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# Call for Papers Winter\_Issue 2016

## Journal of Mathematical Economics and Finance

*Journal of Mathematical Economics and Finance* (JMEF) is a biannually peer-reviewed journal of *Association for Sustainable Education, Research and Science*.

**Aims and Scope.** The primary scope of our Journal is to provide a forum to exchange ideas in economic theory which expresses economic and financial concepts and laws using formal and well-constructed mathematical reasoning, Mathematical Decision Theory, Game Theory, Functional Analysis, Differential Geometry and so on.

Our Journal covers and publishes original researches and new significant results and methods of Mathematical Economics, Finance, Game Theory and applications, mathematical methods of economics, finance and management, Quantitative Decision theory and Risk Theory. The mathematical form of economic and financial laws appears of fundamental importance to the developments and deep understanding of Economics and Finance themselves. Such a translation in mathematical terms can determine whether an economic or financial intuition shows a coherent and logical meaning. Also, a full rational and mathematical development of economic ideas can itself suggest new economic concepts and deeper economic intuitions.

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**Refereeing process.** Invited manuscripts will be due till June 15<sup>th</sup>, 2017, and shall go through the usual double blind refereeing process.

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## Insurance-markets Equilibrium with Sequential Non-convex Private and Public-Sector Labor Supply

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**Abstract.** *This paper describes the lottery- and insurance-market equilibrium in an economy with non-convex private- and public-sector employment. In contrast to Vasilev 2017, 2015, the public-sector labor supply decision is a sequential one. This requires two separate insurance market to operate, one for private-sector work, and one for public-sector employment. In addition, given that the labor choice for private- and public-sector hours is made in succession, the insurance market for public employment needs to open once the other insurance market has closed. This segmentation and sequentiality of insurance markets operation is a new result in the literature and a direct consequence of the double non-convexity, and the sequential nature of the sectoral labor supply decision.*

**Keywords:** Indivisible Labor, Public Employment, Sequential Lotteries, Insurance.

**JEL Classification:** E1, J22, J45.

### 1. Introduction

This paper explores the problem of non-convex labor supply decisions in an economy with both private and public sector jobs. In contrast to Vasilev 2017, the sectoral labor choice is made in a sequential manner. In contrast to this earlier study, the focus of the present note falls on the lottery- and insurance-market equilibrium for the setup in Vasilev 2017.<sup>1</sup> The main result is that in the presence of non-convex

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<sup>1</sup>In an separate line of research, Vasilev 2016 extends Hansen and Sargent 1988 with a sequential overtime decision. More specifically, the problem is one of two-stage non-convex labor supply decisions in an economy where agents first decide whether to participate in the labor market or stay unemployed, and then, conditional on being hired, need to decide whether they will work only the full-time equivalent, or engage in overtime hours. We follow this approach here with the sequential labor choice as well.

labor supply for both private- and public-sector employment, and when the public sector labor decision is assumed to be sequential, the setup requires two separate insurance market to operate, one for private sector work, and one for public sector employment. In addition, given that the sectoral labor choice is made in succession, the insurance market for public-sector hours needs to open only after the insurance market for private-sector employment has already closed. This sequentiality of insurance markets operation is a new result in the literature and a direct consequence of the sequential nature of the sectoral labor decision.

## 2. Model Setup

The theoretical setup follows to a great extent Vasilev 2017, 2015, except for the timing of the sectoral labor supply decisions. The economy is static, there is no physical capital, and agents face a sequential non-convex decision in a two-sector economy.<sup>2</sup> Since the focus is on a one-period world, the model abstracts away from technological progress, population growth and uncertainty. There is a large number of identical one-member households, indexed by  $i$  and distributed uniformly on the  $[0, 1]$  interval. In the exposition below, we will use small case letters to denote individual variables and suppress the index  $i$  to save on notation.

