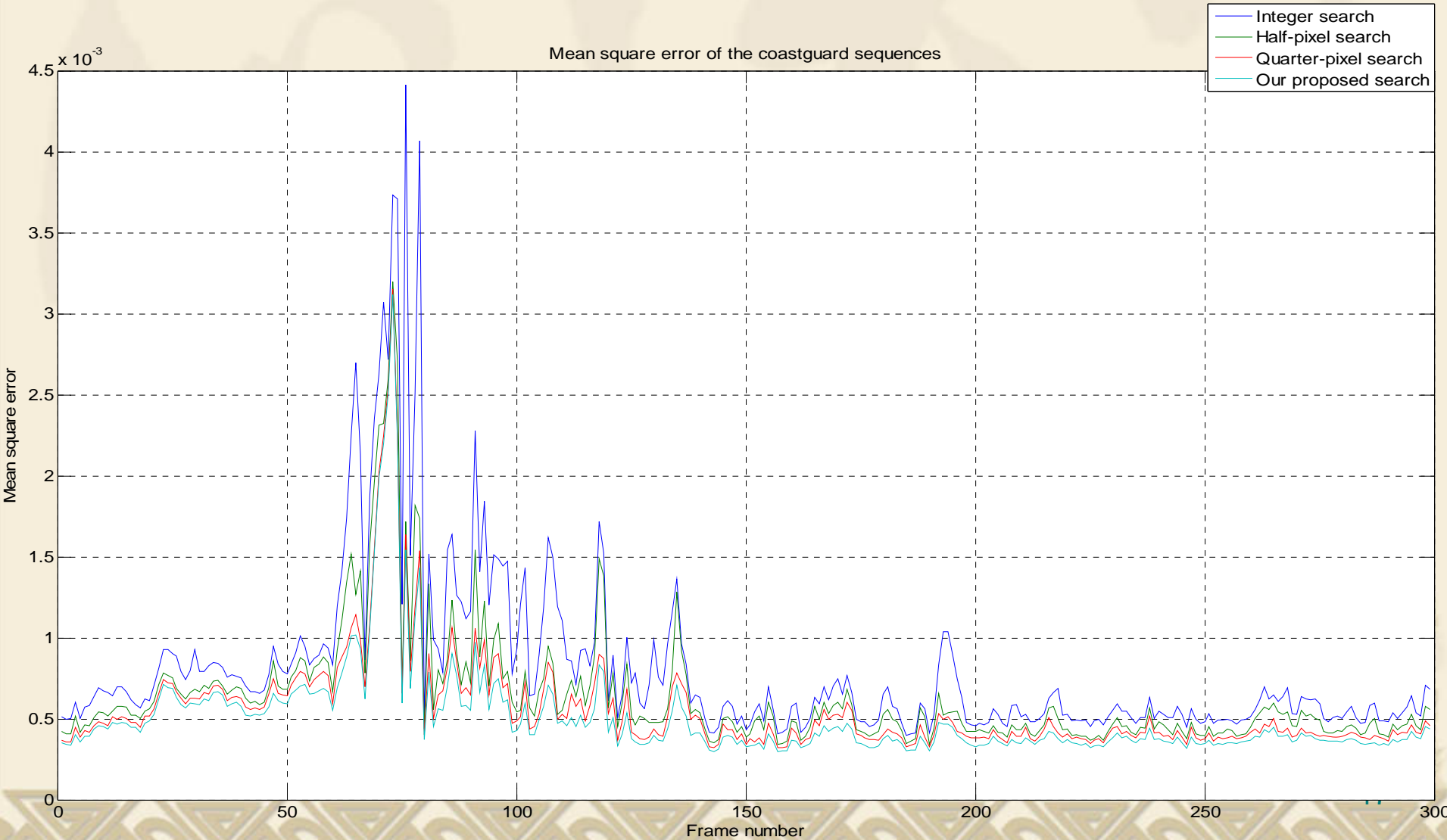
# Motion Estimations

## ❖ Computer Numerical Simulation Results (8x8)



