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Keywords	GMTI radar tracking, Doppler blindness region, miss detection and falase alarms, Gaussian mixtures model
Taxonomy	Signal Processing, Radar Tracking, Tracking

Corresponding Author	Miao Yu
Corresponding Author's Institution	University of Lincoln
Order of Authors	Liyun Gong, Miao Yu
Suggested reviewers	Wenwu Wang, Stefanos Kollias, Jonathon Chambers

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A new Gaussian mixture method with exactly exploiting the negative information for GMTI radar tracking in a low-observable environment

Liyun Gong and Miao Yu

Abstract

This paper investigates the problem of ground vehicle tracking with a Ground Moving Target Indicator (GMTI) radar. In practice, the movement of ground vehicles may involve several different manoeuvring types (acceleration, deceleration, standstill, etc.). Consequently, the GMTI radar may lose measurements when the radial velocity of the ground vehicle is below a threshold when it stops, i.e. falling into the Doppler blind region. Besides, there will be false alarms in low-observable environments where there exist high noises interferences. In this paper, we develop a novel algorithm for the GMTI tracking in a low-observable environment with false alarms while exactly incorporating the 'negative information' (i.e., the target is likely to stop when no measurements are recorded) based on the Bayesian inference framework. For the Bayesian inference implementation, the Gaussian mixture approximation method is adopted to approximate related distributions, while different filtering algorithms (including both extended Kalman filter and its generalization for interval-censored measurements) are applied for updating the Gaussian mixture components. Target state estimation can be directly obtained through the Gaussian mixture model for the GMTI tracking at every time instance. We have compared the developed method with other state-of-the-art ones and the simulation results show that the proposed method substantially outperforms the existing methods for the GMTI tracking problem.

I. Introduction

In this paper, we consider the problem of ground vehicle tracking using a Ground Moving Target Indicator (GMTI) radar which can discriminates a moving target against the static background. Based on the Doppler effect [1], GMTI radar is well suited for detecting targets moving on ground due to its wide-area, all-weather, day/night, and real-time capabilities [2]. Therefore, the GMTI radar based tracking has received a wide range of applications in vehicles tracking to support the surveillance in different environments, such as battlefield and urban.

There have been a number of methods developed to address the GMTI tracking. Kirubarajan et al. in [3] proposed a variable structure interacting multiple model (VS-IMM) algorithm for GMTI tracking based on the extended Kalman filtering (EKF) approach. In order to better overcome the nonlinearity in the measurement model of the GMTI radar, a particle filtering approach was proposed in [4]. In addition, a new approach was proposed in [5] to improve on

+ Liyun Gong and Miao Yu are with School of Computer Science, University of Lincoln, UK, LN6 7TS (e-mail: {lgong,myu}@lincoln.ac.uk).

the performance of [4] in which the traditional particle filter was replaced with a more advanced unscented particle filter developed in [6].

The major limitation of the methods in [3], [4] and [5] is that the GMTI measurements were assumed to be ‘ideal’, which is recorded at every time step and is not the case in the real-world problems. In practice, as pointed out in [7], the GMTI measurements will not always be received; besides, in order to separate out moving objects of interest from heavy, static clutter, the received measurements are deliberately suppressed if they are in the Doppler blind region, i.e. the magnitude of the measured radial velocity drops below the minimum detectable velocity.

A straightforward way to deal with it is that, when no measurements are received,only the state model is applied to estimate the target state. However, this type of method did not consider the ‘negative information’ due to the characteristics of a GMTI radar, that is, when a GMTI radar does not receive any measurements, it is likely that the target stops and its radial velocity falls into the Doppler blindness region. For incorporating the ‘negative information’, [2] and [8] adopted a state-dependent detection probability, which helps to determine the conditional distribution of the target state when no measurements are recorded. To solve the problem that negative weights may possibly arise in the Gaussian mixture approximation, an extra approximation stage was introduced in [9] to replace the resulting negative Gaussian mixture with one having strictly positive mixture weights, and thus improving on algorithmic stability. Besides, the particle filtering method is applied in [10],which treated each non-detection case as evidence. The corresponding likelihood function of the non-detection evidence was formulated and incorporated into the particle filtering procedure to update the target state probability distribution when no measurements are recorded. In this way, the negative information is exploited.

Clark et al. [11] have developed a new Gaussian Mixture Model based algorithm for GMTI tracking. This algorithm was carefully designed based on the exact Bayesian inference and the ‘negative information’ of the GMTI radar can be efficiently incorporated based on a generalized extended Kalman filtering (EKF) algorithm considering interval measurements . The simulation results in [11] showed that this new approach outperformed some existing methods (such as [8]), especially when dealing with the scenario of no measurement. [12] further improves the original algorithm in [11], by exploiting an enhanced particle filtering based approach.