### 2.1 Households

Each household solves the following optimization problem:

$$\max_{(c, S, h^p, h^g) \in \mathcal{K}} \ln[c^\eta + S^\eta]^{\frac{1}{\eta}} + \alpha \ln(1 - h^p - h^g) \quad (1)$$

where  $c, S, h^p, h^g$  denote private consumption, consumption of the public good, hours worked in the private sector, and hours worked in the government sector and their quadruples  $(c, S, h^p, h^g)$  belong to a feasible fixed compact subset  $\mathcal{K}$  of the space  $\mathbb{R}^4$ . The parameter

$$\alpha > 1$$

measures the relative weight of leisure in the utility function. Total consumption is a Constant Elasticity of Substitution (CES) aggregation of private consumption and consumption of government services, where

$$\eta > 0$$

measures the degree of substitutability between private and public consumption.<sup>3</sup>

---

<sup>2</sup>Adding physical capital accumulation decision, and a dynamic structure to the model is then straightforward. Also, the absence of those elements in the current analysis does not affect in any major way the derivation of the optimality conditions characterizing the aggregate labor supply decisions.

<sup>3</sup>The separability of consumption and leisure is not a crucial assumption for the results that follow. A more general, non-separable, utility representation, does not generate new results, while significantly complicates the algebraic derivations, and thus interferes with model tractability.

Each household is endowed with 1 unit of time that can be allocated to work in the private sector, work in the government sector, or leisure

$$h^p + h^g + l = 1. \quad (2)$$

Labor supply in each sector is assumed to be discrete

$$h^p \in \{0, \bar{h}^p\}, \quad h^g \in \{0, \bar{h}^g\}.$$

In contrast to Vasilev 2017, within the period, each household decides first to look for a job in the private sector, and if unsuccessful, will search for work in the public sector. The wage rate per hour worked in the private and public sectors is  $w^p$  and  $w^g$ , respectively.

In addition to labor income income, households hold shares in the private firm and receives an equal profit share  $\pi$ , with

$$\int_0^1 \pi di = \Pi.^4$$

Income is subject to a (equal) lump-sum tax  $t$ , where

$$\int_0^1 t di = T,$$

with  $T$  denoting aggregate tax revenue. Therefore, each household's budget constraint is

$$c^j \leq w^j h^j + \pi - t, \quad \text{with } j = p, g. \quad (3)$$

Households act competitively by taking the wage rates

$$\{w^p, w^g\},$$

aggregate outcomes

$$\{C, S, H^p, H^g\}$$

and lump-sum taxes  $\{T\}$  as given. Each household chooses

$$(c^j, h^p, h^g)$$

to maximize the function

$$U(c, h^p, h^g) = \ln[c^\eta + S^\eta]^{\frac{1}{\eta}} + \alpha \ln(1 - h^p - h^g),$$

upon the constraints given by 2 and 3.

---

<sup>4</sup>This technical assumption guarantees a positive consumption to either of the two types, even if they choose not to work in their sector.

### 3. Firms

There is a representative firm in the private sector producing a homogeneous final consumption good, which uses labor as an only input. The production function is given by

$$Y = F(H^p), \quad F' > 0, \quad F'' < 0, \quad F'(\bar{H}^p) = 0, \quad (4)$$

where the last assumption is imposed to proxy a capacity constraint. The firm acts competitively by taking the hourly wage rate

$$\{w^p\},$$

aggregate outcomes

$$\{C, S, H^g\}$$

and policy variable

$$\{T\}$$

as given. Accordingly,  $\{H^p\}$  is chosen to maximize static aggregate profit:<sup>5</sup>

$$\max_{H^p \geq 0} F(H^p) - w^p H^p. \quad (5)$$

Given the assumption imposed on the production function, in equilibrium, the firm will realize positive economic profit.

### 4. Government

The government hires employees to provide public services and levies lump-sum taxes on households to finance the government wage bill. The technology of the public good provision uses labor  $H^g$  as an input, which is remunerated at a non-competitive wage rate

$$w^g = \gamma w^p.$$

Parameter  $\gamma \geq 1$  will measure the fixed gross mark-up of government sector wage rate over the private sector one.<sup>6</sup> The production function of public services is as follows:

$$S = S(H^g), \quad S' > 0, \quad S'' < 0, \quad S'(\bar{H}^g) = 0, \quad (6)$$

where the last assumption states that due to a capacity constraint, not everyone can work in the production of the public good.

