Prospect Theory in a Dynamic Game: Theory and Evidence from Online Pay-Per-Bid Auctions

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Abundant evidence exists that expected utility theory does not adequately describe decision making under risk. Although prospect theory is a popular alternative, it is rarely applied in strategic situations in which risk arises through individual interactions. This study fills this research gap by incorporating prospect theory preferences into a dynamic game theoretic model. Using a large field data set from multiple online pay-per-bid auction sites, the authors empirically show that their proposed model with prospect theory preferences makes a better out-of-sample prediction than a corresponding expected utility model. Prospect theory also provides a unified explanation for two behavioral anomalies: average auctioneer revenues above current retail prices and the sunk cost fallacy. The empirical results indicate that bidders are loss averse and overweight small probabilities, such that the expected revenue of the auction exceeds the current retail price by 25.46%. The authors illustrate and empirically confirm a managerial implication for how an auctioneer can increase revenue by changing the details of the auction design.

Keywords: internet auction, prospect theory, dynamic game, pricing, penny auction

JEL classification: D44, C73, D81, D91

1. Introduction

As early as Allais (1953), scholars have noted that expected utility theory (EUT) does not adequately describe decision making under risk. The leading alternative to EUT, (cumulative) prospect theory (PT; Kahneman and Tversky 1979; Tversky and Kahneman 1992), assumes that people evaluate gains differently from losses: a loss hurts a person more than a gain of the same size benefits him or her. Furthermore, the weights that a person attributes to outcomes of risky decisions do not coincide with the probabilities of the respective outcomes. This probability weighting typically leads to overweighting of small probabilities and underweighting of large probabilities.

Extant research contains many successful applications of PT, especially in finance, insurance, and betting markets (for overviews of this literature, see Barberis 2013; Camerer 2000). With the exception of Goeree et al. (2003), however, PT studies are similar in that the risk that people face is exogenous. In many real-world situations, in contrast, risk does not come from an exogenously given random device but rather is a result of the interaction of one person with others.

A typical example is a meeting dedicated to find a volunteer for a public service, say, chairing a department. The meeting continues until (at least) one participant of the meeting volunteers. Each participant prefers to free ride over volunteering but dislikes the additional time that must be spent in the meeting to find a department chair. Thus, the decision of each participant to volunteer at a given point in time depends on the probability that one of the other participants will volunteer. This probability is not exogenous; it is determined by the strategies of the other participants in the meeting. This example is known as the dynamic volunteer’s dilemma (Bliss and Nalebuff 1984; Otsubo and Rapoport 2008). It is a typical case of a (general) war of attrition (Bulow and Klemperer 1999). Other applications of the war of attrition include firms that compete to serve a natural monopoly market (Tirole 1988, p. 311f), bargaining (Abreu and Gul 2000), and election campaigns. However, despite the common view that PT is better than EUT at describing decision making under risk and the prevalence of situations in which risk arises through people's interactions, only a few studies apply PT to these situations.

The aim of this study is to propose and empirically apply a dynamic game theoretic model that accounts for endogenous risk that arises from the interaction between people and to use PT preferences for describing their behavior (hereinafter referred to as the "PT model") to adequately capture their decision making under risk. In developing the PT model, we show (1) a solution strategy for incorporating PT into a dynamic game theoretic model and its empirical applicability to field data, (2) the usefulness of PT in describing people's behavior, and (3) managerial implications that result from using PT over the more commonly used EUT.

Several factors explain why it is unclear whether PT as a descriptive theory of decision making under risk extends to situations in which risk arises through individual interaction, as in the preceding dynamic volunteer's dilemma example. First, in a strategic interaction, it is not sufficient that people themselves act according to PT; to form correct beliefs about other people's actions, they must recognize that other people's behavior is also best described by PT. Second, we cannot rely on the solution concepts commonly used to analyze strategic situations theoretically, because when departing from EUT, Nash equilibrium may fail to exist (Dekel et al. 1991). Moreover, behavior of people with PT preferences is not necessarily dynamically consistent (Dekel et al. 1991; Machina 1989). Consequently, a subgame-perfect Nash equilibrium (SPNE) does typically not exist.

For the empirical application, we use pay-per-bid auctions—another variant of the war of attrition. These auctions provide an effective testing ground for PT for several reasons. First, under EUT, pay-per-bid auctions have a SPNE in behavioral strategies; thus, the entire risk that a bidder faces is due to the strategies adopted by other bidders.[1] Second, online pay-per-bid auctions show two bidding behavior anomalies that challenge the application of EUT: (1) average auctioneer revenues well above the current retail price (CRP) and (2) the sunk cost fallacy.[2] Third, online pay-per-bid auction websites generate large amounts of data, which allow us to estimate the model using an extensive field data set.

Our results show that PT describes behavior in pay-per-bid auctions well. Although our PT parameter estimates only capture one person's belief about another person's PT preferences, the estimates are comparable to those found in controlled laboratory experiments in the context of individual decision making under risk. Thus, our results show that PT is an effective descriptive theory for people's behavior and—a unique contribution of our study—that it also adequately describes people's beliefs about other people's behavior. Because a SPNE does not exist, we use backward induction as our solution concept. We explicitly address dynamic inconsistency by considering various ways bidders address this inconsistency in our analysis.

Our findings further show that PT provides a unified explanation for the two bidding behavior anomalies. We first empirically demonstrate the existence of the bidding behavior anomalies by using individual bidding information on more than 140,000 auctions. Our parameter estimates of the PT model, based on 538,045 auctions for 1,261 unique product IDs

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[1] Yet the Federal Trade Commission notes that "in many ways, a penny auction [pay-per-bid auction] is more like a lottery than a traditional online auction" (http://www.consumer.ftc.gov/articles/0037-online-penny-auctions).

[2] Arkes and Blumer (1985, p.124) define the sunk cost effect as "a greater tendency to continue an endeavor once an investment in money, effort, or time has been made."

from 30 different pay-per-bid auction sites, indicate that bidders are loss averse and overweight small probabilities. Applying the median estimates of the PT model to a representative pay-per-bid auction, we show that PT preferences lead to expected revenues that exceed the CRP by 25.46%. Decomposing the overall effect, we find that, relative to the CRP, risk preferences alone increase the expected revenue of the auction by 1.38%, loss aversion reduces the expected revenue by 35.38%, and probability weighting almost doubles expected revenue (91.30%). We further show that the PT model fits the data better than a competing model based on EUT, both in- and out-of-sample.

2. Previous Literature

2.1. Literature on Pay-per-Bid Auctions

Pay-per-bid auctions (Kim et al. 2014; Reiner et al. 2014), also referred to as pay-to-bid auctions (Platt et al. 2013) or penny auctions (Augenblick 2016; Hinnosaar 2016), typically start at a price of $0.00, and each bid increases the price by a small fixed increment (e.g., $.01, $.02, $.12, $.24). A bidder pays a fee, often $.60 for every bid, and each bid extends the auction by a specific time (e.g., up to 20 seconds). The auction has a soft-close ending and ends if no new bid is placed before the countdown clock reaches zero. The bidder who placed the final bid wins the auction. The winner can then buy the product at the price of the final bid (i.e., the final price), which is usually much lower than the CRP.

It is important to stress that, despite superficial similarities, pay-per-bid auctions are substantially different from well-known auction formats such as eBay. Whereas eBay auctions have a fixed duration, pay-per-bid auctions have an open end. They can end quickly if there are only a few bids, but they can also go on for a long time. In eBay auctions, only the winning bidder pays his or her bid, whereas in pay-per-bid auctions, all bidders, whether they win or not, incur bidding fees. Whereas pay-per-bid auctions are modeled as a war of attrition, eBay auctions are theoretically similar to sealed-bid second price auctions, although Zeithammer and Adams (2010) show that the sealed-bid abstraction does not describe actual bidding behavior very well.

The auctioneer is usually the seller of the product, and previous research shows that pay-per-bid auction websites earn considerable profits (Augenblick 2016; Platt et al. 2013). The mirror image of high auctioneer profits is that bidders lose money on average. Given the bad expected outcome for bidders in pay-per-bid auctions and the negative media attention (see, e.g., Ayres 2008; Choi 2011), one would expect that the phenomenon of online pay-per-bid auctions would disappear. Yet pay-per-bid auction sites are still present and continue to enjoy popularity (see also Augenblick 2016).

Independently from one another, Platt et al. (2013), Hinnosaar (2016), and Augenblick (2016) develop models that describe equilibrium bidding strategies in pay-per-bid auctions. Platt et al. (2013) and Augenblick (2016) demonstrate that in any SPNE that results in more than one bid, the bidders must use behavioral strategies. An equilibrium in behavioral strategies implies that a bidder (1) at every point in time is indifferent between placing a bid or not and (2) selects either of these two actions at random, such that the bidders in the preceding period (i.e., round) are indifferent between bidding and not bidding. Consequently, each bidder's expected surplus from participating in the auction is zero, and the full surplus goes to the seller.[3]

Platt et al. (2013) compare the results of actual auctions with the predicted results obtained under equilibrium bidding strategies to show that the actual number of bids is higher than the

[3] Hinnosaar's (2016) model is similar to Augenblick's (2016) and Platt et al.'s (2013) in many respects but differs in how ties are resolved.

predicted number of bids under risk neutrality. They then demonstrate that mild risk-seeking behavior explains this result.

Augenblick (2016) uses the bidding history of pay-per-bid auctions to show that a bidder's behavior depends on how many bids she has already placed earlier in the auction: the more bids placed, the greater the likelihood of placing additional bids in the same auction. This empirical observation is a typical example of the sunk cost fallacy. It contrasts with the SPNE, in which the probability of placing a bid varies only with the current auction price; the bidding fees a bidder has accumulated are considered sunk costs and therefore do not affect the decision to bid again. This characteristic also holds true for Platt et al.'s (2013) risk preference extension, which uses a utility function that assumes constant absolute risk aversion (CARA). Augenblick (2016) uses a sunk cost fallacy model to explain this finding. Whereas Platt et al. (2013) cannot explain the sunk cost fallacy, Augenblick's (2016) model is inconsistent with average revenues below the CRP, a common finding for pay-per-bid auctions (see Platt et al., 2013).

Several studies offer alternative explanations for bidder behavior in pay-per-bid auctions. Byers et al. (2010) demonstrate that introducing asymmetric beliefs into Augenblick's (2016) and Platt et al.'s (2013) risk-neutral models can lead to revenues that are larger than the CRP. This is also true when bidders underestimate the number of bidders in the auction and, therefore, overestimate the probability of winning. Wang and Xu (2016) show empirically that bidders' strategic sophistication and experience positively affect their consumer surplus. Goodman (2012) uses signaling theory to explain aggressive bidding: if a bidder places many bids in auctions, she can acquire a reputation that deters other bidders from placing bids in future auctions in which this bidder is also active. As a result, this bidder could win future auctions at very low prices.

Caldara (2013) investigates pay-per-bid auctions in laboratory experiments. He finds that auction revenues are significantly larger than the values of the products being auctioned, even when the number of bidders, the bidding fee, and the value of the products are revealed to all participants. This result suggests that asymmetric beliefs about the parameters of the auction are not the main driver of auction revenues that are larger than the CRP. Moreover, Caldara finds that risk-seeking behavior does not significantly affect the probability of bidding, which casts doubt on explanations that rely purely on risk-seeking preferences.

Our study extends Platt et al.'s (2013) and Augenblick's (2016) baseline models by incorporating PT, a model that has been successfully employed in the literature to describe decision making under risk in a wide range of applications. Our PT model offers a unified explanation for (1) average revenues that differ from the product's value and (2) the sunk cost fallacy.