However, the aforementioned methods ([2], [8], [9], [10], [11] and [12]) can not deal with false alarms, which can occur in a low-observable environment when the signal-to-noise ratio of a GMTI radar is low. To deal with these false alarms, both data association method ([13]) and random finite set based method ([14] and [15]) can be applied; however, both of them don’t take negative information into account for the GMTI tracking.

In this work, we developed a new GMTI tracking algorithm, which can both exploit the negative information and deal with false alarms in a low-observable environment. This algorithm is based on the exact Bayesian inferencing by considering various of aspects including miss detection, false alarms and Doppler blindness region. The Gaussian mixture method is applied for approximating the target state distribution, from which the target state can also be straightforwardly estimated for the GMTI tracking. Both the EKF filtering algorithm and its generalized version for dealing with interval measurements are applied for the updating of Gaussian mixture models for consecutive time instances. The developed method is compared with the state-of-the-art tracking methods in low-observable environment which can cope with both miss detection and false alarms.

This paper is divided into the following sections: Section II describes the state and measurement models used for the GMTI tracking. The proposed algorithms are mentioned in Section III. The related simulations have been given in Section IV. And finally, we give the conclusion and future works in Section V.

II. Tracking models

For a standard tracking problem, both the state and measurement models are applied. The state model is used to describe the dynamics of the target vehicle movement and measurements. In this work, a simple but general constant velocity (CV) model is applied, which is shown as:

$$\mathbf{x}_t = F\mathbf{x}_{t-1} + G\mathbf{w} \tag{1}$$

where $\mathbf{x}_t = [x_t, y_t, \dot{x}_t, \dot{y}_t]$ is the state vector consisting of the positions (x_t, y_t) and velocities $(\dot{x}_t, \dot{y}_t)$ in x and y directions (here we assume that the vehicle moves on the ground with $z_t \equiv 0$ and $\dot{z}_t \equiv 0$). $\mathbf{w}$ is a 2×1 vector representing the acceleration, which is assumed to follow a Gaussian distribution with $\mathbf{w} \sim N(\mathbf{w}|, \mathbf{0}, P_{\mathbf{w}})$. The parameters F and G are defined as:

$$F = \begin{bmatrix} 1 & 0 & T & 0 \\ 0 & 1 & 0 & T \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad G = \begin{bmatrix} T^2/2 & 0 \\ 0 & T^2/2 \\ T & 0 \\ 0 & T \end{bmatrix}, \tag{2}$$

where $T = 5s$ is the time interval between two consecutive samplings.

With respect to the measurement model, the standard GMTI radar measures the range, azimuth angle and range rate (denoted as z_r, z_θ and $z_{\dot{r}}$ at time step t, respectively) of the ground vehicle relative to the position of the GMTI radar. Following [11], we assume that these measurements are noise-corrupted from actual values:

$$\begin{aligned} \mathbf{z}_t &= \begin{bmatrix} y_r \\ y_\theta \\ z_{\dot{r}} \end{bmatrix} = h(\mathbf{x}_t) + \mathbf{n}_t = \begin{bmatrix} r_t(\mathbf{x}_t) \\ \theta_t(\mathbf{x}_t) \\ \dot{r}_t(\mathbf{x}_t) \end{bmatrix} + \begin{bmatrix} n_{r_t} \\ n_{\theta_t} \\ n_{\dot{r}_t} \end{bmatrix} \\ &= \begin{bmatrix} \sqrt{(x_t - x_{o,t})^2 + (y_t - y_{o,t})^2 + (z_t - z_{o,t})^2} \\ \arctan2(y_t - y_{o,t}, x_t - x_{o,t}) \\ \dfrac{(x_t - x_{o,t}) \cdot (\dot{x}_t - \dot{x}_{o,t}) + (y_t - y_{o,t}) \cdot (\dot{y}_t - \dot{y}_{o,t}) + (z_t - z_{o,t}) \cdot (\dot{z}_t - \dot{z}_{o,t})}{\sqrt{(x_t - x_{o,t})^2 + (y_t - y_{o,t})^2 + (z_t - z_{o,t})^2}} \end{bmatrix} + \begin{bmatrix} n_{r_t} \\ n_{\theta_t} \\ n_{\dot{r}_t} \end{bmatrix}, \end{aligned} \tag{3}$$

where $\arctan2$ denotes the quadrant inverse tangent function, $(x_{o,t}, y_{o,t}, z_{o,t})$ and $(\dot{x}_{o,t}, \dot{y}_{o,t}, \dot{z}_{o,t})$ represent the position and velocity of the observer (GMTI radar) at time t respectively, $h(\mathbf{x}_t) = [r_t, \theta_t, \dot{r}_t]^T$ is the ideal measurement vector without the measurement noises and $\mathbf{n}_t = [n_{r_t}, n_{\theta_t}, n_{\dot{r}_t}]^T$ represents the measurement noise vector which is assumed to be Gaussian with zero-mean vector and a diagonal covariance matrix $diag\{\sigma_r^2, \sigma_\theta^2, \sigma_{\dot{r}}^2\}$.