The government runs a balanced budget: The public sector wage bill is financed by levying a lump-sum tax  $T$  on all households

$$w^g H^g = T. \quad (7)$$

---

<sup>5</sup>This representation can be viewed as being isomorphic to a problem in which capital has already been optimized over.

<sup>6</sup>Such a mark-up is a stylized fact for the major EU economies, e.g. Vasilev 2015

In terms of fiscal instruments available at the government's disposal, the government takes total public sector hours,  $H^g$ , as given, and sets the public sector wage rate,  $w^g$ , as a fixed gross mark-up above the competitive wage rate.  $T$  will be then residually chosen to guarantee that the budget is balanced.

#### 4.1 Insurance Markets

Alternatively, we could regard the labor selection arrangement as follows: the workers are participating in a compound (two-stage) lottery with the proportions representing the probability of being selected for work. Conditional on the sequential labor choice, a household would receive the same income in expected terms. Lastly, we can introduce insurance markets, and allow households to buy insurance, which would allow them to equalize the actual income received. Given the observed difference in the private- and public-sector wages, and the sequential nature of sectoral non-convex labor supply decision, sequential and segmented insurance markets are also needed in order to provide actuarially fair insurance.

There is one representative insurance company for private sector employment, and one for public sector hours. The two companies are segmented and operate in sequence. At the beginning of each period, the households decide if and how much insurance to buy against the probability of being chosen for private-sector work. Then, the company closes, and the insurance company for public-sector work opens. In both cases insurance costs  $q^j$  per unit,  $j = p, g$ , and provides one unit of income if the household is not working. We can think of insurance as bonds that pay out only in case the household is not chosen for work. Thus, household will also choose the quantity of insurance to purchase  $b^j$ ,  $j = p, g$ . This setup requires that the overtime insurance company insures workers who have already been selected for work in the first stage. In this sense, the insurance markets are segmented as well. Without the segmented and sequential nature of the insurance markets described above, insurance will not be actuarially fair, one of the groups will face better odds versus price, the company will not be able to break even, and/or at least one type of households will not be able to buy full insurance, which would completely smooth consumption across employment states, given the non-convexity constraint of labor supply.

**4.1.1 Private-Sector Insurance company.** The insurance company for straight time maximizes profit. The company services all households. It receives revenue if a household is working and makes payment if it is not. More specifically, the proportion of people working in the private sector contribute towards the unemployment benefits pool, which are then distributed of benefits to the unemployed. The amount of insurance sold is a solution to the following problem.

Taking  $q^p(i)$  as given,  $b^p(i)$  solves

$$\max_{b^p(i)} \lambda^p(i)q^p(i)b^p(i) - [1 - \lambda^p(i)]b^p(i). \quad (8)$$



With free entry profits are zero, hence

$$\lambda^p(i)q^p(i)b^p(i) - [1 - \lambda^p(i)]b^p(i) = 0. \quad (9)$$

This condition implicitly clears the insurance market for each household.

**4.1.2 Public-sector insurance company.** The insurance company for public sector employment maximizes profit as well. The company opens once the other insurance company has already closed, and services only the households that have been not been selected for private-sector work in the first stage. It receives revenue if a household is working in the ublic sector, and makes payment if it is not. More specifically, the proportion of people working in the public sector contribute towards the unemployment benefits pool. The amount of insurance sold is a solution to the following problem.

Taking  $q^p(i)$  as given,  $b^g(i)$  solves

$$\max_{b^g(i)} \lambda^g(i)q^g(i)b^g(i) - [1 - \lambda^g(i)]b^g(i). \quad (10)$$

With free entry profits are zero, hence

$$\lambda^g(i)q^g(i)b^g(i) - [1 - \lambda^g(i)]b^g(i) = 0. \quad (11)$$

This condition implicitly clears the insurance market for each household, conditional on not being selected in the first stage for work in the private sector.