2.2. Literature on Prospect Theory

Previous research has successfully used PT to explain a variety of phenomena in the field that cannot be explained with EUT (for an overview, see Camerer 2000). For example, Jullien and Salanié (1997) show that PT performs better than EUT in explaining horse race bettors' behavior. In particular, probability weighting makes long shots more attractive and favorites less attractive relative to unweighted probabilities, the so-called favorite–long shot bias. Barberis (2012) uses PT to explain why a person enters a casino and gambles longer than initially intended. The author explicitly addresses the dynamic inconsistency and distinguishes three ways people address it: naive people are unaware of the dynamic inconsistency; sophisticated people without commitment know that their actions are not dynamically consistent, but they cannot commit to a specific plan of action; and sophisticated people with commitment are aware of the dynamic inconsistency and can commit to a plan of action. We adopt this classification for our analysis.

The majority of PT applications address situations in which people face exogenous risk. Only two studies examine PT in a strategic context in which risk arises endogenously as a result of strategic interaction: Goeree et al. (2003) use PT to explain individual behavior in experiments with asymmetric matching penny games, and Metzger and Rieger (2010) study the existence of Nash equilibria and their properties in games in which participants have PT preferences.

The present study is the first to include PT (including probability weighting) in a dynamic *and* game theoretic model. A notable exception is Gnutzmann (2014) which also studies pay-per-bid auctions using PT. The key differences between this study and ours is that in Gnutzmann (2014) (1) past bidding costs are immediately integrated in the reference point and do not affect current or future decisions; and (2) dynamic inconsistency is not explicitly addressed. Presumably, the lack of studies addressing PT preferences in dynamic games is due to the dynamic inconsistency that arises when departing from EUT and the nonexistence of the SPNE. Our study demonstrates that the backward solution, in combination with the assumption of sophisticated people without commitment, is a tractable solution concept. We show that sophisticated people with commitment do not take part in the auction in the first place and naive people behave similarly to sophisticated people without commitment.

Our study also contributes to an extensive literature stream that involves estimating the parameters of PT. Beginning with Tversky and Kahneman (1992), numerous researchers have attempted to estimate the parameters for various PT specifications (for an overview of many of these parameter estimates, see Booij et al. 2010, Table 1). The majority of these studies use data from controlled laboratory experiments. Notable exceptions include Jullien and Salanié’s

(1997) work, which uses data from the betting market for horse races, and Gurevich et al. (2009), who employ U.S. stock option data. The present study applies a PT model of pay-per-bid auctions to a large empirical data set of online auctions and as such is one of the few studies to empirically estimate PT parameter values in the field.

3. The Model

In this section, we develop a pay-per-bid auction model that extends Platt et al.'s (2013) and Augenblick's (2016) models by considering that people (hereinafter labeled "bidders") have PT preferences. The model assumes that one product is being sold in an auction. The product has a known common value to the bidders, denoted by υ. Note that we do not assume that all *people* value the product equally; in fact, people are likely to hold very different valuations for this product. But only those people with the highest valuation have an incentive to participate in the auction and, therefore, become *bidders*.[4] Let n denote the number of bidders in the auction.

The auction starts in period 0 at an auction price of 0. In period 0, bidders decide simultaneously whether to place a bid or not. Placing a bid entails a bidding fee b and increases the auction price by increment $d > 0$. We assume that the maximum number of bids represents a natural number, which means that $(\upsilon - b)/d$ is a natural number. If several bidders choose to bid in period 0, a fair random device determines which of the bidders makes the first bid and thereby incurs bidding fee b. The bidder who places the first bid becomes the leader in the next period (period 1). In period 1, the leader remains inactive, and the other

[4] This is analogous to the "one too many property" in the general war of attrition (Bulow and Klemperer, 1999). If $N + K$ firms compete for N prizes, the $K - 1$ least able firms will drop out immediately and $N + 1$ will compete for the N prizes.

bidders choose whether to bid. If one of the nonleaders chooses to bid, she incurs bidding fee *b*, the auction price increases by increment *d*, she becomes the leader in the next period, and the process described for period 1 is repeated. If none of the nonleaders chooses to bid, the auction ends, and the leader receives the product and pays the current auction price (i.e., the number of bids times the increment *d*).

Although this auction can go on indefinitely, a critical number of bids exists after which choosing to bid is strictly dominated by not bidding. Consider a bidder who contemplates placing a bid at period t. If this bidder places a bid, her best outcome is that there are no more bids and she wins the auction and gains the right to buy the product for a price of td. When this price exceeds the value of the product net of the bidding fee (i.e., $td > \upsilon - b$), the bidder does not want to place another bid even if she is sure of winning the auction. Thus, if the auction reaches period $T = (\upsilon - b)/d$, the bidders who are not the current leader will not bid, and the auction ends. This observation leads to our first proposition:

Proposition 1: The number of bids in a pay-per-bid auction does not exceed $T = (\upsilon - b)/d$.

When bidders' preferences can be described by EUT, we follow Platt et al. (2013) and focus on symmetric subgame-perfect Nash equilibrium.

3.1. Solution with Expected Utility Theory

Under EUT and risk neutrality, several equilibria exist in which the auction does not continue beyond period 1; in other words, there is either no bid at all or only one bid in the auction (namely, the one in period 0 because the auction ends in period 1). The most relevant SPNE for analyzing the empirical outcomes of online pay-per-bid auctions is the equilibrium that, at each period $t < T$, continues with positive probability p_t.

Proposition 2: In the unique symmetric SPNE that reaches period T with positive probability, the probability that at least one bidder chooses to bid at period $0 < t < T$ is $p_t = 1 - \frac{b}{\upsilon - t \cdot d}$*. In the general case, the probability* p_0 *is not pinned down in equilibrium and can take any value between 0 and 1.*

For a proof, see Platt et al. (2013) and Augenblick (2016). Intuitively, a bidder is indifferent between placing a bid or not, if the probability of another bid in the subsequent period, p_t, satisfies

(1) $$0 = p_t \cdot (-b) + (1 - p_t) \cdot (\nu - t \cdot d - b)$$

Solving Equation (1) for p_t yields the probability given in Proposition 2.

If the probability to bid at the beginning of the auction, p_0, is strictly below 1, there is a positive probability that the auction ends without a single bid and that the product remains with the auctioneer. These auctions are not represented in our data set. Therefore, we set $p_0 = 1$, thereby restricting attention to auctions that attracted at least one bid. In the SPNE described in Proposition 2, the probability that the auction ends after t bids given that it reaches t bids is equal to $\frac{b}{\upsilon - t \cdot d}$. If $p_0 = 1$, the expected revenue for the auctioneer is υ.

If bidders have CARA preferences, the probability to bid is decreasing in the degree of risk aversion; i.e., risk seeking (averse) bidders, make more (fewer) bids and the expected revenue is above (below) υ (Platt et al., 2013).

3.2. Derivation of Backward Solution with Prospect Theory

Kahneman and Tversky (1979) and Tversky and Kahneman (1992) propose PT as an alternative to EUT. Let the potential outcomes of a gamble be $x_{-m}, \ldots, x_{-1}, x_0, x_1, \ldots, x_n$ and let

$p_{-m},\ldots,p_{-1},p_0,p_1,\ldots,p_n$ be the respective probabilities. The outcomes are arranged in increasing order such that $x_i > x_j$, for $i > j$, and $x_0 = 0$. Under cumulative PT, a bidder's valuation of such a gamble is as follows:

(2) $$\text{EV}=\sum_{i=-m}^{n}\pi_i\cdot\nu(x_i),$$

where

(3) $$\nu(x_i)=\begin{cases} f(x_i) & for\, x_i \geq 0, \\ -\lambda\cdot f(-x_i) & for\, x_i < 0, \end{cases}$$

(4) $$\pi_i=\begin{cases} \varpi(p_i), & for\, i=n, \\ \varpi(p_i+\ldots+p_n)-\varpi(p_{i+1}+\ldots+p_n), & for\, 0\leq i<n, \\ \varpi(p_{-m}+\ldots+p_i)-\varpi(p_{-m}+\ldots+p_{i-1}), & for\, -m<i<0, \\ \varpi(p_i), & for\, i=-m, \end{cases}$$

and

(5) $$\varpi(p_i)=\frac{p_i^{\delta}}{\left(p_i^{\delta}+(1-p_i)^{\delta}\right)^{1/\delta}}.$$

The value function in Equation (3) captures the bidder's loss aversion (λ) and risk preferences. The function *f* is strictly increasing and *f(0) = 0*. For concave *f*, the bidder is risk averse in the positive domain and risk seeking in the negative domain. A convex function implies risk seeking in the positive domain and risk aversion in the negative domain. However, loss aversion, λ > 1, implies that a loss decreases a bidder's utility more than a gain of the same size increases her utility.

Equation (3) implicitly assumes that the bidder's reference point for determining whether an outcome is a gain or a loss is 0. In general, defining the right reference point is difficult in many situations (see Barberis 2013). Here, $0 is a natural candidate for the reference point: a "gain" ("loss") implies buying the product for less (more) than the CRP, which is prominently posted on the auction website. For our qualitative results, the assumption that the reference point is 0 can be relaxed. All theoretical results hold as long as the reference point is not fully readjusted after each bid to equal total bidding fees.

Equation (4) defines the decision weights, which use the probability weighting function in Equation (5), where $0 < \delta \leq 1$. The probability weighting function acknowledges the common observation that bidders overweight small and underweight large probabilities. As δ becomes lower, the overweighting of probabilities becomes greater. Note that the decision weights do not necessarily represent biased beliefs about the objective probabilities; PT is reduced to risk neutral EUT if f is linear, $\lambda = 1$, and $\delta = 1$ in Equations (2)–(5).

It is well known that departing from EUT may induce a dynamic inconsistency (for a survey, see Machina 1989), which means that the intended actions at one point can differ across time. Consider the following example for a bidder with probability weighting. Let's assume that after two bids have been made in an auction, the bidder is indifferent between making the third bid and not bidding. If that same bidder compared all possible plans of action before the start of the auction, the plan in which the bidder does not make the third bid is strictly preferred to plans that entail making the third bid. This bidder might make the third bid, although she ruled it out before the start of the auction.

Thus, when studying PT in a dynamic situation, we must specify how bidders address this inconsistency in behavior across time. Following Barberis (2012), we distinguish three types: (1) *naive bidders* (i.e., bidders who are unaware of the dynamic inconsistency); (2) *sophisticated bidders without commitment* (i.e., bidders who know about the dynamic inconsistency but are unable to commit to their intended actions in the future); and (3) *sophisticated bidders with commitment* (i.e., bidders who are aware of the dynamic inconsistency and commit to their intended actions in the future).

Sophisticated bidders without commitment. We consider a pay-per-bid auction played by sophisticated bidders with the same PT preferences. Bidders are sophisticated in the sense that they know that they will make decisions in future periods that are not optimal from their current point of view. Moreover, they have no means to commit to the optimal strategy. As a result, they determine their strategy using a recursive process starting from the last period. In each period, nonleading bidders choose their probability of bidding such that the other bidders are indifferent in the preceding period. Let C_{t-1} denote the bidding costs incurred up to period t. Note that, when $n>2$, there are several nonleading bidders and they will typically have accumulated different bidding costs. We will later show that under certain conditions the bidder with the highest accumulated bidding costs is most eager to bid and, thus, needs to be made indifferent. In that case C_{t-1} represents the maximum accumulated bidding costs among the $n-1$ nonleading bidders.