A. Miss detection and false alarms

In practice, a target will not be always detected by the GMTI radar but with a detection probability P_d; besides,according to the characteristic of the GMTI radar, a target will be mistaken as the background environment if it is within the Doppler blindness region. The target is within the Doppler blindness region when $|z_{\dot{r}}| \leq \kappa$, where κ denotes the minimum detectable velocity of the GMTI radar and $z_{\dot{r}}$ represents the measured radial velocity (i.e.,the projection of a target's velocity along the range direction). The actual target measurement $\mathbf{z}_t^{target}$ then follows the following definition:

$$\mathbf{z}_t^{target} = \begin{cases} \emptyset & |z_{\dot{r}}| \leq \kappa \text{ or } |v_t^r| > \kappa \text{ with probability } 1 - P_D \\ \mathbf{y}_t & |z_{\dot{r}}| > \kappa \text{ with probability } P_D. \end{cases} \tag{4}$$

Not only the target measurement, in the realistic scenario there are also false alarms due to the interferences caused by the background environment. Generally, it is modelled that the number k of false alarms follows a Poisson distribution $P(k)$ with:

$$P(k) = \frac{\lambda^k}{k!} e^{-\lambda} \tag{5}$$

where λ^k represents the expected number of false alarms. Besides, each false measurement is modelled as a uniform distribution across the measurement space with a volume of V.

III. The proposed algorithm

It is assumed at time instance $t-1$ the target follows a distribution of $p(\mathbf{x}_{t-1}|\mathbf{Z}_{1:t-1})$. We can estimate the distribution $p(\mathbf{x}_t|\mathbf{Z}_{1:t})$ at time t,according to the Bayesian inference rule from the following two steps:

Prediction:

$$p(\mathbf{x}_t|\mathbf{Z}_{1:t-1}) = \int p(\mathbf{x}_{t-1}|\mathbf{Z}_{1:t-1}) p(\mathbf{x}_t|\mathbf{x}_{t-1}) d\mathbf{x}_{t-1} \tag{6}$$

Updation:

$$p(\mathbf{x}_t|\mathbf{Z}_{1:t}) \propto p(\mathbf{x}_t|\mathbf{Z}_{1:t-1}) p(\mathbf{Z}_t|\mathbf{x}_t) \tag{7}$$

where $\mathbf{Z}_{1:t} = \{\mathbf{Z}_1, ..., \mathbf{Z}_t\}$ represents the measurements assemble from initial time instance denoted by $t = 1$ until the current time instance t. An element $\mathbf{Z}_t$ represents the measurements at a particular time instance t, which can be represented as $\mathbf{Z}_t = \{\mathbf{z}_{t,1}, ..., \mathbf{z}_{t,N}\}$ if assumed that there are N GMTI measurements obtained at the time instance t.

The $p(\mathbf{x}_t|\mathbf{x}_{t-1})$ is determined through the prediction model (6). By applying the total probability theorem while considering the doppler blindness region information, $p(\mathbf{Z}_t|\mathbf{x}_t)$can be derived as:

$$p(\mathbf{Z}_t|\mathbf{x}_t) = p(\mathbf{Z}_t, \emptyset|\mathbf{x}_t) + p(\mathbf{Z}_t, DBR|\mathbf{x}_t) + \sum_{i=1}^{i=N} p(\mathbf{Z}_t, \theta_i|\mathbf{x}_t) \tag{8}$$

where $\emptyset$ represents the event that the object is not detected, DBR represents the event that the object is in the Doppler blindness zone. And θ_i represents that the object is associated with the $i-th$ measurement.

By substituting (8) into (7), we can obtain that:

$$\begin{aligned} & p(\mathbf{x}_t|\mathbf{Z}_{1:t}) \propto p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\mathbf{Z}_t,\emptyset|\mathbf{x}_t) + p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\mathbf{Z}_t,DBR|\mathbf{x}_t) + \sum_{i=1}^{i=N} p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\mathbf{Z}_t,\theta_i|\mathbf{x}_t) \\ & = p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\},\emptyset|\mathbf{x}_t) + p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\},DBR|\mathbf{x}_t) \\ & + \sum_{i=1}^{i=N} p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\},\theta_i|\mathbf{x}_t) \\ & = p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\emptyset|\mathbf{x}_t)p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\}|\emptyset,\mathbf{x}_t) + p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(DBR|\mathbf{x}_t)p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\}|DBR,\mathbf{x}_t) \\ & + \sum_{i=1}^{i=N} p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\theta_i|\mathbf{x}_t)p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\}|\theta_i,\mathbf{x}_t) \end{aligned} \tag{9}$$