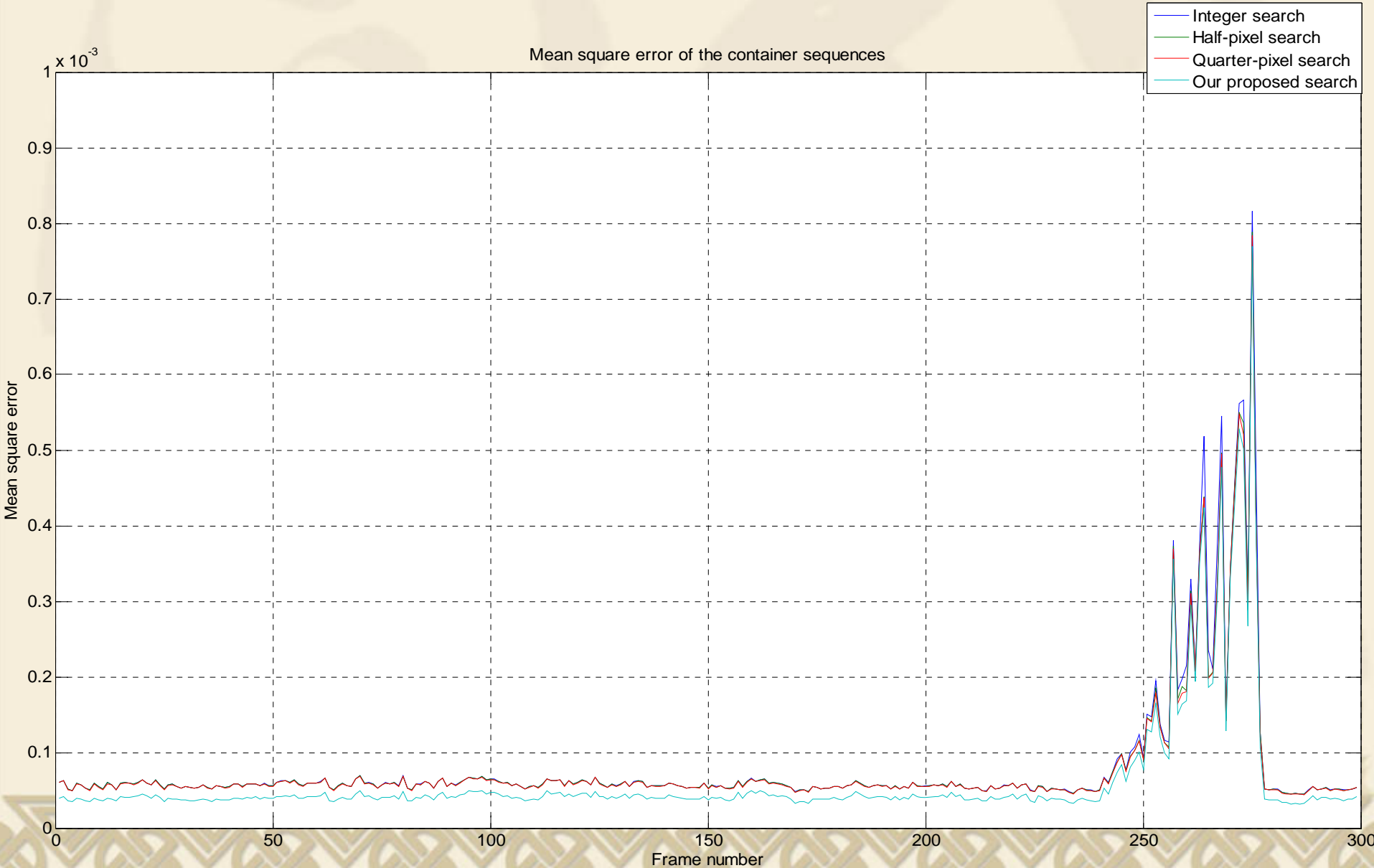
# Motion Estimations

## ❖ Computer Numerical Simulation Results (16×16)