Proposition 3: In the backward solution that reaches period T with positive probability, the probability that a bid is placed at period $0 < t < T$, p_t, solves

(6)
$$-\lambda \cdot f\left(C_{t-1}\right) = \varpi\left(p_t\right)\cdot\left(-\lambda\right)\cdot f\left(C_{t-1}+b\right) + \varpi\left(1-p_t\right)\cdot f\left(\upsilon - d\cdot t - C_{t-1} - b\right), \quad \text{for } \upsilon - d\cdot t - C_{t-1} - b > 0$$

and $f\left(C_{t-1}\right) = \varpi\left(p_t\right)\cdot f\left(C_{t-1}+b\right) + \left(1-\varpi\left(p_t\right)\right)\cdot f\left(-\left(\upsilon - d\cdot t - C_{t-1} - b\right)\right)$ *otherwise.*

If f is linear, $\lambda = 1$, and $\delta = 1$, Equation (6) simplifies to Equation (1). The following proposition summarizes the effects of the PT parameters on the bidding probabilities obtained by backward induction:

Proposition 4: In the backward solution in the neighborhood of risk neutral EUT, the probability that bidders with PT preferences place a bid is

- *increasing for concave f at the beginning of the auction and decreasing for the remainder of the auction,*

- *decreasing in the loss aversion parameter (λ) at the beginning of the auction and unaffected by λ for the remainder of the auction, and*

- *decreasing in the probability weighting parameter (δ) except the last b/d periods, when it can be increasing in δ.*

The Appendix provides the proof for Proposition 4, and the following example illustrates its implications.

Consider a pay-per-bid auction with two bidders, product value $\upsilon = 6$, bidding fee $b = 2$, and price increment $d = 1$. According to Proposition 1, this auction has at most $T = (6 - 2)/1 = 4$ periods. Both bidders have the same value function with $f(x) = x^{\alpha}$. We assume that both bidders choose to bid in period 0 and that one of them is randomly chosen to be the first leader. Table 1 presents the probabilities of bidding in periods 1–3. The first row presents the bidding behavior of a risk-neutral expected-utility maximizer. The parameter values for the PT models (PT1, PT2, and PT4) are based on the estimates ($\alpha = .88$, $\delta = .65$, $\lambda = 2.25$) obtained from Tversky and Kahneman's (1992) experimental study.[5] The second-to-last row represents the behavior for risk-seeking preferences in the positive domain (PT3).

[Insert Table 1 about here]

Under EUT with risk neutrality, the expected revenue of the auctioneer is 6. However, if bidders overweight small probabilities and underweight large probabilities (PT1), they bid

[5] Tversky and Kahneman (1992) obtain these estimates from experiments in which participants had to state a number that made them indifferent between two binary lotteries. Thus, we acknowledge the context is very different from the auction setting considered in the current study. Most notably, in Tversky and Kahneman's experiment, the experimenter exogenously induced the risk that participants face and it does not depend on the choices of other participants. Nevertheless, we use these parameter estimates because they are often used in the literature.

more aggressively. Consequently, the auctioneer's expected revenue is well above 6. Bidders who are risk averse for gains and risk seeking for losses (PT2) show a lower probability of bidding compared with risk-neutral bidders in periods 1 and 2. However, when bidders accumulate bidding fees, the risk-seeking part of the value function dominates, and the probability of bidding is higher than for risk-neutral bidders. Compared with risk-neutral expected-utility maximization, risk seeking for gains and risk aversion for losses (PT3) leads to a higher probability of bidding in the first period and to a lower probability of bidding in the final period. The expected revenue is higher than 6. Loss-averse bidders (PT4) also bid with smaller probabilities in periods 1 and 2; however, in period 3, all relevant payoffs are negative, and the effect of loss aversion is zero. Expected revenue is lower than 6 in this case.

Naive bidders. For the analysis of a naive bidder's behavior, let us assume that at least one bidder is unaware of the dynamic inconsistency and the other bidders are sophisticated without commitment. Let us fix the bidding behavior of these bidders at the bidding probabilities obtained in the previous section. How does a naive bidder respond? Ultimately, she behaves exactly like a sophisticated bidder: in each period in which the naive bidder is the nonleading bidder, she is indifferent between placing one more bid and not bidding. The crucial difference from a sophisticated bidder is that the naive bidder intends in period t to make exactly one bid and then stop bidding thereafter. However, in the next period in which the naive bidder is the nonleading bidder, period $t + 2$, she is again indifferent between making another bid and stopping. So, she might bid another time, and another time in the next nonleading period after that, and so on. However, had she known in period t that she would make two or more bids in future periods, she would have chosen not to bid and thereby exit the auction in period *t*.[6]

Proposition 5: Consider a pay-per-bid auction played by bidders with the same PT preferences. At least one of these bidders is naïve. If the probability that at least one bid is placed is given by Proposition 3, then

- *a naive bidder is indifferent between placing exactly one more bid and exiting the auction, and*
- *a naive bidder prefers placing exactly one more bid to placing two more bids.*

The Appendix provides proof of Proposition 5. The intuition for the result that the naive bidder prefers placing only one bid to placing two or more bids is that a bidder with PT preferences likes the skewness of the payoff distribution and making only a single bid maximizes this skewness. Comparing the PT values associated with making one more bid and not bidding at all does not depend on the bidder being naive or sophisticated; thus, a naive bidder is indifferent between placing exactly one more bid and exiting the auction entirely.

Sophisticated bidders with commitment. A sophisticated bidder with commitment chooses the plan of action over all periods up to period *T* that maximizes her PT valuation. The optimal strategy is to bid in the current period and then to stop bidding, because this plan of actions offers the highest skewness of the payoff distribution. Thus, a sophisticated bidder with commitment acts like a naive bidder intends to act, but unlike a naive bidder, she sticks to her intentions. Thus, we do not expect to encounter many sophisticated bidders with

[6] This distinction between naïve and sophisticated people is familiar from the present bias literature, pioneered in Strotz (1956). Naïve and sophisticated people have the same preferences over consumption in the present (holding future consumption constant), but they have different beliefs about their future consumption patterns.

commitment in pay-per-bid auctions for two reasons: First, there may not be good commitment devices that keep bidders from bidding again, so sophisticated bidders with commitment will choose not to participate at all, adhering to the philosophy of Gimein (2009) from the *Washington Post*, who writes, "The only winning strategy is not to play in the first place."[7] Second, even if they can commit to their strategy, they only make at most one bid and then exit.

3.3. Effect of Past Bids on Bidding Behavior

An important implication of EUT in combination with either risk neutrality or CARA is that a bidder's behavior in the current period of the auction is independent of the bidding fees that she has already incurred in previous periods of the same auction, because accumulated bidding fees represent sunk costs that should not affect future decisions. Under PT, this is no longer true, as is summarized in the next proposition. This proposition uses a very general specification for the probability weighting function, $\varpi(p)$. We make the following assumption about this function.

Assumption (Subcertainty): The probability weighting function satisfies $\varpi(p) + \varpi(1-p) \leq 1$ *for all* $p \in [0, 1]$.

The probability weighting function in Equation (5) satisfies the subcertainty assumption.

Proposition 6: Suppose bidders' PT preferences satisfy subcertainty. For any given probability of winning with the next bid, $1 - p_{t+1}$*, a bidder who has already placed bids in*

[7] In the case of casino gambling, a sophisticated person might only bring a limited amount of cash and no credit card as a commitment device (Barberis 2012). However, because bidding in online auctions is typically done at home with electronic payments, similar devices are not likely for pay-per-bid auctions.

previous periods of the auction is strictly more likely to place another bid than a bidder who has not yet placed a bid in the same auction if

- *f(x) is linear and* $\lambda \cdot \frac{1-\varpi\left(p_{t+1}\right)}{\varpi\left(1-p_{t+1}\right)} > 1,$
- *f(x) is concave and* $\lambda \cdot \frac{1-\varpi\left(p_{t+1}\right)}{\varpi\left(1-p_{t+1}\right)} > \rho_1$, *or*
- *f(x) is convex and* $\lambda \cdot \frac{1-\varpi\left(p_{t+1}\right)}{\varpi\left(1-p_{t+1}\right)} > \rho_2$.

The Appendix provides expressions for ρ_1 and ρ_2. The ρs are ratios of the slopes of *f* at different points and, thus, are a measure of the curvature of *f*. Proposition 6 states that the sunk cost fallacy occurs if loss aversion and/or probability weighting is sufficiently strong.

Note that Proposition 6 is derived holding the bidding probabilities of the other bidder constant; that is, for any given probability of winning with the next bid, a bidder's expected valuation of the next bid is increasing in the number of bids the bidder has already made in that same auction.

We next investigate how well this proposed PT model can explain bidder behavior in real-world pay-per-bid auctions—that is, examine the applicability of the PT model.

4. Data and Evidence for Pay-Per-Bid Auction Anomalies

4.1. Data

For our empirical analyses, we use a data set that tracked the activities of a large number of pay-per-bid auction websites.[8] The data cover 965,403 auctions that occurred between November 19, 2009, and March 12, 2011, on 138 pay-per-bid auction sites with more than 1

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[8] We thank the owner of the (currently inactive) website allpennyauctions.com, Mark Streich, for providing us with the data.

million unique bidders who placed over 100 million bids for 35,039 items. The data provide not only information about the auctions such as the CRP, the final price, the bid fee, and the number of bids (i.e., the price increment equals final price/number of bids), but also individual bidding histories for many of the auctions including bidder usernames.

From these data, we use a subset, named Data Set 1—the auction data, which consists of all unique product IDs that show at least 75 auctions. By “unique product IDs,” we mean that we additionally split the original product IDs into different unique product IDs if the auctions contain different CRPs, increments, or bidding fees. In doing so, we ensure that the CRP, the increment, and the bidding fee are constant across the auctions for a certain unique product ID. We further discard all auctions that did not use U.S. dollars as currency and auctions for which the bidding increment and the final price were inconsistent.

In total, Data Set 1 contains 538,045 auctions representing 1,261 products (as identified by the unique product ID) from 30 auction sites. Thus, we have on average 427 auctions per product. The CRPs (according to the pay-per-bid auctioneer) are in the range of $3.60 to $1,500, and the mean (median) CRP is at $56.93 ($37). Notably, the overall average revenue per auction is $131.98; thus, the total revenue over all 538,045 auctions corresponds to ≈ $71 million.

Table 2 summarizes several descriptive statics, such as the increments and bidding fees that are present in our data. In Table 2, we further distinguish between the average values in our data (i.e., “All Auctions”), the top five auction sites, which accumulate 89.1% of the observations, the average values of the data except for the top five sites (i.e., “All others”). The substantial differences across the auction sites in Table 2 (e.g., for the number of bidders, CRP, revenue) motivate a more disaggregate view across the auction sites.

[Insert Table 2 about here]

From the individual bidding histories, we derived Data Set 2—the bidding data. For Data Set 2, we use 142,650 auctions in which 285,312 unique bidders participated and placed more than 10 million bids. Data Set 2 contains detailed information corresponding to the bidders in each auction (e.g., the username associated with each individual bid). The availability of the individual bidding histories in Data Set 2 enables us to investigate the bidding behavior of bidders. In Data Set 2, we observe an average (median) number of 8.2 (3) bidders per auction, and the number of bidders per auction ranges from 1 to 1,020. Furthermore, Data Set 2 allows us to compute the average consumer surplus per auction and per bidder. The results illustrate that 21.06% of the bidders achieve a positive consumer surplus, and the majority (78.94%) of bidders realizes a negative consumer surplus (i.e., a loss). Further investigation of the data reveals that, on average, each bidder in our Data Set 2 loses $6.57 per auction. This finding provides evidence that the average bidder loses money when participating in pay-per-bid auctions.

4.2. Evidence for Pay-per-Bid Auction Anomalies

In the following subsections, we provide empirical evidence for the two aforementioned pay-per-bid auctions anomalies we use to illustrate the favorability of using PT over EUT.