From (9), we can see that the posterior distribution is composed of three terms: miss detection related term (denoted as T_{MD}), doppler blindness zone (DBR) related term (denoted as T_{DBR}) and GMTI measurements related term (denoted as T_{GMTI}), which can be represented as follows:

1).miss detection related term:

$$T_{MD} = p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\emptyset|\mathbf{x}_t)p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\}|\emptyset,\mathbf{x}_t) \tag{10}$$

2). doppler blindness zone (DBR) related term:

$$T_{DBR} = p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(DBR|\mathbf{x}_t)p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\}|DBR,\mathbf{x}_t) \tag{11}$$

3). GMTI measurements related term:

$$T_{GMTI} = \sum_{i=1}^{i=N} p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})p(\theta_i|\mathbf{x}_t)p(\{\mathbf{z}_{t,1},...,\mathbf{z}_{t,N}\}|\theta_i,\mathbf{x}_t) \tag{12}$$

In the following, we will introduce the derivation of the aforementioned three terms in details. There is no exact solution for (10), (11) and (12) and in this work, we derive mixture of Gaussians model to approximate these terms. Compared with the time-consuming particle filtering based approach, the derivation of mixture of Gaussians model for approximation is much more efficiency with respect to the computational costs. Initially, it is assumed that the distribution $p(\mathbf{x}_{t-1}|\mathbf{Z}_{1:t-1})$ at time instance $t-1$ can be approximated by a mixture of Gaussians distribution as:

$$p(\mathbf{x}_{t-1}|\mathbf{Z}_{1:t-1}) = \sum_{i=1}^{m} w_i N(\mathbf{x}_{t-1}|\mathbf{u}_{t-1}^i, P_{t-1}^i) \tag{13}$$

A. Miss detection related term

Based on (13), the component $p(\mathbf{x}_t|\mathbf{Z}_{1:t-1})$ in the miss detection related term (10) can be represented as:

$$p(\mathbf{x}_t|\mathbf{Z}_{1:t-1}) = \sum_{i=1}^{m} w_i N(\mathbf{x}_{t-1}|\bar{\mathbf{u}}_t^i, \bar{P}_t^i) \tag{14}$$

where $\bar{\mathbf{u}}_t^i$ and $\bar{P}_t^i$ can be derived according to the state model (1) as:

$$\bar{\mathbf{u}}_t^i = F\mathbf{u}_{t-1}^i \tag{15}$$

$$\bar{P}_t^i = FP_{t-1}^i F^T + GQG^T \tag{16}$$

The probability $p(\emptyset|\mathbf{x}_t)$ in (10) represents the probability that $\mathbf{x}_t$ is either not detected or not within the gating region,which is calculated as $1 - P_d$.

$p(\{\mathbf{z}_{t,1}, ..., \mathbf{z}_{t,N}\}|\emptyset, \mathbf{x}_t)$ represents the false alarms distribution of $\{\mathbf{z}_{t,1}, ..., \mathbf{z}_{t,N}\}$. According to the assumption of the uniform distribution of the false alarms, it can be represented as:

$$p(\{\mathbf{z}_{t,1}, ..., \mathbf{z}_{t,N}\}|\emptyset, \mathbf{x}_t) = V^{-m} \tag{17}$$

In combination, the non-detection term can be derived as:

$$T_{MD} = V^{-m}(1 - P_d) \sum_{i=1}^{m} w_i N(\mathbf{x}_t|\bar{\mathbf{u}}_t^i, \bar{P}_t^i) \tag{18}$$

B. GMTI measurements related term

For the GMTI measurements related term as in (12), by assuming the independence between all the measurements (including both the target measurement and false alarms) and exploiting the Bayesian rules, it can be re-written as:

$$\begin{aligned} T_{GMTI} &= \sum_{i=1}^{i=N} p(\mathbf{x}_t|\mathbf{Z}_{1:t-1}) p(\theta_i|\mathbf{x}_t) p(\{\mathbf{z}_{t,1}, ..., \mathbf{z}_{t,N}\}|\theta_i, \mathbf{x}_t) \\ &= \sum_{i=1}^{i=N} p(\mathbf{x}_t|\mathbf{Z}_{1:t-1}) p(\theta_i|\mathbf{x}_t) p(\mathbf{z}_{t,i}|\theta_i, \mathbf{x}_t) \Pi_{i=1,..i-1,i+1,..,N} f(\mathbf{z}_{t,i}) \\ &= \sum_{i=1}^{i=N} p(\theta_i|\mathbf{x}_t) V^{-N+1} p(\mathbf{z}_{t,i}) p(\mathbf{x}_t|\mathbf{Z}_{1:t-1}, \mathbf{z}_{t,i}) \end{aligned} \tag{19}$$