# Motion Estimations

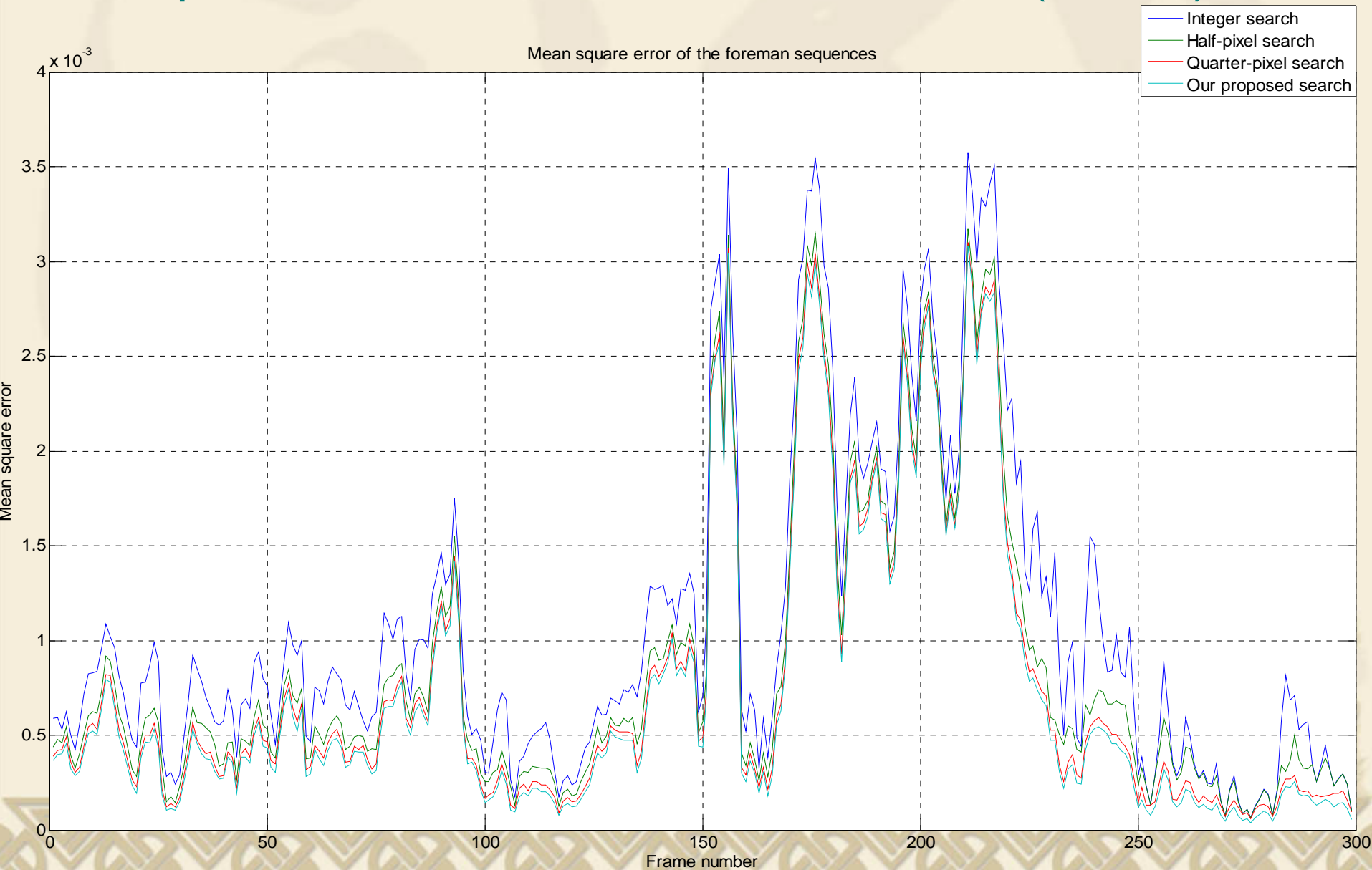
## ❖ Computer Numerical Simulation Results (16×16)





