# Fast Implementation of a General *L/M* Rate Changer by a Filter Bank Structure


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***Abstract*—In this paper, we show that an *L/M* rate changer can be realized as a discrete time SISO *(L,M)* shift invariant system in form of a two-dimensional kernel function or a filter bank structure. Based on this realization, we can implement an *L/M* rate changer by a bank of filters with the average number of the coefficients in the filters in each channel is *1/L* of the original *L/M* rate changer. Hence, the system is speed up by *L*. This helps the designer to design a sharp cutoff discrete time FIR filters in an *L/M* rate changer for some real time applications in video systems.**



***Index Terms*—*L/M* rate changer, discrete time SISO linear *(L,M)* shift invariant system, kernel function, filter bank structure, sharp cutoff discrete time FIR filters**


## I. INTRODUCTION

An *L/M* rate changer shown in figure 1 plays an important role in the audio, image and video systems [1]. A good *L/M* rate changer sometimes requires a sharp cutoff discrete time FIR filter *h[n]*, especially those applied in the digital image and the digital video systems. However, a sharp cutoff discrete time FIR filter always contains a lot of coefficients. As a result, it requires a very long processing time and restricts in some of the real time applications in video systems.

In this paper, a parallel processing technique is proposed to break down a single *L/M* rate changer into a multi-channel system shown in figure 2, such that the average number of coefficients of the filters in each channel $h_j[n]$, for $j=0,1,\ldots,L-1$, is *1/L* that of the original *L/M* rate changer. This will speed up the system by *L*.

## II. REALIZATION OF AN *L/M* RATE CHANGER AS A DISCRETE TIME SISO LINEAR *(L,M)* SHIFT INVARIANT SYSTEM

When the input of an *L/M* rate changer shifts by *M* samples, the output will shift by *L* samples. Hence, an *L/M* rate changer is a discrete time SISO linear *(L,M)* shift invariant system. As a discrete time SISO linear *(L,M)* shift invariant system can be characterized by a two-dimensional kernel function *g[n,k]*, where $g[n,k]=g[n-L,k-M]$, $\forall k,n\in\mathbf{Z}$ [2], or by a filter bank structure shown in figure 2 [3], so we can realize an *L/M* rate changer *h[n]* in terms of a two-dimensional kernel function or a filter bank structure. The transformation from an *L/M* rate changer to a discrete time SISO linear *(L,M)* shift invariant system is summarized in the following two theorems:

**Theorem 1**

Given an *L/M* rate changer shown in figure 1, there exists a discrete time SISO linear *(L,M)* shift invariant system, characterized by a two-dimensional kernel function *g[n,k]* with $g[n,k]=h[n\cdot M-k\cdot L]$, $\forall k,n\in\mathbf{Z}$, such that the input output relationship of the discrete time SISO linear *(L,M)* shift invariant system will be exactly the same as that of an *L/M* rate changer. The proof is as follows:

The input output relationship of the discrete time SISO linear *(L,M)* shift invariant system is governed by [2]:

$$y[n]=\sum_{k\to-\infty}^{+\infty} g[n,k]\cdot x[k], \quad \forall n\in Z \qquad (1).$$

Let $g[n,k]=h[n\cdot M-k\cdot L]$, $\forall k,n\in\mathbf{Z}$, we have:

$$y[n]=\sum_{k\to-\infty}^{+\infty} h[n\cdot M-k\cdot L]\cdot x[k], \quad \forall n\in Z \qquad (2),$$

which is the same input output relationship of the *L/M* rate changer [1], so this proves the theorem. ■

For example, if the upsampler and the downsampler of an *L/M* rate changer is *2* and *3* respectively, and the filter is:

$$h[n]=\begin{cases}\frac{1}{2}\cdot \operatorname{sin}c\left(\frac{n}{2}\right) & ;\text{for } -10\le n\le 10,\\ 0 & ;\text{ otherwise}\end{cases} \qquad (3),$$

then the two-dimensional kernel function of the discrete time SISO linear *(L,M)* shift invariant system is:

$g[0,k]=0.5\cdot\delta[k]$,

$$g[1,k]=\begin{cases}\frac{1}{2}\cdot \operatorname{sin}c\left(\frac{3-2\cdot k}{2}\right) & ;\text{for } -3\le k\le 6,\\ 0 & ;\text{ otherwise}\end{cases} \qquad (4),$$

and $g[n,k]=g[n-2,k-3]$ for other integer values of *n*.