Based on the mixture of Gaussian representation in (13)and by applying on the extended Kalman based filtering algorithm, we can obtain that:

$$p(\mathbf{x}_t|\mathbf{Z}_{1:t-1}, \mathbf{z}_{t,i}) = \sum_{j=1}^{m} w_j N(\mathbf{x}_t|\mathbf{u}_t^{i,j}, P_t^{i,j}) \tag{20}$$

with $\mathbf{u}_t^{i,j}$ and $P_t^{i,j}$ for every j based on the $i-th$ measurement being calculated as follows:

$$
\begin{aligned}
\tilde{\mathbf{z}}_t^{i,j} &= \mathbf{z}_{t,i} - h(\bar{\mathbf{u}}_t^j) \\
\mathbf{H}_t^j &= \nabla_{\mathbf{x}_t} h(\mathbf{x}_t)|_{\mathbf{x}_t = \bar{\mathbf{u}}_t^j} \\
\mathbf{S}_t^j &= \mathbf{H}_t^j \bar{P}_t^j (\mathbf{H}_t^j)^T + \mathbf{R}, \\
\mathbf{K}_t^j &= \bar{P}_t^j (\mathbf{H}_t^j)^T (\mathbf{S}_t^j)^{-1}, \\
\mathbf{u}_t^{i,j} &= \bar{\mathbf{u}}_t^j + \mathbf{K}_t^j \tilde{\mathbf{y}}_t^{i,j}, \\
P_t^{i,j} &= (\mathbf{I} - \mathbf{K}_t^j \mathbf{H}_t^j) \bar{P}_t^j
\end{aligned}
\tag{21}
$$

The term $p(\mathbf{z}_{t,i})$ can be calculated as:

$$
p(\mathbf{z}_{t,i}) = \sum_{j=1}^{m} w_j N(\tilde{\mathbf{z}}_t^{i,j} | \mathbf{0}, \mathbf{S}_t^j) \tag{22}
$$

where $\mathbf{0}$ is a vector with the same dimensionality as $\tilde{\mathbf{z}}_t^{i,j}$.

The term $p(\theta_i|\mathbf{x}_t)$ represents the prior probability that the target is associated with the $i-th$ measurement. According to the derivations in [16], it can be obtained as:

$$
p(\theta_i|\mathbf{x}_t) = \frac{P_d}{P_d N + (1 - P_d) V \lambda} \tag{23}
$$

By combing (20), (22) and (23), the T_{GMTI} as in (19) can be finaly represented as:

$$
T_{GMTI} = \sum_{i=1}^{i=N} \frac{P_d}{P_d N + (1 - P_d) V \lambda} V^{-m+1} p(\mathbf{z}_{t,i}) \sum_{j=1}^{m} w_j N(\mathbf{x}_t | \mathbf{u}_t^{i,j}, P_t^{i,j}) \tag{24}
$$

C. DBR related term

For the DBR related term as in (11), we have:

$$
\begin{aligned}
T_{DBR} &= p(\mathbf{x}_t | \mathbf{Z}_{1:t-1}) p(DBR|\mathbf{x}_t) p(\{\mathbf{z}_{t,1}, ..., \mathbf{z}_{t,N}\} | DBR, \mathbf{x}_t) \\
&= V^{-N} p(DBR) p(\mathbf{x}_t | DBR)
\end{aligned}
\tag{25}
$$

Based on the prior mixture of Gaussian distribution as in (13), the posterior $p(\mathbf{x}_t|DBR)$ also holds a same mixture of Gaussian representation as:

$$
p(\mathbf{x}_t | DBR) = \sum_{j=1}^{m} w_j N(\mathbf{x}_t | \mathbf{u}_t^{j,DBR}, \mathbf{P}_t^{j,DBR}) \tag{26}
$$