# Motion Estimations

## ❖ Computer Numerical Simulation Results (16×16)



# Digital Forensics

- ❖ What is digital forensics?
  - ⌘ A branch of forensic science pertaining to legal evidence found in computers and digital storage media.
- ❖ Interests in digital forensics
  - ⌘ **Device identification**
  - ⌘ Device linking
  - ⌘ Recovery of processing history
  - ⌘ Detection of digital forgeries

# Digital Forensics

## ❖ Problem statement

- ❧ Identify the corresponding mobile handsets based on images and video sequences downloaded from facebook or internet.

## ❖ Assumption

- ❧ Images and video sequences downloaded from facebook or internet are taken from mobile handsets.

## ❖ Technique involved

- ❧ Optimization
- ❧ Statistical signal processing
- ❧ Neural networks
- ❧ Pattern recognition
- ❧ Image and video signal processing

# Digital Forensics

## ❖ Working Principles

- ❧ Each mobile handset has its own noise profile.
- ❧ By computing the correlation coefficient between the noise profile of two different sets of mobile handsets, two probability density functions are obtained.
- ❧ Recognition and classification can be performed.

# Digital Forensics

## ❖ Open Problems in Digital Forensics

- ❧ The effects of different video frames (I frames, B frames and P frames) on the probability density functions are unknown.
- ❧ The effects of different video contents (fast video motion sequences, slow video motion sequences, flat field motion sequences and complete black video motion sequences) on the probability density functions are unknown.
- ❧ The effects of different video formats (avi, 3gpp and mpeg) on the probability density functions are unknown.
- ❧ The effects of different image and video signal processing techniques (resizing and cropping) on the probability density functions are unknown.



# Digital Forensics

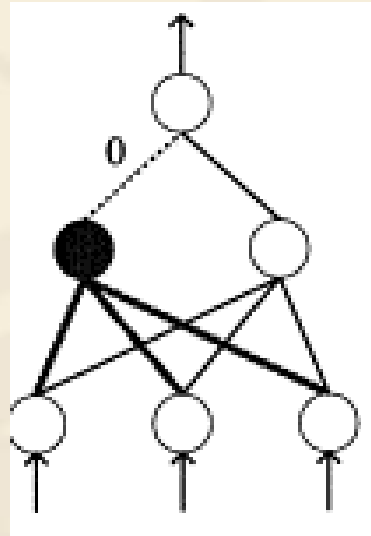
## ❖ Open Problems in Digital Forensics

- ❧ The effects of different colour planes (red, green and blue) on the probability density functions are unknown.
- ❧ The thresholds on the probability density functions for classifying different types of mobile handsets are unknown.
- ❧ Whether new techniques, such as using multi-layer perceptrons, could improve the classification rate is unknown.

# Digital Forensics

## ❖ Optimization for Digital Forensics

- ∞ Design a multi-layer perceptron for the classification of mobile handsets.
- ∞ Structure of multi-layer perceptron



# Digital Forensics

## ❖ Optimization for Digital Forensics

∞ Output at the first layer of the multi-layer perceptron:

$$\mathbf{y}_i = \mathbf{f}(\mathbf{W}\mathbf{x}_i)$$

∞ Objective: minimize the interclass separation and maximize the intraclass separation

$$\min_{\mathbf{W}} J'(\mathbf{W}) \equiv \frac{\sum_{\mathbf{x}_i \in S_0} \left\| \mathbf{f}(\mathbf{W}\mathbf{x}_i) - \frac{1}{N_0} \sum_{\mathbf{x}_i \in S_0} \mathbf{f}(\mathbf{W}\mathbf{x}_i) \right\|^2 + \sum_{\mathbf{x}_j \in S_1} \left\| \mathbf{f}(\mathbf{W}\mathbf{x}_j) - \frac{1}{N_1} \sum_{\mathbf{x}_j \in S_1} \mathbf{f}(\mathbf{W}\mathbf{x}_j) \right\|^2}{\left\| \frac{1}{N_0} \sum_{\mathbf{x}_i \in S_0} \mathbf{f}(\mathbf{W}\mathbf{x}_i) - \frac{1}{N_1} \sum_{\mathbf{x}_j \in S_1} \mathbf{f}(\mathbf{W}\mathbf{x}_j) \right\|^2}$$