Average auctioneer revenues above the CRP. From Data Set 1, we can compute the average final price as well as the average revenue per auction. The ratio of average revenue and CRP is greater than 1 over all auction sites and for all auction sites individually. This finding indicates that pay-per-bid auctions raise revenues that are, on average, greater than the products' CRPs, though there is considerable variation across auction sites: for the auction site QuiBids, the ratio is, for example, only 1.02, while for Beezid it is 4.45. Overall, we find for Data Set 1 that 46.54% of the auctions show average revenue above the CRP, while

53.46% of the auctions show revenues smaller than the CRP. In the PT model, average revenues above the CRP can be explained by risk-loving and probability weighting. High degrees of loss aversion can lead to average revenues below the CRP.

Sunk cost fallacy. We use the individual bidding histories of Data Set 2 to document the sunk cost fallacy. To this end, we estimate a binary probit model using all auctions (N = 14,790) that have at least 100 bids. The dependent variable is a dummy variable (D(id90 = id100)) that equals 1 if the identity of the bidder who made the 90th bid (id90) in a specific auction equals the identity of the bidder who made the 100th bid (id100) (i.e., both bids are placed by the same bidder). Independent variables are the bidding fees that bidder id90 has accumulated in the first 99 periods (C_id90) and the number of unique bidders (n) during the first 99 periods (i.e., modeled as 1/(n – 1)).[9]

Under EUT with CARA preferences (with risk neutrality as a special case), the unique symmetric SPNE implies that each nonleading bidder is equally likely to make the 100th bid. The fees accumulated in the previous periods should not matter. Thus, under EUT with CARA preferences, we expect that $\beta_{(1/(n-1))} = 1$ and $\beta_{C\_id90} = 0$. A positive value for $\beta_{C\_id90} > 0$ is evidence for the sunk cost fallacy. The results (see Table 3) show that β_{C_id90} is significantly positive (.05, $p < .01$), confirming that bidders' bidding fees accumulated in previous periods increase the probability of placing a bid.

[Insert Table 3 about here]

In our PT model, the sunk cost fallacy can be explained by loss aversion and/or probability weighting.

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[9] We choose the 90th and 100th bid because it strikes a good balance between keeping the sample of auctions in the regression representative and generating enough variation in the accumulated bidding fees. The regression results are not sensitive to this choice.

4.3. Number of Bidders in Pay-per-Bid Auctions

An important characteristic of pay-per-bid auctions for our proposed PT model is the number of bidders per auction. Table 2 shows that the average number of bidders per auction site varies from 2.5 to 18.8 bidders. To shed light on the distribution of the activity of bidders in the auctions, Table 4 illustrates the average percentages of bids per auction that come from the five top bidders—that is, the five bidders with the highest number of bids. To account for the variation of the number of bids per auction (mean = 76.12, median = 5, max = 27,380) in Data Set 2, we report the percentages for different minimum number of bids per auction. We find across all auctions, including those with only a few bids, that the top two bidders account for the vast majority of bids (Table 4, columns 2 and 3) and that these bids also translate into winning the most auctions (Table 4, column 7). However, observing the auctions that show a larger number of bids (≥75 and ≥100), we find that the percentages of bids from the top two bidders decrease to a range of approximately 40% and the top two bidders win approximately 50% of the auctions.

A further inspection of the data shows that 45.28% of all bidders across all auctions in Data Set 2 only place a single bid. These bidders could be classified as sophisticated bidders with commitment. In addition, 76.67% of all bidders in an auction place less than or equal to five bids. The fact that bidders who made only a few bids exit the auction altogether is consistent with Proposition 6. Due to loss aversion and/or probability weighting, bidders who have placed more bids are more eager to bid again and will drive bidders with fewer bids out of the auction.

[Insert Table 4 about here]

5. Bringing the Model to the Data

5.1. Bidder assumptions

Bidders' sophistication. We make no assumption about the bidders' sophistication. However, the theoretical analysis has shown that naïve bidders' behavior is observational equivalent to the behavior of sophisticated bidders without commitment (Proposition 5) and that sophisticated bidders with commitment make at most one bid. We, therefore, use the backward solution obtained for sophisticated bidders without commitment for the estimation.

Specification of the bidding fee function. Proposition 6 implies that bidders that have placed the most bids will be the most eager to place another bid. Consequently, the probabilities, p_t, for $t= 1, ..., T-1$ in Proposition 3 are chosen to make the bidders with the highest bidding costs indifferent between bidding or not. Table 4 shows that the top two bidders place on average 40% of all bids in an auction. We, therefore, assume that the actual bidding cost is one-fifth of the number of bids placed times the bidding fee, $C_t = 0.2 \cdot t \cdot b$. Robustness checks reveal that modifying the share of the bids placed by the top two bidders in the bidding fee function does not change our main results fundamentally.

Specification of bidders' valuations of the auctioned product. The products in our pay-per-bid auctions are new products also offered by many other retailers. Therefore, it is unlikely that there are significant idiosyncratic differences in bidders' valuations of the products. If bidders valued the product differently, the "one too many" property (see footnote 4) implies that all but the two bidders who value the product the most would drop out of the auction immediately. Consequently, we assume that bidders attach the same value to the product.

Moreover, we assume that the bidders' value is equal to the CRP. The CRP constitutes a clear upper bound for the value. No bidder will pay more for the product than she would pay at another retailer. In addition, pay-per-bid websites typically display the CRP prominently for the duration of auction, suggesting that the CRP may serve as an anchor. We believe, therefore, that the CRP is a good proxy for the value. However, we are aware that the CRP reported on auction sites can overestimate the value of the product and we will discuss how inflating the value of the product affects our estimates. Alternatively, we could estimate the willingness to pay jointly with the other parameters of interest. Platt et al. (2013), however, show that this joint consideration does not improve the model fit considerably.

Buy-Now option. By late 2009, most auction platforms added a new feature to their auctions called the Buy-Now option (Stix, 2012). With the Buy-Now option losing bidders can use their accumulated bidding fees to pay for the item they were bidding for. In other words, a bidder who has spent $*C* worth of bids in an auction that she eventually lost, can acquire the item for a price of *$(CRP-C)*. Precise information on which auctions offered the Buy-Now option and how often it has been used is not available. But since our data starts in November 2009, the vast majority of auctions in our sample is likely to offer the Buy-Now option. The Buy-Now option changes the dynamics of the auction; it becomes a “game of chicken” (Byers et al., 2010). In the game of chicken, the auction end is not random. A bidder will continue to bid until the sum of her accumulated bidding fees and the current auction price are equal to the CRP at which point, she will use the Buy-Now option to buy the item. Thus, the bidder with the lowest accumulated bidding fees will be the last to bid and win the auction. All other bidders should either use the Buy-Now option or should not have taken part in the auction in the first place.

These implications are not supported by the data: The auction end in our data appears to be random; the winning bidder is typically one of the bidders that have placed the most bids (not the least bids, see Table 4); and previous literature shows that only a few bidders exercise the Buy-Now option (on average 1.26 bidders per auction, Reiner et al., 2014).

There are (at least) two reasons why the game of chicken does not describe the data well. First, auction sites frequently offer auctions for voucher bid packs. Voucher bids can be used to bid in auctions just like purchased bids, the only difference being that voucher bids cannot be used towards buying the product using the Buy-Now option. Second, bidders might exhibit “narrow framing”. Kahneman and Lovallo (1993) argue that “people tend to consider decision problems one at a time, often isolating the current problem from other choices that may be pending.” This tendency has been confirmed in financial markets (Barberis et al., 2006 and Barberis and Huang, 2009). Thus, although bidders are aware of the Buy-Now option, they ignore it for the moment and concentrate on the pay-per-bid auction itself.

As the game of chicken is inconsistent with several empirical facts of pay-per-bid auctions, we base our analysis on the traditional model of pay-per-bid auctions proposed by Augenblick (2016) and Platt et al. (2013). We are aware that ignoring the Buy-Now option can bias our estimates and we will return to this issue in the next section.

Having made these assumptions, the value of placing a bid in period *t* is as follows:

(7) $$v(bid)=\begin{cases} v\left(CRP-d\cdot t-0.2\cdot(t-1)\cdot b-b\right), & \textit{if winning bid, then auction stops,} \\ v\left(-0.2\cdot(t-1)\cdot b-b\right), & \textit{if not winning bid, then auction continues,} \end{cases}$$

and the value of not bidding is equal to the forgone bidding fees, here defined as

(8) $$v(no\,bid)=v\left(-0.2\cdot(t-1)\cdot b\right).$$

For the empirical application, we use the exponential value function

(9) $$v(x)=\begin{cases} \dfrac{1-e^{-\alpha\cdot x}}{\alpha} & \text{for } x\geq 0, \\ \dfrac{-\lambda\cdot\left(1-e^{-\alpha\cdot|x|}\right)}{\alpha} & \text{for } x<0, \end{cases}$$

The general results of the analysis are not affected by choosing Equation (9) over Tversky and Kahneman's (1992) original specification. However, we choose the exponential value function because it is closely related to the constant absolute risk aversion function used by Platt et al. (2013).[10] Thus, any differences between the PT and the Platt model can then be attributed to PT rather than the functional form. For the probability weighting function $\varpi(p)$, and the decision weights π, we rely on Equations (2), (4), and (5).

5.2. Description of Estimation Procedure

We estimate the parameters (α, λ, δ) for each unique product ID using a maximum likelihood procedure. For a given unique product ID, we observe A different auctions. The a^{th} auction ends after T_a bids. The likelihood of observing the final number of bids, $(T_a)_{a=1\ldots A}$, for this product is then given by

(10)
$$L = \prod_{a=1}^{A} \left(1 - p_{T_a+1}\right) \prod_{t=1}^{T_a} p_t \,,$$

where the probabilities p_t, $t = 1, \ldots, T_a + 1$, are characterized by Proposition 3. We then determine the set of parameters (α, λ, δ) by minimizing the negative of the log-likelihood by means of a simulated annealing optimization (Xiang et al. 2013) that allows for a robust estimation of the three parameters.

Note that the likelihood function in Equation (10) treats each auction as an independent observation. This assumption seems strong because identical products are also auctioned off in simultaneous auctions. Evidence from eBay shows that approximately 19% of bidders switch between simultaneous auctions (Anwar et al. 2006; Haruvy and Popkowski Leszczyc 2010).

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[10] Moreover, De Giorgi and Hens (2006) show that the exponential value function has several advantages over the power function; for example, the St. Petersburg paradox does not arise with the exponential value function, and the disposition effect can be explained with the exponential value function but not with the power function.

However, in our Data Set 2, only approximately 7% of bidders participate in more than one auction of identical products on the same day. Thus, cross-bidding on simultaneous auctions is less common in pay-per-bid auctions than in eBay auctions, presumably because bidding in pay-per-bid auctions requires much more active participation throughout the auction.

6. Empirical results

6.1. Empirical Results for the PT Model

PT model results. Table 5 summarizes the parameter estimates for the PT model that yield the best fit for each of the 1,261 unique product IDs and displays the median, mean, and standard deviation of the estimated parameters α (risk preference), λ (loss aversion), and δ (probability weighting). We find that the median bidder is risk seeking for gains and risk averse for losses (see Table 5, column "All Auctions") because the median value of the risk preference parameter α is –.002. Across the estimates for risk preferences, we find that 45.4% of estimates for α are positive, indicating that bidders for these products are risk averse for gains and risk seeking for losses. However, the majority of estimates, namely 688 (54.6%), are negative. For the loss aversion parameter, we find a median value of λ = 1.742, providing evidence that bidders are in general loss averse. This estimate implies that bidders perceive the pain of losing one dollar as strongly as the pleasure of gaining $1.74. For the probability weighting parameter (δ), we obtain a median value of δ = .864, indicating that the bidders overweight the small probability of winning the auction. The following example illustrates the impact of probability weighting: For a product with a CRP of $37, a bidding fee of $.6 and an increment of $.01, the probability of winning with the 3000st bid is 5.7%. A PT bidder with δ = .864 perceives this probability to be 8.1%. We observe that 11.4% of the estimates for probability weighting are smaller than .7, indicating a strong overweighting of the probabilities of winning the auction. On the opposite side, we find that 20.7% of the estimates are very close to one (>.99).