In the next section, the equilibrium with lotteries and no insurance markets is presented and discussed first, and then the setup is extended to incorporate a regime with insurance.

## 5. Decentralized Competitive Equilibrium (DCE) with lotteries

### 5.1 Definition of the DCE with lotteries

A competitive Equilibrium **with sequential Lotteries** for this economy is a list

$$(c^u(i), c^p(i), c^g(i), q^p(i), q^g(i), \lambda^p(i), \lambda^g(i), \bar{h}^p, \bar{h}^g, w^p, w^g, \pi)$$

such that the following conditions are fulfilled.

1. **Consumers maximization condition.** Taking prices  $w^p, w^g, \pi$  as given, for each  $i$ , the sequence

$$\sigma = (c^u(i), c^p(i), c^g(i), q^p(i), q^g(i), \lambda^p(i), \lambda^g(i))$$

solves the maximization problem

$$\max_{\sigma \in \Sigma} q(i) \left\{ \lambda^p(i) \left[ \ln(c^p) + \alpha \ln(1 - \bar{h}^p) \right] + \lambda^g(i) [1 - \lambda^p(i)] \left[ \ln(c^g) + \alpha \ln(1 - \bar{h}^g) \right] \right\} + (1 - \lambda^g(i)) [1 - \lambda^p(i)] \ln(c^u), \quad (12)$$

where

$$\begin{aligned} \lambda^p(i)c^p + \lambda^g(i)(1 - \lambda^p(i))c^g + (1 - \lambda^g(i))(1 - \lambda^p(i))c^u = \\ = \lambda^p(i)w^p\bar{h}^p + \lambda^g(i)[1 - \lambda^p(i)]w^g\bar{h}^g + \pi, \end{aligned} \quad (13)$$

with

$$c^p, c^g, c^u \geq 0, \quad 0 < \lambda^p(i), \lambda^g(i) < 1, \quad (14)$$

where  $\Sigma$  is the constraint defined by the above relations 13, 14.

2. **Firm maximization condition.** Taking prices  $w^p, w^g, \pi$  as given,

$$\max_{\bar{H}^p} F(\bar{H}^p) - w^p \bar{H}^p \quad \text{s.t.} \quad \bar{H}^p \geq 0. \quad (15)$$

3. **Government sector condition.** Taking prices  $w^p, w^g, \pi$  as given,  $T$  is chosen to balance the government budget

$$T = w^g \bar{H}^g \quad (16)$$

and

$$w^g = \gamma w^p, \quad (17)$$

$$S^g = S(\bar{H}^g). \quad (18)$$

4. **Market clearing condition.** We have

$$\int_i \lambda^p(i) \bar{h}^p di = \bar{H}^p \quad (19)$$

$$\int_i \lambda^g(i) (1 - \lambda^p(i)) \bar{h}^g di = \bar{H}^g \quad (20)$$

$$\int_i \left\{ \lambda^p(i)c^p + \lambda^g(i)[1 - \lambda^p(i)]c^g + (1 - \lambda^g(i))[1 - \lambda^p(i)]c^u \right\} di = F(\bar{H}^p). \quad (21)$$

## 5.2 Characterizing the equilibrium

The household's problem is as follows:

$$\begin{aligned} \mathcal{L} = \max_{\sigma \in \Sigma} \lambda^p(i) & \left[ \ln(c^p) + \alpha \ln(1 - \bar{h}^p) \right] + \lambda^g(i)[1 - \lambda^p(i)] \left[ \ln(c^g) + \alpha \ln(1 - \bar{h}^g) \right] + \\ & + (1 - \lambda^g(i))[1 - \lambda^p(i)] \ln(c^u) - \mu \left[ \lambda^p(i)c^p + \lambda^g(i)(1 - \lambda^p(i))c^g + \right. \\ & \left. + (1 - \lambda^g(i))(1 - \lambda^p(i))c^u - \lambda^p(i)w^p\bar{h}^p - \lambda^g(i)[1 - \lambda^p(i)]w^g\bar{h}^g - \pi \right] \end{aligned} \quad (22)$$

where  $\mu$  is the Lagrangian multiplier in front of the households' budget constraint.