# Digital Forensics

## ❖ Optimization for Digital Forensics

∞ Define  $\mathbf{W} \equiv \begin{bmatrix} \mathbf{w}_1^T \\ \vdots \\ \mathbf{w}_N^T \end{bmatrix}$  and  $\mathbf{w} \equiv \begin{bmatrix} \mathbf{w}_1 \\ \vdots \\ \mathbf{w}_N \end{bmatrix}$

∞ Then  $\mathbf{W}\mathbf{x}_i = \text{diag}(\mathbf{x}_i^T, \dots, \mathbf{x}_i^T)\mathbf{w}$

$$\min_{\mathbf{w}} J(\mathbf{w}) \equiv \frac{\sum_{\mathbf{x}_i \in S_0} \left\| \mathbf{f}(\text{diag}(\mathbf{x}_i^T, \dots, \mathbf{x}_i^T)\mathbf{w}) - \frac{1}{N_0} \sum_{\mathbf{x}_i \in S_0} \mathbf{f}(\text{diag}(\mathbf{x}_i^T, \dots, \mathbf{x}_i^T)\mathbf{w}) \right\|^2 + \sum_{\mathbf{x}_j \in S_1} \left\| \mathbf{f}(\text{diag}(\mathbf{x}_j^T, \dots, \mathbf{x}_j^T)\mathbf{w}) - \frac{1}{N_1} \sum_{\mathbf{x}_j \in S_1} \mathbf{f}(\text{diag}(\mathbf{x}_j^T, \dots, \mathbf{x}_j^T)\mathbf{w}) \right\|^2}{\left\| \frac{1}{N_0} \sum_{\mathbf{x}_i \in S_0} \mathbf{f}(\text{diag}(\mathbf{x}_i^T, \dots, \mathbf{x}_i^T)\mathbf{w}) - \frac{1}{N_1} \sum_{\mathbf{x}_j \in S_1} \mathbf{f}(\text{diag}(\mathbf{x}_j^T, \dots, \mathbf{x}_j^T)\mathbf{w}) \right\|^2}$$

# Digital Forensics

## ❖ Optimization for Digital Forensics

∞ Output at the second layer of the multi-layer perceptron:

$$Q(\tilde{\mathbf{w}}^T \mathbf{y}_i)$$

∞ Objective: maximize the separation between two different classes:

subject to 
$$\begin{aligned} & \max_{\tilde{\mathbf{w}}} \delta \\ & t_i \mathbf{y}_i^T \tilde{\mathbf{w}} \geq \delta \end{aligned}$$



# Conclusions

- ❖ Many image and video signal processing problems can be formulated as optimization problems.
- ❖ Solving these optimization problems could improve performances of the corresponding image and video signal processing systems. Hence, it is important to the image and video signal processing community.

# References

- ❖ [1] Charlotte Yuk-Fan Ho, Bingo Wing-Kuen Ling, Blasi S. Giovanni, Zhi-Wei Chi and Wan-Chi Siu, “Single Step Optimal Block Matched Motion Estimation with Motion Vectors Having Arbitrary Pixel Precisions,” *International Symposium on Communication Systems, Networks, and Digital Signal Processing, CSNDSP*, pp. 364-374, 21-23 July, 2010. (Newcastle).
- ❖ [2] Jessica Fridrich, “Digital Image Forensics,” *IEEE Signal Processing Magazine*, vol. 26, no. 2, pp. 26-37, 2009.

# Questions and Answers