According to a generalized EKF filtering based on the derivations on the mathematical expectation and variance conditioned on interval-censored measurements as in [11] (related derivations are also given in the appendix), we can obtain that:

$$\begin{aligned} \mathbf{u}_t^{j,DBR} &= \bar{\mathbf{u}}_t^j + \mathbf{K}_t^{j,DBR}(m_A^j - \dot{r}_t(\bar{\mathbf{u}}_t^j)), \\ \mathbf{P}_t^{j,DBR} &= \bar{P}_t^j - \mathbf{K}_t^{j,DBR}\mathbf{H}_t^{j,DBR}\bar{P}_t^j + \mathbf{K}_t^{j,DBR}V_A^j(\mathbf{K}_t^{j,DBR})^T, \end{aligned} \tag{27}$$

with related parameters being calculated as:

$$\begin{aligned} &\mathbf{H}_t^{j,DBR} = \nabla_{\mathbf{x}_t}\dot{r}_t|_{\mathbf{x}_t=\bar{\mathbf{u}}_t^j} \\ &\bar{\sigma}^j = (\mathbf{H}_t^{j,DBR}\mathbf{P}_t^{j,DBR}(\mathbf{H}_t^{j,DBR})^T)^{1/2} + \sigma_v^2 \\ &\gamma_t^j = \int_{-\kappa}^{\kappa}\mathcal{N}(x|\dot{r}_t(\bar{\mathbf{x}}_t), \bar{\sigma}^j)dx \\ &m_A^j = (\gamma_t^j)^{-1}(\bar{\sigma}^j)^2[\mathcal{N}(-\kappa|\dot{r}_t(\bar{\mathbf{x}}_t), (\bar{\sigma}^j)^2) - \mathcal{N}(\kappa|\dot{r}_t(\bar{\mathbf{x}}_t), (\bar{\sigma}^j)^2)] + \dot{r}_t(\bar{\mathbf{x}}_t) \\ &V_A^j = (\gamma_t^j)^{-1}(\bar{\sigma}^j)^2[(-\kappa + \dot{r}_t(\bar{\mathbf{x}}_t))\mathcal{N}(-\kappa|\dot{r}_t(\bar{\mathbf{x}}_t), (\bar{\sigma}^j)^2) \\ &- (\kappa + \dot{r}_t(\bar{\mathbf{x}}_t))\mathcal{N}(\kappa|\dot{r}_t(\bar{\mathbf{x}}_t), (\bar{\sigma}^j)^2)] + (\dot{r}_t(\bar{\mathbf{x}}_t)^2 + (\bar{\sigma}^j)^2) - (m_A^j)^2 \\ &\mathbf{K}_t^{j,DBR} = P_t^j(\mathbf{H}_t^{j,DBR})^T\mathbf{H}_t^{j,DBR}P_t^{j,r}(\mathbf{H}_t^{j,DBR})^T \end{aligned} \tag{28}$$

Based on calculated $\bar{\sigma}^j$ (for $j = 1, ..., m$), the term $p(DBR)$ is calculated as:

$$p(DBR) = \sum_{j=1}^{m} w_i\bar{\sigma}^j \tag{29}$$

Until now, we have obtained all the three terms of T_{MD}, T_{DBR} and T_{GMTI}. According to (18), (24) (25) and (26), the posterior $p(\mathbf{x}_t|\mathbf{Z}_{1:t})$ can then be obtained as:

$$\begin{aligned} &p(\mathbf{x}_t|\mathbf{Z}_{1:t}) = T_{MD} + T_{DBR} + T_{GMTI} \\ &= \sum_{j=1}^{m} w_j^{DBR}N(\mathbf{x}_t|\mathbf{u}_t^{j,DBR}, \mathbf{P}_t^{j,DBR}) + w_j^{MD}N(\mathbf{x}_t|\bar{\mathbf{u}}_t^i, \bar{P}_t^i) + \sum_{i=1}^{i=N} w_{i,j}^{GMTI}N(\mathbf{x}_t|\mathbf{u}_t^{i,j}, P_t^{i,j})) \end{aligned} \tag{30}$$

where the weights being calculated as:

$$\begin{aligned} &w_j^{DBR} \propto V^{-N}p(DBR)w_j \\ &w_j^{MD} \propto V^{-N}(1 - P_d)w_j \\ &w_{i,j}^{GMTI} \propto \frac{P_d}{P_dN + (1 - P_d)V\lambda}V^{-N+1}p(\mathbf{z}_{t,i})w_j \end{aligned} \tag{31}$$

with these weights being normalized to be summed up to be 1. And the estimation of the target (denoted as $\hat{\mathbf{x}}_t$) can be finally obtained as:

$$\hat{\mathbf{x}}_t = \sum_{j=1}^{m}(w_j^{DBR}\mathbf{u}_t^{j,DBR} + w_j^{MD}\bar{\mathbf{u}}_t^i + \sum_{i=1}^{i=N} w_{i,j}^{GMTI}\mathbf{u}_t^{i,j}) \tag{32}$$

One practical problem is that as time evolves the number of Gaussian components representing posterior distribution will increase, which will lead to the expansion to a large number of Gaussian components after a certain number of times,which will incur high computational burden. In order to overcome it, a Gaussian components reduction technique as in [11] is applied to keep the number of Gaussian components to be a fixed value.