The first-order optimality conditions are as follows:

$$c^u : \frac{(1 - \lambda^g(i))[1 - \lambda^p(i)]}{c^u} = \mu(1 - \lambda^g(i))[1 - \lambda^p(i)] \quad (23)$$

$$c^p : \frac{q\lambda^p(i)}{c^p} = \mu\lambda^p(i) \quad (24)$$

$$c^g : \frac{\lambda^g(i)[1 - \lambda^p(i)]}{c^g} = \mu\lambda^g(i)[1 - \lambda^p(i)]. \quad (25)$$

It follows that

$$c^u = c^p = c^g = 1/\mu.$$

We simplify the Lagrangian by suppressing all consumption superscripts and  $i$  notation in the derivations to follow

$$\lambda^p(i) : \alpha \ln(1 - \bar{h}^p) - \lambda^g \alpha \ln(1 - \bar{h}^g) = -\mu[w^p\bar{h}^p - \lambda^g w^g \bar{h}^g] \quad (26)$$

$$\lambda^g(i) : \alpha \ln(1 - \bar{h}^g) = \mu w^g \bar{h}^g. \quad (27)$$

This equation is a discrete version of the marginal product of labor equals the marginal rate of substitution. It implicitly characterizes optimal  $\lambda^g$ .

Note that it is optimal from the benevolent planner/government point of view to choose randomly  $\lambda^p$  and  $\lambda^g$  and to introduce uncertainty. With randomization, choice sets are convexified, and thus market completeness is achieved. A household is exposed to risk: first, it can be chosen to work with some probability; second, conditional on not being chosen to work in the private sector, it can be picked to provide government labor services. Given the risk in the economic environment, it would be optimal to have insurance. The government sells employment lotteries, and individuals will buy insurance to cover any risk exposure. With insurance, the employer pays wage to individuals only if they work. Now we extend the commodity space a little

bit to include insurance markets explicitly.

## 6. Decentralized Competitive Equilibrium with lotteries and insurance markets

### 6.1 Definition of the equilibrium with insurance markets

A competitive Equilibrium **with sequential Lotteries and insurance markets** for this economy is a list

$$(c^u(i), c^p(i), c^g(i), q^p(i), q^g(i), \bar{h}^p, \bar{h}^g, b^p, b^g, w^p, w^g, \pi, \lambda^p(i), \lambda^g(i))$$

such that the following conditions are fulfilled.

1. **Consumers maximization condition.** Taking prices  $w^p, w^g, \pi$  as given, for each  $i$ ,

$$\sigma = (c^u(i), c^p(i), c^g(i), q^p(i), q^g(i), \lambda^p(i), \lambda^g(i))$$

solves

$$\begin{aligned} \max_{\sigma \in \Sigma} \lambda^p(i) & \left[ \ln(c^p) + \alpha \ln(1 - \bar{h}^p) \right] + \lambda^g(i)[1 - \lambda^p(i)] \left[ \ln(c^g) + \right. \\ & \left. + \alpha \ln(1 - \bar{h}^g) \right] + (1 - \lambda^g(i))[1 - \lambda^p(i)] \ln(c^u) \end{aligned} \quad (28)$$

such that

$$c^p + b^p q^p = w^p \bar{h}^p + \pi \quad (29)$$

$$c^g + b^g q^g = b^p + w^g \bar{h}^g + \pi \quad (30)$$

$$c^u = b^g + \pi \quad (31)$$

$$c^p, c^g, c^u \geq 0, \quad 0 < \lambda^p(i), \quad \lambda^g(i) < 1. \quad (32)$$