**Theorem 2**

Given an *L/M* rate changer shown in figure 1, there exists a discrete time SISO linear *(L,M)* shift invariant system, characterized by a filter bank structure shown in figure 2 with $h_j[n]=h[j\cdot M+n\cdot L]$, for $\forall n\in\mathbf{Z}$ and $j=0,1,\ldots,L-1$, such that the input output relationship of the discrete time SISO linear *(L,M)* shift invariant system will be exactly the same as that of an *L/M* rate changer. The proof is as follows:

The input output relationship of the discrete time SISO linear *(L,M)* shift invariant system is governed by:

$$y[n]=\begin{cases}\sum_{k\to-\infty}^{+\infty} x[k]\cdot h_0\left[\frac{n\cdot M}{L}-k\right] & ;\text{if } \bmod(n,L)=0,\\ \sum_{k\to-\infty}^{+\infty} x[k]\cdot h_1\left[\frac{(n-1)\cdot M}{L}-k\right] & ;\text{if } \bmod(n,L)=1,\\ \vdots & \vdots\\ \sum_{k\to-\infty}^{+\infty} x[k]\cdot h_{L-1}\left[\frac{(n-L+1)\cdot M}{L}-k\right] & ;\text{if } \bmod(n,L)=L-1\end{cases} \quad (5).$$

Let $h_j[n]=h[j\cdot M+n\cdot L]$, for $\forall n\in\mathbf{Z}$ and $j=0,1,\ldots,L-1$, we have:

$$y[n]=\sum_{k\to-\infty}^{+\infty} h[n\cdot M-k\cdot L]\cdot x[k],\quad \forall n\in Z \quad (6),$$

which is also the same input output relationship of the *L/M* rate changer, hence, the theorem is proved. ■

For the same example described after theorem 1, the filters of the filter bank structure of the discrete time SISO linear *(L,M)* shift invariant system are:

$h_0[n]=0.5\cdot\delta[n]$ and

$$h_1[n]=\begin{cases}\frac{1}{2}\cdot \operatorname{sin}c\left(\frac{3\cdot n+2}{2}\right) & ;\text{for } -6\le n\le 3,\\ 0 & ;\ \text{otherwise}\end{cases} \quad (7).$$

## III. REALIZATION OF A DISCRETE TIME SISO LINEAR *(L,M)* SHIFT INVARIANT SYSTEM AS AN *L/M* RATE CHANGER

It can been shown that an *L/M* rate changer is input output equivalent to a discrete time SISO linear *(L,M)* shift invariant system if and only if *L* and *M* is co-prime. Hence, only the discrete time SISO linear *(L,M)* shift invariant system with *L* and *M* is co-prime can be realized as an *L/M* rate changer. Under this condition, the transformation from a discrete time SISO linear *(L,M)* shift invariant system to an *L/M* rate changer is summarized in the following two theorems:

**Theorem 3**

Given a discrete time SISO linear *(L,M)* shift invariant system with *L* and *M* is co-prime, there exists an *L/M* rate changer shown in figure 1 with $h[n\cdot M-k\cdot L]=g[n,k]$, $\forall n,k\in\mathbf{Z}$, such that the input output relationship of the *L/M* rate changer will be exactly the same as that of the discrete time SISO linear *(L,M)* shift invariant system. The proof is as follows:

The input output relationship of the *L/M* rate changer is governed by [1]:

$$y[n]=\sum_{k\to-\infty}^{+\infty} h[n\cdot M-k\cdot L]\cdot x[k],\quad \forall n\in Z \quad (8).$$

Since $h[n\cdot M-k\cdot L]=g[n,k]$, $\forall n,k\in\mathbf{Z}$, we have:

$$y[n]=\sum_{k\to-\infty}^{+\infty} g[n,k]\cdot x[k],\quad \forall n\in Z \quad (9),$$

which is the same input output relationship of the discrete time SISO linear *(L,M)* shift invariant system, so this proves the theorem. ■

For example, if *L* and *M* of a discrete time SISO linear *(L,M)* shift invariant system are *2* and *3* respectively, and the two-dimensional kernel function is:

$g[0,k]=k$, $g[1,k]=2\cdot k$ and $g[n,k]=g[n-2,k-3]$ for other integer values of $n$, then the filter in an *L/M* rate changer is:

$$h[n]=\begin{cases}-\frac{n}{2} & ;\text{for n is even,}\\ 3-n & ;\text{for n is odd}\end{cases} \quad (10).$$