[Insert Table 5 about here]

When looking at the disaggregated results for the top five auction sites, QuiBids stands out. QuiBids has a positive median value for the risk preferences, indicating that bidders are risk averse on this auction site, whereas we observe risk-seeking behavior on all other sites. QuiBids also shows the highest estimate for the probability weighting parameter and a rather low estimate for loss aversion. Notably, QuiBids is, in contrast to many other auction sites in our sample, still in business. Overall, the empirical parameter estimates of the PT model, combined with the results of the theoretical analysis, can explain the two anomalies observed for pay-per-bid auctions.

Revenues well above the CRP. Bidders overweight small probabilities (median δ =.864). Proposition 4 indicates that this overweighting increases the probability of placing a bid and therefore increases the expected auction revenue. Moreover, bidders are loss averse (median λ = 1.742), which leads to fewer bids in the beginning of the auction and lower expected auction revenue. For the majority of products, the effect of probability weighting dominates the effect of loss aversion, such that the average revenues per auction exceed the CRP, but there are also products for which the average revenue per auction is below the CRP.

Sunk cost fallacy. Proposition 6 states that the sunk cost fallacy occurs when loss aversion and probability weighting are large relative to the curvature of the value function. Our estimates support this proposition. The median estimate for λ is well above 1 and the median estimate for δ well below 1, whereas the median for α is close to 0, suggesting a value function that is almost linear.

Comparing our PT parameter estimates with previous estimates, we find that the average loss aversion parameter is between Tversky and Kahneman's (1992) original estimate (λ = 2.25) and Harrison and Rutström's (2009) estimate (λ = 1.38). Our estimates for the probability weighting parameter are larger, and therefore, our probability weighting is less pronounced than in most laboratory studies, in which δ ranges from .56 to .76 (see Booij et al. 2010, Table 1). One exception is Harrison and Rutström (2009), who find an estimate of δ = .91. The only other estimates of the probability weighting parameter using field data come from Gurevich et al. (2009) and are comparable to ours (δ = .77 and δ = .91).

Estimates of the parameter of the value function typically support the traditional S-shaped value function, which is concave in gains and convex in losses. Our estimates, however, imply the opposite pattern for the majority of auctions: the value function is convex in gains and concave in losses. The finding that bidders in pay-per-bid auctions are risk seeking in gains is consistent with Platt et al.'s (2013) results and implies that bidders in pay-per-bid auctions are more risk seeking than participants in laboratory studies. Moreover, even some laboratory studies (e.g., Abdellaoui et al. 2008; Bruhin et al. 2010) find that people are risk averse in losses.

Decomposing the overall effect of Prospect Theory into its individual components. Proposition 4 provides the qualitative effects of the PT parameters on bidding behavior and, consequently, expected revenues. Using the parameter estimates from the previous subsection, we can now quantify these effects on expected revenues. To this end, we perform the following simulation exercise. We consider a product with a CRP of \$37 that is auctioned off in a pay-per-bid auction with a bidding fee of \$.60 and an increment of \$.01. (These numbers represent the median values for CRP, bidding fee, and increment in Data Set 1, respectively.) We next calculate the expected revenues that result from setting the PT parameters to their median estimates one at a time, while leaving the two remaining parameters at their risk-neutral EUT values. In addition, we calculate expected revenues assuming that all PT parameters are set to their median estimates. If all PT parameters are at their risk-neutral EUT levels, expected revenue is equal to the CRP. Table 6 presents the results of this decomposition.

[Insert Table 6 about here]

Risk preferences alone, with $\alpha = -.002$, lead to expected revenues that exceed the CRP by 1.38%, a very modest effect on expected revenues. A median level of loss aversion, however, reduces expected revenues by 35.38% compared with the CRP. Probability weighting has by far the largest individual impact. When setting δ to its median estimate of .864, expected revenues amount to almost twice the CRP (91.30%). Taken together, the three components of PT lead to expected revenues that exceed the CRP by 25.46%.

Figure 1 illustrates how the auction duration (measured by the number of bids) implied by the estimated PT model compares with the empirical auction duration.

[Insert Figure 1 about here]

Although the two items (a temperature gauge and a voucher bid pack) have differently skewed empirical distributions, the PT model maps them well.

6.2. Sensitivity Analysis

In the following we will analyze how sensitive our parameter estimates are with respect to some of the assumptions we made in Section 5.

Bidders' valuation of the product. The CRPs that auction sites set are typically higher than the prices on other online shopping websites, like Amazon (Augenblick, 2016, Platt et al., 2013). A sensitivity analysis shows that if a product's CRP is biased upwards by 25%, α is biased upwards by 21%, δ is biased upwards by 4% and λ is biased downwards by .8% (the changes for λ are insignificant). We, therefore, believe that our estimates for α and δ are biased upwards and provide an upper bound for the PT parameters.

The bias in the bidders' valuation can also shed some light on the heterogeneity of the parameter estimates across items. Many items that are auctioned off are vouchers. These can be gift cards or voucher bid packs that can be used to bid on that same auction website. The CRPs of voucher auctions are usually set at the nominal value of the gift card or voucher bid pack and are, therefore, good proxies for the bidders' valuation, whereas the CRPs of product auctions tend to be inflated. Consistent with this, we find that voucher auctions show significantly lower estimates for α and δ than product auctions; this is particularly pronounced for voucher bid auctions. There is no significant difference in the estimates for λ between voucher auctions and product auctions. Further evidence for this difference between voucher auctions and product auctions is the auction site Beezid. In our sample 99.4% of the auctions on Beezid are voucher bid auctions. In line with the result that voucher bid auctions show significantly lower estimates for α and δ than product auctions, Beezid is the auction site with the lowest α and δ estimates (see Table 5).

Buy-Now option. To understand how sensitive our parameter estimates are to the Buy-Now option, we compare our parameter estimates with estimates using a data set of auctions without the Buy-Now option. This data set for the auction site Swoopo is described in Byers et al. (2010). The data in Byres et al. (2010) contains auctions from before and after the introduction of the Buy-Now option and thus allows for dividing the data into two subsets with and without the Buy-Now option. Using a regression approach that controls for factors such as CRP, bidding fee and increment, we find no significant difference for α and δ between our estimates and the estimates using the Swoopo data before Buy-Now was introduced. Our estimates for λ are significantly higher than those for the Swoopo data without Buy-Now option. But there is no significant difference between the Swoopo auctions with and without Buy-Now option, suggesting that the difference to our results is not directly attributable to the Buy-Now option.

6.3. Comparison of Results of PT and EUT Models

To illustrate the suitability of using PT preferences for describing bidder behavior in pay-per-bid auctions, we compare the PT model and the model proposed by Platt et al. (2013), an EUT model using a CARA utility function to incorporate risk preferences (hereinafter, the Platt model). The Platt model predicts the patterns of pay-per-bid auctions well and therefore serves as a strong benchmark.

Therefore, we estimate the Platt model for the 1,261 unique product IDs in our Data Set 1. For the estimation of the risk preference parameter (denoted by α_P for the Platt model), we again assume that the product value corresponds to the CRP and use the same estimation procedure used for the PT model. The results for the Platt model show, in the median, risk-seeking preferences (median $\alpha_P = -.004$ [mean $\alpha_P = .017$]); however, we find that 39.65% of the estimates are positive, therefore indicating risk-averse preferences.

The PT model results in a better log-likelihood than the Platt model for 1,226 of the 1,261 unique product IDs (97.22%). Note that the PT model has three parameters, whereas the Platt model has only one. We, therefore, use the AIC and BIC criteria to compare the two models. Based on AIC (BIC), the PT model describes 77.4% (59.7%) of the unique product IDs better.

In addition, we conducted a Kolmogorov-Smirnov test. To do so, we use the model estimations to generate the corresponding distributions of ending bids described by the PT and

Platt models. We then compared these distributions against the empirical observed data distributions. We find that the PT model fits the observed data distributions better than the Platt model. The PT model maps 80.65% of the observed data distributions (p > .05), while the Platt model only maps 43.3%. Comparing the two models, we find that the PT model maps 472 products that are not explained by the Platt model, the Platt model maps only one product that is not explained by the PT model, and the models describe 545 products equally well.

6.4. Managerial Implications

Auction increments. One of the auctioneer's key decisions is to specify the increment for the auctions, aiming for the maximum revenue. Under EUT with risk neutrality, the auctioneer's expected revenue is the CRP and, hence, independent of the bidding fee and the increment. Under PT, these details of the auction design do affect the auctioneer's expected revenue, as we illustrate with the following simulation.

We assume a product with a CRP of $37 and a bidding fee of $.60. The product is auctioned off with two different increments, $.01 and $.02. We further assume that bidders have PT preferences best described by the median parameter estimates provided in Table 5. For both increments, the revenues of these simulated auctions show average revenues above the CRP, and they are higher for the $.01 increment (25.47% above the CRP, or $46.42) than for the $.02 increment (24.37% above the CRP, or $46.02).

We next analyze whether our data support the prediction of the PT model, that average revenues are ceteris paribus higher for the $.01 increment than the $.02 increment. There are 16 products that are auctioned off with $.01 and $.02 increments and for which we observe more than 75 auctions for each increment. The bidding fee is $.6 for all these auctions.

Averaging over these 16 products, the average revenue of the auctions with a $.01 increment is 23.19% (median = 24.91%) above the CRP. For the auctions with a $.02 increment, the average revenue is 9.05% (median = 19.90%) above the CRP. A t-test shows that the average revenues are significantly greater for the $.01 increments (*p*-value: .006). This finding confirms the implication of the PT model, that the auctioneer can increase her expected revenue by reducing the increment.

These 16 items offer a good opportunity to study the consistency of the parameter estimates with respect to changes in the auction design details.[11] Table 7 shows that the differences between parameter estimates for the $.01 and $.02 increments are small (especially for δ and λ) and insignificant. Thus, an auctioneer can indeed estimate the parameters for one increment and use these estimates to make predictions for different levels of the increment.

[Insert Table 7 about here]

[11] We thank an anonymous reviewer for suggesting this consistency check.

Prediction of revenue of auctions. Auctioneers are also interested in predicting revenues for upcoming auctions. To test the predictive power of our PT model, we split the data from Data Set 1 in two parts: a training data set (i.e., 80% of the auctions belonging to each unique product) and a holdout sample (i.e., the remaining 20% of the auctions belonging to each unique product). We use the training data to estimate the PT and Platt models as explained previously. We then use the model estimates per unique product to predict the average auction revenue. Last, we compare the predicted auction revenues with the observed revenues of the holdout sample (normalized by the CRP). We find that the RMSE of the PT model prediction is .616, which is superior to the RMSE prediction of the Platt model (mean = .672).

7. Summary and conclusions

We incorporate PT preferences into a dynamic game theoretic model of pay-per-bid auctions. The novel feature of this application of PT is that risk arises endogenously as bidders employ mixed strategies. We derive the effects of the elements of PT (risk preference [α], loss aversion [λ], and probability weighting [δ]) on bidding behavior using a backward solution. Probability weighting implies that bidders overestimate the probability to win and therefore bid more often than a bidder with EUT preferences would bid. Loss aversion, in contrast, makes bidders bid more cautious. Both probability weighting and loss aversion are responsible for the sunk cost fallacy (i.e., the behavior that bidders who have already placed bids during an auction are more likely to bid in subsequent periods than bidders who have not yet bid in the auction).