The interpretation of the constraints is as follows: In the first stage, workers buy unemployment insurance, while unemployed households will receive the payout (unemployment benefits, denoted by  $b^p$ ). Then, conditional on not being employed in the private sector, those households will buy public-sector insurance (in case they are not chosen to work in the public sector), and those

who remain unemployed after phase 2 will receive the payout  $b^g$ . Thus, public-sector workers need to buy two types of insurance. Also, in equilibrium, it must be that

$$b^p = \lambda^p w^p \bar{h}^p,$$

and

$$b^g = (1 - \lambda^p)(1 - \lambda^g)w^g h^g.$$

2. **Firm maximization condition.** Taking prices  $w^p, w^g, \pi$  as given,

$$\max_{\bar{H}^p} F(\bar{H}^p) - w^p \bar{H}^p \quad \text{s.t.} \quad \bar{H}^p \geq 0. \quad (33)$$

3. **Government sector conditions.** Taking prices  $w^p, \pi$  as given,  $T$  is chosen to balance the government budget

$$T = w^g \bar{H}^g \quad (34)$$

and

$$w^g = \gamma w^p, \quad (35)$$

$$S^g = S(\bar{H}^g). \quad (36)$$

4. **Insurance company profit-maximization condition.** The insurance is sequential. In stage 1, by taking  $q^p(i)$  as given,  $b^p(i)$  solves

$$\max_{b^p} \lambda^p q^p(i) b^p - (1 - \lambda^p) b^p, \quad (37)$$

i.e. the revenue from insurance premia paid by workers who end up employed in the private sector exceeds the total payout the insurance company makes to the unemployed.

In the second stage, a separate insurance scheme is run among those that are selected for private sector employment. Taking  $q^g(i)$  as given,  $b^g(i)$  solves

$$\max_{b^g} (1 - \lambda^p) \lambda^g q^g(i) b^g - (1 - \lambda^p)(1 - \lambda^g) b^g \quad (38)$$

i.e. the revenue if, conditional on not being employed in the private sector, an individual is working in the public sector minus payment is s/he is not, or the proportion of people working in the public sector and contributing towards the benefits pool for those who are not selected for work. This implicitly clears the insurance market for each individual.

In equilibrium, the price of insurance depends on the probability of the event the household is insuring against. We cannot enforce

$$q^p(i) = q^p$$

and

$$q^g(i) = q^g$$

although ex post (in equilibrium) that would indeed be the case. For the insurance firm, the profits are linear in  $q^p$  and  $q^g$ . This implies that profits cannot be positive or negative in equilibrium, but have to be zero. Zero profits means that

$$q^g = \frac{1 - \lambda^g}{\lambda^g},$$

and

$$q^p = \frac{1 - \lambda^p}{\lambda^p}.$$

A common interpretation is that for insurance companies the price of the insurance is the odds ratio, or the ratio of probabilities of the two events.  $\lambda^p$  and  $\lambda^g$  are the same for all employed households.

5. **Market clearing condition.** We have

$$\int_i \lambda^p(i) \bar{h}^p di = \bar{H}^p \quad (39)$$

$$\int_i \lambda^g(i) (1 - \lambda^p(i)) \bar{h}^g di = \bar{H}^g \quad (40)$$

$$\int_i \left\{ \lambda^p(i) c^p + \lambda^g(i) [1 - \lambda^p(i)] c^g + (1 - \lambda^g(i)) [1 - \lambda^p(i)] c^u \right\} di = F(\bar{H}^p). \quad (41)$$

## 6.2 Characterization of the equilibrium with insurance markets

Before optimizing, we can simplify the constraint set by substituting out  $b^g$  from the budget constraint in the state the household is unemployed to obtain

$$b^g = c^u - \pi. \quad (42)$$

Next, plug the obtained expression in the budget constraint in the state when the household is employed in the public sector to obtain

$$c^g + q^g (c^u - \pi) = b^p + w^g \bar{h}^g + \pi. \quad (43)$$

Now substitute out  $b^p$  from the budget constraint in the state the household is employed in the public sector to obtain

$$b^p = c^g + q^g (c^u - \pi) - w^g \bar{h}^g - \pi. \quad (44)$$

Next, plug the obtained expression in the budget constraint in the state when the household is employed in the private sector to obtain

$$c^p + q^p[c^g + q^g(c^u - \pi) - w^p\bar{h}^p - \pi] = w^p\bar{h}^p + \pi. \quad (45)$$