**Theorem 4**

Given a discrete time SISO linear *(L,M)* shift invariant system with *L* and *M* is co-prime, there exists an *L/M* rate changer shown in figure 1 with $h[j\cdot M+n\cdot L]=h_j[n]$, for $\forall n\in\mathbf{Z}$ and $j=0,1,\ldots,L-1$, such that the input output relationship of the *L/M* rate changer will be exactly the same as that of the discrete time SISO linear *(L,M)* shift invariant system. The proof is as follows:

The input output relationship of the *L/M* rate changer is governed by [1]:

$$y[n]=\sum_{k\to-\infty}^{+\infty} h[n\cdot M-k\cdot L]\cdot x[k],\quad \forall n\in Z \quad (11).$$

Since $h[j\cdot M+n\cdot L]=h_j[n]$, for $\forall n\in\mathbf{Z}$ and $j=0,1,\ldots,L-1$, we have:

$$y[n]=\begin{cases}\sum_{k\to-\infty}^{+\infty} x[k]\cdot h_0\left[\frac{n\cdot M}{L}-k\right] & ;\text{if } \bmod(n,L)=0,\\ \sum_{k\to-\infty}^{+\infty} x[k]\cdot h_1\left[\frac{(n-1)\cdot M}{L}-k\right] & ;\text{if } \bmod(n,L)=1,\\ \vdots & \vdots\\ \sum_{k\to-\infty}^{+\infty} x[k]\cdot h_{L-1}\left[\frac{(n-L+1)\cdot M}{L}-k\right] & ;\text{if } \bmod(n,L)=L-1\end{cases} \quad (12),$$

which is also the same input output relationship of the discrete time SISO linear *(L,M)* shift invariant system, hence, the theorem is proved. ■

For example, if the upsampler and the downsampler of the discrete time SISO linear *(L,M)* shift invariant system are *2* and *3* respectively, and the filters of the filter bank structure are $h_0[n]=n$ and $h_1[n]=2\cdot n$ respectively, then the filter in an *L/M* rate changer is:

$$h[n]=\begin{cases}\frac{n}{2} & ;\text{for n is even,}\\ n-3 & ;\text{for n is odd}\end{cases} \quad (13).$$

## IV. APPLICATION ON FAST IMPLEMENTATION OF AN *L/M* RATE CHNAGER

By realizing an *L/M* rate changer as a discrete time SISO linear *(L,M)* shift invariant system in form of a filter bank structure shown in figure 2, the average number of coefficients of the filters in each channel $h_j[n]$, for $j=0,1,\ldots,L-1$, is *1/L* that of the original *L/M* rate changer, this will speed up the system by *L*.

The transformation helps the user to design an *L/M* rate changer with a sharp cutoff discrete time FIR filter *h[n]*, and can be applied in real time applications in video systems.

## V. CONCLUSION

In this paper, we show that a discrete time SISO *(L,M)* shift invariant system with *L* and *M* is co-prime can be realized as an *L/M* rate changer. Also, we show that an *L/M* rate changer can be realized as a discrete time SISO *(L,M)* shift invariant system in form of a two-dimensional kernel function or a filter bank structure shown in figure 2. Based on this realization, we can implement an *L/M* rate changer by a bank of filters with the average number of the coefficients in the filters in each channel is *1/L* of the original *L/M* rate changer. This can speed up the system by *L* and helps the designer to design a sharp cutoff discrete time FIR filters in an *L/M* rate changer for some real time applications in video systems.

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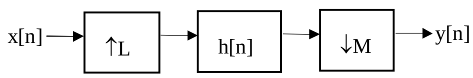


**Fig. 1.** An *L/M* rate changer

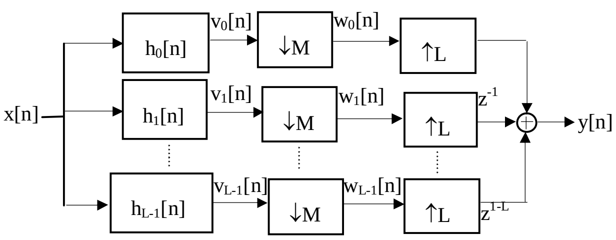


**Fig. 2.** Filter bank realization of an *L/M* rate changer