Using a data set covering 538,045 pay-per-bid auctions, we show that PT provides a good description of the observed bidding behavior and better fits the data than a competing model based on EUT. Our (median) parameter estimates of the PT model confirm that bidders are risk seeking in gains ($\alpha = -.002$) and loss averse (λ = 1.742) and that they overweight small probabilities ($\delta = .864$). Taking all three PT components together, for a representative pay-per-bid auction we observe a 25.46% increase in revenues. A decomposition of the overall effect of PT into its individual components highlights that this increase in revenues is predominantly attributable to probability weighting. The parameter estimates from the PT model can further be used to predict future auction results for the same product well. Moreover, we find that lower increments, on average, are associated with higher revenues.

The broader question that our study addresses is that of the universality of PT and its applicability for modeling behavior in strategic situations. Is PT only useful to describe risk attitudes toward lottery-like, exogenous risk, or is it also an effective descriptive theory of behavior in strategic situations in which risk arises endogenously? The encouraging findings of this study indicate that the success of PT as a descriptive theory of decision making under risk extends to situations in which risk arises through the interactions of different people.

Moreover, many worthwhile applications of PT in dynamic games remain for researchers to explore. Returning to our introductory volunteer's dilemma example, a PT model similar to that presented here predicts that the expected duration of a meeting is longer than one would expect using standard models with risk-neutral EUT preferences. The PT model also offers guidance for how to reduce the expected duration of the meeting: if a manager wants to decrease meeting time by finding a volunteer faster, then she should make sitting in the meeting as costly as possible for the members. An increase in "cost" could be achieved by scheduling the meeting near the end of the working day or by providing uncomfortable chairs or even no chairs at all.

Our study also has implications for the online pay-per-bid auction industry. Several studies argue that pay-per-bid auction websites generate average revenues above the CRP because bidders are inexperienced or naive (Augenblick 2016; Wang and Xu 2016). Under this assumption, as soon as bidders gain experience and no new inexperienced bidders arrive, pay-per-bid auctions will cease to be profitable. One might, therefore, conclude that online pay-per-bid auctions are a short-lived phenomenon. However, traffic data from the largest pay-per-bid auction sites indicate that they still enjoy popularity (for further evidence, see Augenblick 2016 and quibids.com). We show that bidding behavior in online pay-per-bid auctions is consistent with the backward solution of a dynamic game with sophisticated PT bidders. Therefore, online pay-per-bid auctions with substantial positive profits for the auctioneer are not a transitory phenomenon.

Acknowledgments

The third author was supported by the University Research PriorityProgram “Social Networks” at the University of Zurich. We thank the editor, two reviewers, Thomas Otter and Julia Wamsler for their supporting comments.

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Table 1: Bidding Probabilities and Expected Revenue according to Expected Utility Theory and Prospect Theory for a Pay-per-Bid Auction with υ = 6, b = 2, and d = 1

	p_1	p_2	p_3	*Expected Revenue*
EUT (α = 1, δ = 1, λ = 1) (bidders are risk neutral)	60.0%	50.0%	33.3%	6.00
PT1 (α = 1, δ = 0.65, λ = 1) (bidders weight probabilities)	65.1%	50.0%	31.5%	6.24
PT2 (α = 0.88, δ = 1, λ = 1) (bidders are risk-averse)	58.8%	50.0%	35.2%	5.96
PT3 (α = 1.12, δ = 1, λ = 1) (bidders are risk-seeking)	61.2%	50.0%	31.5%	6.04
PT4 (α = 1, δ = 1, λ = 2.25) (bidders are loss-averse)	57.1%	30.8%	33.3%	4.69

Notes: p_1–p_3 refer to the equilibrium bidding probabilities (i.e., the probability of bidding in period t).

Table 2: Descriptive Statistics for Data Sets 1 and 2

Auction Site	All Auctions	Quibids	BidCactus	BigDeal	SkoreIt	Beezid	All Others
Number of auctions	538,045	240,176	102,482	54,071	49,254	33,333	58,729
Number of unique products	1261	614	112	165	120	13	237
Average number of bidders[a]	8.2	2.5	4	18.8	5.5	4.4	9.2
Average CRP (in $)	56.93	28.79	67.41	92.94	83.06	53.57	100.62
Median CRP (in $)	37	21	50	37.5	50	40	45
Average revenue (in $)	131.98	41.53	98.8	580.19	165.58	133.7	117.98
Median revenue (in $)	29.44	12.81	47.12	39.52	58.88	88.81	39.05
Average revenue/CRP[b]	1.64	1.02	1.22	4.22	1.64	4.45	1.49
Median revenue/CRP[b]	1.15	.91	1.1	2.98	1.55	4.73	1.32
Prob(Rev/CRP)>1	.6	.43	.64	.84	.82	1	.72
Increments (in cents)	1; 2; 37	1; 2	1	1	1; 2	1; 37	1; 2
Bidding fees (in cents)	14; 48; 50; 55; 57; 60; 63; 65; 70; 72; 75; 87; 89; 100	60	75	75	63	70	14; 48; 50; 55; 57; 60; 63; 65; 70; 72; 75; 87; 89; 100

[a]Average number of bidders is calculated using the bidding histories of Data Set 2, [b]Average and median revenue is calculated per unique product ID, all other numbers are on the auction level.

Notes: CRP = current retail price, "All auctions" covers all auctions of all auction sites, "All Others" are all auctions except the top five sites (Quibids, BidCactus, BigDeal, SkoreIt, and Beezid).

Table 3: Probit Estimation Results Providing Evidence for the Sunk Cost Fallacy

	Estimate	SE	z Value	*p*-Value	sig.
Intercept	–1.2856	.0253	–50.90	<.001	***
1/(n – 1)	3.8996	.3412	11.43	<.001	***
C_id90	.0503	.0012	41.44	<.001	***

***$p < .01$.

Notes: SE = standard error, C_id90 = bidding fees that bidder id90 has accumulated in the first 99 periods, 1/(n – 1) = number of unique bidders (n) during the first 99 periods, Number of observations = 14,790.

Table 4: Percentage of Bids of Top Five Bidders

		Percentage of Bids of Top Five Bidders (%)[a]					Percentage of Auctions Won by Top Two Bidders (%)
	N	1st	2nd	3rd	4th	5th	
All auctions	142,650	54.54	28.80	15.45	10.28	7.46	89.61
Auctions with ≥75 bids	17,407	27.00	18.09	10.49	7.39	5.53	52.20
Auctions with ≥100 bids	14,790	26.04	17.35	10.36	7.40	5.57	49.61

[a]Percentage of the top bidders = number of bids from one bidder per auction / number of observed bids from all bidders per auction, N = number of observations.

Notes: This table is based on Data Set 2. Averaging across auctions can lead to row sums greater than one.

Table 5: Parameter Estimates of PT Model

Auction Site		All Auctions	QuiBids	BidCactus	BigDeal	SkoreIt	Beezid	All Others
Number of Unique Products		1,261	614	112	165	120	13	237
Probability Weighting (δ)	Mdn	0.864	0.946	0.814	0.712	0.835	0.704	0.838
	M	0.853	0.909	0.820	0.749	0.837	0.686	0.812
	SD	0.128	0.098	0.076	0.114	0.086	0.080	0.166
Risk Preference (α)	Mdn	-0.002	0.011	-0.012	-0.004	-0.005	-0.054	-0.004
	M	0.025	0.049	-0.008	0.012	-0.009	-0.064	0.008
	SD	0.095	0.110	0.020	0.065	0.016	0.053	0.102
Loss Aversion (λ)	Mdn	1.742	1.199	4.533	3.574	2.281	1.045	2.963
	M	3.718	2.591	5.582	4.648	3.927	4.135	4.981
	SD	3.498	2.822	3.735	3.432	3.455	3.978	3.983

Notes: Data Set 1 serves to estimate parameters, Mdn = median, M = mean, SD = standard deviation, “All Auctions” covers all auctions of all auction sites, “All Others” are all auctions except the top five sites (Quibids, BidCactus, BigDeal, SkoreIt, and Beezid).

Table 6: Decomposition of the Overall Effect of Prospect Theory into its Individual Components

	Risk Preference (α)	Loss Aversion (λ)	Probability Weighting (δ)	Expected Revenue ($)	Change in Revenue (in %)
Risk neutral EUT model	0	1	1	37.00	.00
Bidders are risk seeking	-.002	1	1	37.51	1.38
Bidders are loss averse	0	1.742	1	23.91	–35.38
Bidders’ weight probabilities	0	1	.864	70.78	91.30
PT-Model (Median results)	-.002	1.742	.864	46.42	25.46

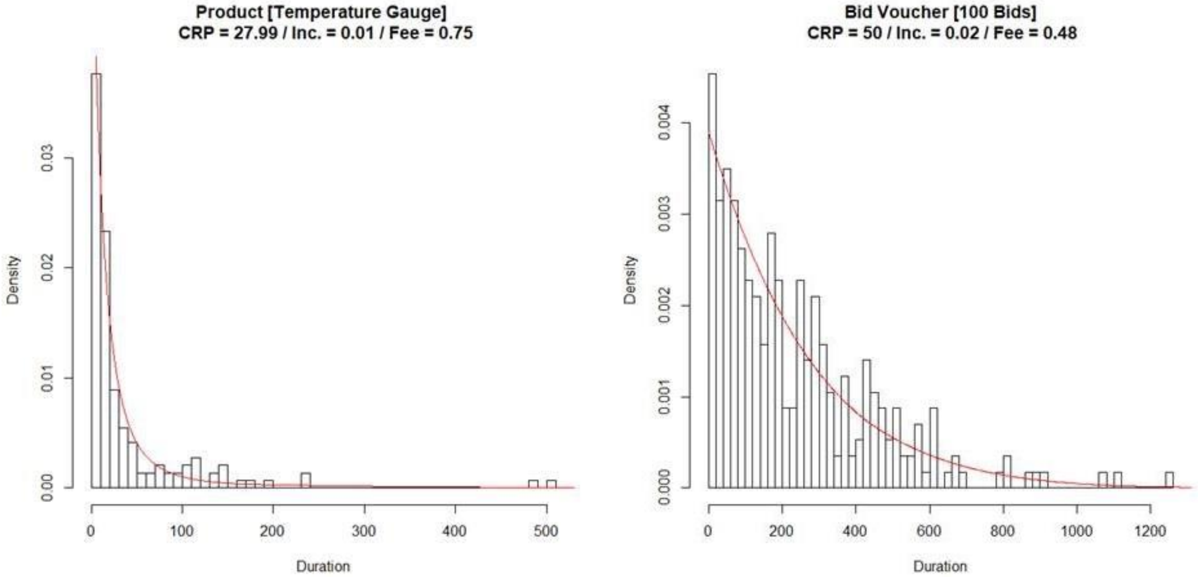
Notes: Current Retail Price = $37, bidding fee = $.60, increment = $.01.

Table 7: Average Parameter Estimates for Different Increments

	Increment		
	$0.01	$0.02	p-value
Risk preference (α)	-0.0294	0.0007	0.2542
Probability weighting (δ)	0.821	0.8025	0.2099
Loss aversion (λ)	3.4055	3.3904	0.1964

Notes: The p-value is based on a paired Wilcox rank sum test.

Figure 1: Empirical versus Estimated Distribution of Auction Duration (in Number of Bids)



Notes: The empirical distribution is indicated by bars and the solid line represents the estimated distribution.

APPENDIX

Proof of Proposition 3

The general principle for the solution with PT preferences is the same as for the case with risk neutral EUT preferences. In each period, bidders choose their probabilities to bid such that bidders in the previous period are indifferent between bidding and not bidding. Equation (4) gives these indifference conditions. The conditions in Equation (4) implicitly assume that preferences satisfy compound independence. Compound independence allows us to replace the PT value of a subgame that follows a decision node with the PT value of its certainty equivalent. Dekel et al. (1991) show that compound independence guarantees dynamic consistency.