IV. Simulation Study

In this section, we use a simulation study to evaluate the numerical performance of the developed algorithm. A scenario similar to [11] is simulated that GMTI sensor is mounted on an airborne platform employed to track the motion of a moving ground vehicle. The target moving eastbound started at a constant speed of 10 m/s and maintained for 180s before it accelerated at a rate of 1 m/s^2 up to a speed of 25 m/s. After travelling at this constant speed for 180s, the target started to decelerate for 25s until it came to a standstill. The target remained stationary for 60s, before accelerating again to a speed of 15 m/s. Target speed as a function of time is plotted in Figure 1.

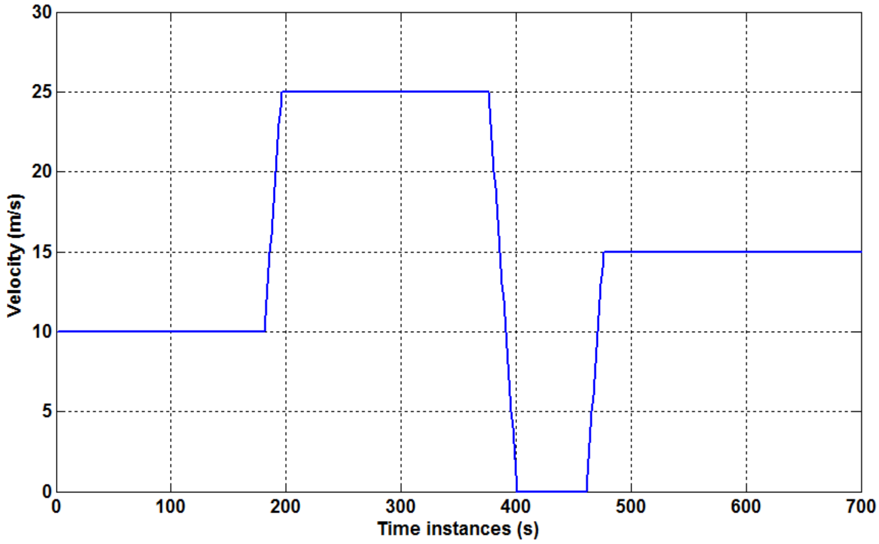


Fig. 1. The speed of the manoeuvring ground vehicle.

The sensor platform travelled northbound at a constant speed of 120 m/s and at an altitude of 10 km. The moving sensor platform took noisy measurements (including the 3-D range, azimuth angle and range rate) of the ground-moving target every 5 seconds. The movements of the target and sensor platform are displayed in Figure 2.

A. The state and measurement models

We applied a constant velocity (CV) model as in (6) to describe the target movement. The covariance $P_{\mathbf{w}}$ of the noise vector $\mathbf{w}$ following:

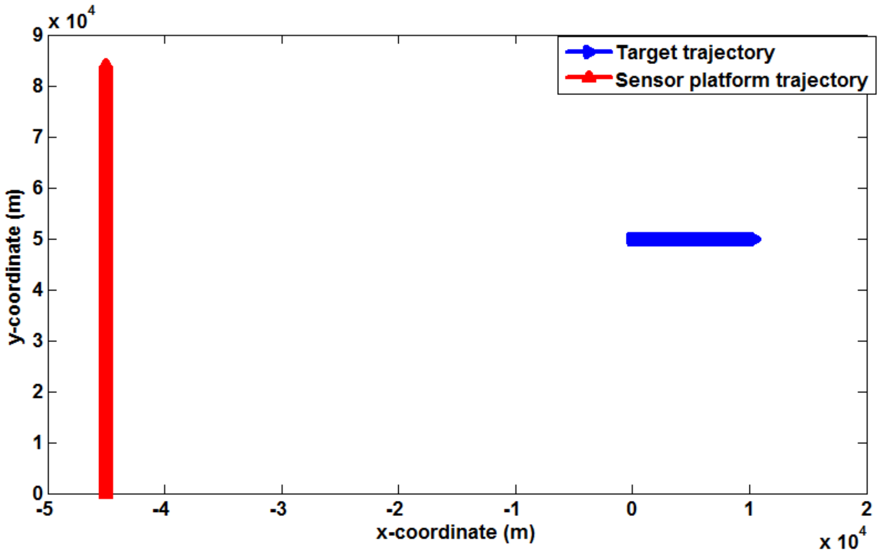


Fig. 2. The trajectories of the ground target and sensor platform.

$$P_{\mathbf{w}} = \begin{bmatrix} 0.05^2 & 0 \\ 0 & 0.05^2 \end{bmatrix} (m/s^2)^2 \tag{33}$$

We followed [11] and set the parameters of the measurement equation (7), including σ_r and σ_θ, as $20m$ and $0.001radians$ respectively.When the target is not detected or the detected radial velocity is within the Doppler blindness region $[-3, +3](m/s)$ as shown in Figure 3, then no target measurements will be available. For the false alarms, it is assumed that the expected number of false alarms λ is 50 and the false alarms follow a uniform distribution across the measurement space. Examples of false alarms together with detected target measurements in the position Cartesian coordinate system are shown in Figure 4.