The problem now simplifies to

$$\begin{aligned} \max_S \lambda^p(i) \left[ \ln(c^p) + \alpha \ln(1 - \bar{h}^p) \right] + \lambda^g(i)[1 - \lambda^p(i)] \left[ \ln(c^g) + \right. \\ \left. + \alpha \ln(1 - \bar{h}^g) \right] + (1 - \lambda^g(i))[1 - \lambda^p(i)] \ln(c^u) \end{aligned} \quad (46)$$

s.t.

$$c^p + q^p[c^g + q^g(c^u - \pi) - w^p\bar{h}^p - \pi] = w^p\bar{h}^p + \pi. \quad (47)$$

The first-order optimality conditions are:

$$c^p : \frac{\lambda^p}{c^p} = \mu \quad (48)$$

$$c^g : \frac{\lambda^g(1 - \lambda^p)}{c^g} = \mu q^g \quad (49)$$

$$c^u : \frac{(1 - \lambda^p)(1 - \lambda^g)}{c^u} = \mu q^p q^g. \quad (50)$$

Since we already established that consumption will be equalized across all states, we can obtain (divide the optimality conditions for public sector employees and unemployed)

$$q^g = \frac{1 - \lambda^g}{\lambda^g}, \quad (51)$$

that is, the price of insurance in the public sector is fair, that is, it equals the odds ratio of being chosen to work in the public sector.

Similarly, divide side by side the optimality condition for private-sector employees and unemployed to obtain

$$q^p = \frac{1 - \lambda^p}{\lambda^p}, \quad (52)$$

that is, the price of insurance in the private sector is also fair, as it equals the odds ratio of being chosen to work in the private sector.

Next,

$$\lambda^p : \ln(c) + \alpha \ln(1 - \bar{h}^p) - \lambda^g[\ln(c) + \alpha \ln(1 - \bar{h}^g)] - (1 - \lambda^g) \ln(c) = 0. \quad (53)$$

Hence,

$$\alpha \ln(1 - \bar{h}^p) - \lambda^g \alpha \ln(1 - \bar{h}^g) = 0 \quad (54)$$

or

$$\lambda^g = \frac{\ln(1 - \bar{h}^p)}{\ln(1 - \bar{h}^g)} \in (0, 1). \quad (55)$$

With the obtained value for  $\lambda^g$  we can solve for  $q^g$ , and then compute  $\lambda^p$  and  $q^p$ . As we already showed, households will buy full insurance to equalize consumption in all states. Since labor income is stochastic, *i.e.*, it is uncertain whether the individual will be employed in either of the two sectors, we need an institution that could offer insurance. More specifically, sequential lotteries then can be introduced to achieve market completeness.<sup>7</sup>

## 7. Conclusions

This note describes the lottery- and insurance-market equilibrium in an economy with private-sector and public-sector employment. In contrast to Vasilev 2017, 2015, the public-sector labor supply decision is assumed to be a sequential one. This requires two separate insurance market to operate, one for private-sector work, and one for public-sector employment. In addition, given that the non-convex sectoral labor choice is made in succession, the insurance market for public-sector employment needs to open only after the insurance market for the private sector has closed, so that in equilibrium, each household would fully insure against the employment status uncertainty. This segmentation and sequentiality features of insurance markets operation is a new result and a direct consequence of both the non-convexity of the labor supply decision and the sequential nature of the sectoral labor decision. Whether this insurance-market sequentiality can be implemented in reality is not clear, as it would require that not only probabilities  $\lambda^p$  and  $\lambda^g$  to be perfectly observable to everyone, but also the winners from each lottery to be perfect knowledge. In addition, everyone should always announce truthfully the same  $\lambda^p$  and  $\lambda^g$  to the insurance companies, and all contracts written have to be perfectly enforceable.

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<sup>7</sup>Having a sequential non-convexity of the labor choice set is similar to having incomplete markets. Therefore, randomization may be optimal in a sequential non-convex environment even though there is no aggregate uncertainty. In equilibrium, not everyone will work in the private sector, and not everyone will work in the public sector, but everyone will consume the same.



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