Proof of Proposition 4

Part 1

Assume that $\delta = \lambda = 1$. Consider first the case in which t is small, such that $\upsilon - td - b - C_t \geq 0$. A risk neutral bidder is indifferent between bidding or not when

(11) $$-C_{t-1} = p_t\left(-C_{t-1} - b\right) + \left(1 - p_t\right)\cdot\left(\upsilon - d\cdot t - C_{t-1} - b\right).$$

For any strictly concave function f, we have

(12) $$f\left(-C_{t-1}\right) > p_t \cdot f\left(-C_{t-1} - b\right) + \left(1 - p_t\right)\cdot f\left(\upsilon - d\cdot t - C_{t-1} - b\right).$$

Rearranging this equation leads to

(13) $$\begin{aligned}&-p_t \cdot f\left(C_{t-1} + b\right) + \left(1 - p_t\right)\cdot f\left(\upsilon - d\cdot t - C_{t-1} - b\right) + f\left(C_{t-1}\right)\\ &< f\left(C_{t-1}\right) + f\left(-C_{t-1}\right) - p_t \cdot\left(f\left(C_{t-1} + b\right) + f\left(-C_{t-1} - b\right)\right)\end{aligned}$$

The left-hand side is the valuation a PT bidder with $\delta = \lambda = 1$, and concave f attaches to making another bid over not bidding. When the right-hand side is negative, inequality (13) implies that the bidder's valuation of another bid is negative. To make the bidder indifferent between bidding and not bidding, p_t needs to decrease. The right-hand side is negative when

(14) $$p_t < \frac{f\left(C_{t-1}\right) + f\left(-C_{t-1}\right)}{f\left(C_{t-1} + b\right) + f\left(-C_{t-1} - b\right)}.$$

This condition is typically satisfied, especially after a few bids have been placed.

Now consider the case in which t is large, such that $v - t\cdot d - b - C_t < 0$. For any strictly concave function f; the function $g(x) \equiv -f(-x)$ is strictly convex. Thus, we have

(15) $$-p_t \cdot f\left(C_{t-1}+b\right)-\left(1-p_t\right)\cdot f\left(-\left(\upsilon-d\cdot t-C_{t-1}-b\right)\right)>-f\left(C_{t-1}\right).$$

The valuation a PT bidder with δ = λ = 1 and concave *f* attaches to making another bid is greater than the valuation of not bidding. To make the bidder indifferent between bidding and not bidding, p_t needs to increase. The effects of a convex function *f* on the bidding probability can be derived similarly.

Part 2

Assume that δ = 1. Consider first the case in which *t* is small, such that $\upsilon-d\cdot t-b-C_t\geq 0$. The probability to bid is characterized by the following indifference condition:

(16) $$-\lambda\cdot f\left(C_{t-1}\right)=p_t\cdot\left(-\lambda\right)\cdot f\left(C_{t-1}+b\right)+\left(1-p_t\right)\cdot f\left(\upsilon-d\cdot t-C_{t-1}-b\right).$$

Rearranging Equation (16) yields

(17) $$p_t=\frac{\lambda\cdot f\left(C_{t-1}\right)+f\left(\upsilon-d\cdot t-C_{t-1}-b\right)}{\lambda\cdot f\left(C_{t-1}+b\right)+f\left(\upsilon-d\cdot t-C_{t-1}-b\right)}.$$

Taking the derivative with respect to λ yields the following:

(18) $$\frac{\partial p_t}{\partial\lambda}=-\frac{\left(f\left(C_{t-1}+b\right)-f\left(C_{t-1}\right)\right)\cdot f\left(\upsilon-d\cdot t-C_{t-1}-b\right)}{\left(\lambda\cdot f\left(C_{t-1}+b\right)+f\left(\upsilon-d\cdot t-C_{t-1}-b\right)\right)^2},$$

which is negative. When *t* is large, such that $\upsilon-d\cdot t-C_t-b<0$, all possible outcomes are in the loss domain Therefore, λ cancels out and has no effect on the bidding probability p_t.

Part 3

Assume that *f(x)* = *x* and λ = 1. Consider first the case in which *t* is small, such that $\upsilon-d\cdot t-C_t-b\geq 0$. The probability of bidding is characterized by the following indifference condition:

(19) $$-C_{t-1}=-\frac{\left(p_t\right)^{\delta}}{\left(p_t^{\delta}+\left(1-p_t\right)^{\delta}\right)^{1/\delta}}\cdot\left(C_{t-1}+b\right)+\frac{\left(1-p_t\right)^{\delta}}{\left(p_t^{\delta}+\left(1-p_t\right)^{\delta}\right)^{1/\delta}}\cdot\left(\upsilon-d\cdot t-C_{t-1}-b\right).$$

Using the implicit-function rule, we obtain the derivative of the probability p_t with respect to δ around the point $\delta = 1$:

(20) $$\frac{\partial p_t}{\partial\delta}\Big|_{\delta=1}=\frac{\left(1-p_t\right)\cdot\log\left(1-p_t\right)\cdot\left(\upsilon-d\cdot t-b\right)-p_t\cdot\log\left(p_t\right)\cdot b}{\upsilon-d\cdot t}.$$

This derivative is negative if $\left(1-p_t\right)\cdot log\left(1-p_t\right)\cdot\left(\upsilon-d\cdot t-b\right)<p_t\cdot log\left(p_t\right)\cdot b$. Using $p_t=1-\frac{b}{\upsilon-d\cdot t}$ for δ = 1, this condition simplifies to $\upsilon-d\cdot t-2\cdot b>0$. Because we consider the case in which $\upsilon-d\cdot t-C_t-b\geq 0$, this condition is satisfied.

Now consider the case in which t is large, such that $\upsilon-d\cdot t-b-C_t<0$. The indifference condition is then given by

(21) $$-C_{t-1}=-\frac{p_t^{\delta}}{\left(p_t^{\delta}+\left(1-p_t\right)^{\delta}\right)^{1/\delta}}\cdot\left(C_{t-1}+b\right)+\left(1-\frac{p_t^{\delta}}{\left(p_t^{\delta}+\left(1-p_t\right)^{\delta}\right)^{1/\delta}}\right)\cdot(\upsilon-d\cdot t-C_{t-1}-b),$$

which simplifies to

(22) $$\frac{p_t^{\delta}}{\left(p_t^{\delta}+\left(1-p_t\right)^{\delta}\right)^{1/\delta}}=1-\frac{b}{\upsilon-d\cdot t}.$$

Using the implicit-function rule, we obtain the derivative of the probability p_t with respect to δ around the point δ = 1:

(23) $$\frac{\partial p_t}{\partial\delta}\Big|_{\delta=1}=p_t\cdot\left(1-p_t\right)\cdot log\left(\left(1-p_t\right)/p_t\right).$$

This derivative is negative if $1-p_t>p_t$ or $p_t<1/2$. Using $p_t=1-\frac{b}{\upsilon-d\cdot t}$ for δ = 1, this condition becomes $t<\frac{\upsilon-2\cdot b}{d}$. Therefore, $\frac{\partial p_t}{\partial\delta}\Big|_{\delta=1}<0$, for $t\in\left[0,\frac{\upsilon-2\cdot b}{d}\right]$, and $\frac{\partial p_t}{\partial\delta}\Big|_{\delta=1}>0$, for $t\in\left[\frac{\upsilon-2\cdot b}{d},\frac{\upsilon-b}{d}\right]$.

Proof of Proposition 5

Part 1

We show that a naïve bidder prefers to make exactly one bid instead of placing two bids; that is,

(24) $$EV_t\left(p_t=1,p_{t+2}=0\right)\geq EV_t\left(p_t=1,p_{t+2}=1,p_{t+4}=0\right).$$

Consider first the case in which t is small such that $\upsilon-\left(t+3\right)\cdot d-2\cdot b-C_t\geq 0$:

(25) $$EV_t\left(p_t=1,p_{t+2}=0\right)=\varpi\left(1-p_{t+1}\right)\cdot f\left(\upsilon-\left(t+1\right)\cdot d-C_t-b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(C_t+b\right)$$

and

(26)
$$EV_t\left(p_t=1,p_{t+2}=1,p_{t+4}=0\right)=\varpi\left(1-p_{t+1}\right)\cdot f\left(\upsilon-(t+1)\cdot d-C_t-b\right)$$
$$+\left[\varpi\left(1-p_{t+1}+p_{t+1}\cdot\left(1-p_{t+3}\right)\right)-\varpi\left(1-p_{t+1}\right)\right]\cdot f\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)$$
$$+\varpi\left(p_{t+1}\cdot p_{t+3}\right)\cdot(-\lambda)\cdot f\left(C_t+2\cdot b\right).$$

Inequality (24) becomes

(27)
$$\varpi\left(p_{t+1}\right)\cdot(-\lambda)\cdot f\left(C_t+b\right)\geq\left[\varpi\left(1-p_{t+1}+p_{t+1}\cdot\left(1-p_{t+3}\right)\right)-\varpi\left(1-p_{t+1}\right)\right]$$
$$\cdot f\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)+\varpi\left(p_{t+1}\cdot p_{t+3}\right)\cdot(-\lambda)\cdot f\left(C_t+2\cdot b\right).$$

We know from Proposition 3 that a sophisticated bidder without commitment chooses p_{t+3} such that

(28)
$$-\lambda\cdot f\left(C_t+b\right)=\varpi\left(p_{t+3}\right)\cdot(-\lambda)\cdot f\left(C_t+2\cdot b\right)+\varpi\left(1-p_{t+3}\right)\cdot f\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right).$$

Multiplying Equation (28) by $\varpi\left(p_{t+1}\right)$ and plugging into Inequality (27) yields

(29)
$$\left[\varpi\left(1-p_{t+1}\right)+\varpi\left(p_{t+1}\right)\cdot\varpi\left(1-p_{t+3}\right)-\varpi\left(1-p_{t+1}+p_{t+1}\cdot\left(1-p_{t+3}\right)\right)\right]\cdot f\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)$$
$$\geq\left[\varpi\left(p_{t+1}\cdot p_{t+3}\right)-\varpi\left(p_{t+1}\right)\cdot\varpi\left(p_{t+3}\right)\right]\cdot(-\lambda)\cdot f\left(C_t+2\cdot b\right).$$

Since $\varpi\left(p\cdot q\right)\geq\varpi\left(p\right)\cdot\varpi\left(q\right)$, for every p, $q\in$ [0,1], the right-hand side of this inequality is negative. Thus,

(30)
$$\varpi\left(1-p_{t+1}\right)+\varpi\left(p_{t+1}\right)\cdot\varpi\left(1-p_{t+3}\right)\geq\varpi\left(1-p_{t+1}+p_{t+1}\cdot\left(1-p_{t+3}\right)\right)$$

is a sufficient condition for this inequality to hold. For the case $\upsilon-(t+3)\cdot d-C_t-2\cdot b\geq 0$, inequality (30) holds for all reasonable levels of probability weighting. E.g. when δ =0.7 inequality (30) holds when the weighted probabilities $\varpi\left(p_{t+1}\right)$ and $\varpi\left(p_{t+3}\right)$ are greater than 0.348. When $\upsilon-(t+3)\cdot d-C_t-2\cdot b\geq 0$, $\varpi\left(p_{t+1}\right)$ and $\varpi\left(p_{t+3}\right)$ cannot be below $f(C_t+b)/f(C_t+2b)$, which is greater than 0.348, unless the function f is highly convex.

Now consider the case where $\upsilon-d\cdot(t+1)-C_t-2\cdot b\leq 0$.

(31)
$$EV_t\left(p_t=1,p_{t+2}=0\right)=\left(1-\varpi\left(p_{t+1}\right)\right)\cdot(-\lambda)\cdot f\left(-\left(\upsilon-(t+1)\cdot d-C_t-b\right)\right)+\varpi\left(p_{t+1}\right)\cdot(-\lambda)\cdot f\left(C_t+b\right)$$

and

(32)
$$EV_t\left(p_t=1,p_{t+2}=1,p_{t+4}=0\right)=\left(1-\varpi\left(p_{t+1}\right)\right)\cdot(-\lambda)\cdot f\left(-\left(\upsilon-(t+1)\cdot d-C_t-b\right)\right)$$
$$+\left[\varpi\left(p_{t+1}\right)-\varpi\left(p_{t+1}\cdot p_{t+3}\right)\right]\cdot(-\lambda)\cdot f\left(-\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)\right)$$
$$+\varpi\left(p_{t+1}\cdot p_{t+3}\right)\cdot(-\lambda)\cdot f\left(C_t+2\cdot b\right).$$

Inequality (24) becomes

(33) $$\varpi(p_{t+1})\cdot(-\lambda)\cdot f(C_t+b)\geq\left[\varpi(p_{t+1})-\varpi(p_{t+1}\cdot p_{t+3})\right]\cdot(-\lambda)\cdot f\left(-\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)\right)$$
$$+\varpi(p_{t+1}\cdot p_{t+3})\cdot(-\lambda)\cdot f(C_t+2\cdot b).$$

We know from Proposition 3 that a sophisticated bidder without commitment chooses p_{t+3} such that

(34) $$-\lambda\cdot f(C_t+b)=\varpi(p_{t+3})\cdot(-\lambda)\cdot f(C_t+2\cdot b)+\left(1-\varpi(p_{t+3})\right)\cdot(-\lambda)\cdot f\left(-\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)\right)$$

Multiplying Equation (34) by $\varpi(p_{t+1})$ and plugging into Inequality (33) yields

(35) $$\varpi(p_{t+1})\cdot\varpi(p_{t+3})\cdot(-\lambda)\cdot f(C_t+2\cdot b)+\varpi(p_{t+1})\cdot\left(1-\varpi(p_{t+3})\right)\cdot(-\lambda)\cdot f\left(-\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)\right)$$
$$\geq\left[\varpi(p_{t+1})-\varpi(p_{t+1}\cdot p_{t+3})\right]\cdot(-\lambda)\cdot f\left(-\left(\upsilon-(t+3)\cdot d-C_t-2\cdot b\right)\right)+\varpi(p_{t+1}\cdot p_{t+3})\cdot(-\lambda)\cdot f(C_t+2\cdot b).$$

Simplifying Inequality (35) yields the following condition:

(36) $$(t+3)\leq\frac{\upsilon}{d},$$

which is satisfied for all $t\leq T$.

Part 2

Given part (1) a naïve bidder's decision to bid or not is the same as that of a sophisticated bidder without commitment. Consequently, the naïve bidder is also indifferent between bidding or not.

Proof of Proposition 6

Part 1

$f(x)=x$: A bidder who has not yet placed a bid in the auction is indifferent between bidding at time t or not if

(37) $$\varpi(1-p_{t+1})\cdot(\upsilon-d\cdot t-b)+\varpi(p_{t+1})\cdot(-\lambda)\cdot b=0.$$

If a bidder has already accumulated bidding fees of C_t, with $\upsilon-d\cdot t-b-C_t\geq 0$, her valuation of another bid is

(38) $$\begin{aligned}&\varpi(1-p_{t+1})\cdot(\upsilon-d\cdot t-C_t-b)+\varpi(p_{t+1})\cdot(-\lambda)\cdot(C_t+b)\\&=\varpi(1-p_{t+1})\cdot(\upsilon-d\cdot t-b)-\varpi(1-p_{t+1})\cdot C_t+\varpi(p_{t+1})\cdot(-\lambda)\cdot b+\varpi(p_{t+1})\cdot(-\lambda)\cdot C_t\\&=-\varpi(1-p_{t+1})\cdot C_t+\varpi(p_{t+1})\cdot(-\lambda)\cdot C_t\\&\quad\geq\left[\varpi(1-p_{t+1})+\varpi(p_{t+1})\right]\cdot(-\lambda)\cdot C_t\\&\quad\geq(-\lambda)\cdot C_t.\end{aligned}$$

Inserting Equation (37) into Inequality (38) yields the second equality, and the two inequalities use loss aversion, $\lambda \geq 1$, and subcertainty, respectively.

If a bidder has already accumulated bidding fees of C_t, with $\upsilon - d \cdot t - b - C_t \geq 0$, her valuation of another bid is

(39)
$$\begin{aligned} &\left(1-\varpi\left(p_{t+1}\right)\right)\cdot\lambda\cdot\left(\upsilon-d\cdot t-C_t-b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot\left(C_t+b\right) \\ &=\left(1-\varpi\left(p_{t+1}\right)\right)\cdot\lambda\cdot\left(\upsilon-d\cdot t-b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot b-\lambda\cdot C_t \\ &=\left[\left(1-\varpi\left(p_{t+1}\right)\right)\cdot\lambda-\varpi\left(1-p_{t+1}\right)\right]\cdot\left(\upsilon-d\cdot t-b\right)-\lambda\cdot C_t \\ &\geq\left(-\lambda\right)\cdot C_t. \end{aligned}$$

Inequality (39) uses loss aversion, $\lambda \geq 1$, and subcertainty. Thus, the value of placing another bid is strictly greater than the value of not bidding if at least one of these inequalities (loss aversion and subcertainty) is strict.

Part 2

$f(x)$ is concave: A bidder who has not yet placed a bid in the auction is indifferent between bidding at time *t* or not if

(40)
$$\varpi\left(1-p_{t+1}\right)\cdot f\left(\upsilon-d\cdot t-b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(b\right)=0.$$

If a bidder has already accumulated bidding fees of C_t, with $\upsilon - d \cdot t - b - C_t \geq 0$, her valuation of another bid is

(41)
$$\begin{aligned} &\varpi\left(1-p_{t+1}\right)\cdot f\left(\upsilon-d\cdot t-C_t-b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(C_t+b\right) \\ &\geq\varpi\left(1-p_{t+1}\right)\cdot f\left(\upsilon-d\cdot t-b\right)-\varpi\left(1-p_{t+1}\right)\cdot f\left(C_t\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(C_t\right) \\ &=-\varpi\left(1-p_{t+1}\right)\cdot f\left(C_t\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(C_t\right) \\ &\geq\left[\varpi\left(1-p_{t+1}\right)+\varpi\left(p_{t+1}\right)\right]\cdot\left(-\lambda\right)\cdot f\left(C_t\right) \\ &\geq\left(-\lambda\right)\cdot f\left(C_t\right). \end{aligned}$$

The first inequality uses the concavity of f; inserting Equation (40) yields the equality, and the last two inequalities use loss aversion, $\lambda \geq 1$, and subcertainty, respectively. Thus, the value of placing another bid is strictly greater than the value of not bidding if at least one of these inequalities is strict.

If the accumulated bidding fees of C_t exceed $\upsilon - d \cdot t - b$, then the valuation of another bid is

(42)
$$\begin{aligned} &\left(1-\varpi\left(p_{t+1}\right)\right)\cdot\left(-\lambda\right)\cdot f\left(C_t-\left(\upsilon-d\cdot t-b\right)\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(C_t+b\right) \\ &\geq\left(1-\varpi\left(p_{t+1}\right)\right)\cdot\left(-\lambda\right)\cdot f\left(C_t-\left(\upsilon-d\cdot t-b\right)\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(C_t\right) \\ &=\left(1-\varpi\left(p_{t+1}\right)\right)\cdot\left(-\lambda\right)\cdot f\left(C_t-\left(\upsilon-d\cdot t-b\right)\right)-\varpi\left(1-p_{t+1}\right)\cdot f\left(\upsilon-d\cdot t-b\right)+\varpi\left(p_{t+1}\right)\cdot\left(-\lambda\right)\cdot f\left(C_t\right). \end{aligned}$$

The inequality uses the concavity of f; inserting Equation (40) yields the equality. This expression is greater than or equal to $(-\lambda)\cdot f(C_t)$, if

(43) $$\lambda\cdot\frac{1-\varpi(p_{t+1})}{\varpi(1-p_{t+1})}\geq\frac{f(\upsilon-d\cdot t-b)}{f(C_t)-f(C_t-(\upsilon-d\cdot t-b))}\equiv\rho_1.$$

Part 3

$f(x)$ is convex: If a bidder has already accumulated bidding fees of C_t and $\upsilon-d\cdot t-b-C_t\geq 0$, her valuation of another bid is

(44) $$\begin{aligned}&\varpi(1-p_{t+1})\cdot f(\upsilon-d\cdot t-C_t-b)+\varpi(p_{t+1})\cdot(-\lambda)\cdot f(C_t+b)\\ &\geq\varpi(1-p_{t+1})\cdot\left[f(\upsilon-d\cdot t-b)-C_t\cdot f'(\upsilon-d\cdot t-b)\right]+\varpi(p_{t+1})\cdot(-\lambda)\cdot\left[f(b)+C_t\cdot f'(\upsilon-d\cdot t-b)\right]\\ &=-\varpi(1-p_{t+1})\cdot C_t\cdot f'(\upsilon-d\cdot t-b)+\varpi(p_{t+1})\cdot(-\lambda)\cdot C_t\cdot f'(\upsilon-d\cdot t-b).\end{aligned}$$

The inequality sign follows from the convexity of f; inserting Equation (40) yields the equality. This expression is greater than or equal to $(-\lambda)\cdot f(C_t)$, if

(45) $$\lambda\cdot\frac{1-\varpi(p_{t+1})}{\varpi(1-p_{t+1})}\geq\frac{1-\varpi(p_{t+1})}{\frac{f(C_t)}{C_t}\Big/f'(\upsilon-d\cdot t-b)-\varpi(p_{t+1})}\equiv\rho_a.$$

If the accumulated bidding fees of C_t exceed $\upsilon-d\cdot t-b$, her valuation of another bid is

(46) $$\begin{aligned}&(1-\varpi(p_{t+1}))\cdot(-\lambda)\cdot f(C_t-(\upsilon-d\cdot t-b))+\varpi(p_{t+1})\cdot(-\lambda)\cdot f(C_t+b)\\ &\geq(1-\varpi(p_{t+1}))\cdot(-\lambda)\cdot\left[f(C_t)-f(\upsilon-d\cdot t-b)\right]+\varpi(p_{t+1})\cdot(-\lambda)\cdot f(C_t+b)\\ &=(1-\varpi(p_{t+1}))\cdot(-\lambda)\cdot f(C_t)+(1-\varpi(p_{t+1}))\cdot(-\lambda)^2\cdot\frac{\varpi(p_{t+1})}{\varpi(1-p_{t+1})}\cdot f(b)+\varpi(p_{t+1})\cdot(-\lambda)\cdot f(C_t+b).\end{aligned}$$

The inequality sign follows from the convexity of f; inserting Equation (40) yields the equality. This expression is greater than or equal to $(-\lambda)\cdot f(C_t)$, if

(47) $$\lambda\cdot\frac{1-\varpi(p_{t+1})}{\varpi(1-p_{t+1})}\geq\frac{f(C_t+b)-f(C_t)}{f(b)}\equiv\rho_b.$$

Thus, the value of placing another bid is strictly greater than the value of not bidding if

$$\lambda\cdot\frac{1-\varpi(p_{t+1})}{\varpi(1-p_{t+1})}\geq max\{\rho_a,\rho_b\}\equiv\rho_2.$$