B. Tracking performance analysis

Based on the state and measurement models, the proposed algorithm was applied for the tracking of the moving target in the set scenario and the corresponding tracking performance was evaluated. As in [11], we assumed the initial target state distribution followed a single Gaussian prior density obtained by two-point difference [17] for the proposed approach and for the other algorithms used for comparison below.

We analyse the tracking accuracy of the proposed method and draw a comparison with some other state-of-the-art approaches for object tracking in a low-observable environment with false alarms, including both the Kalman filtering based approach with probabilistic data association (PDA) (denoted as PDA-EKF) and particle filtering based approach with a generalized likelihood function for dealing with miss detection and false alarms (denoted as G-PF). For each approach, 100 Monte-Carlo simulation experiments were carried out and the root mean square

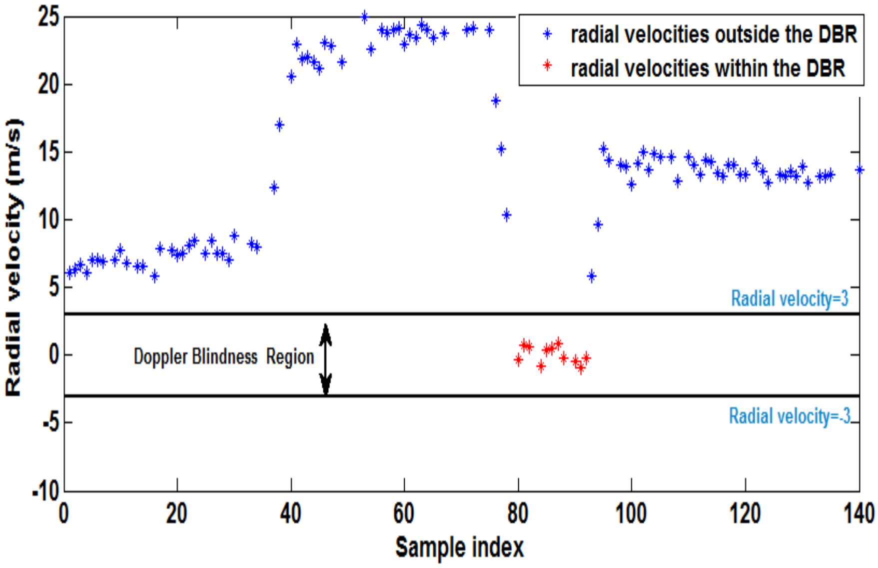


Fig. 3. The detected target radial velocities with/without the Doppler blindness region (DBR) during the target movement period.

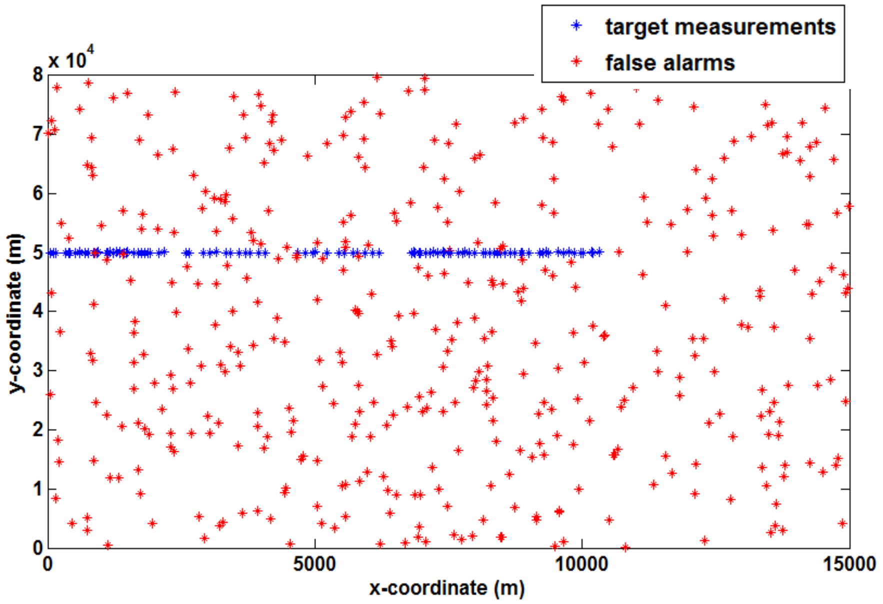


Fig. 4. The target measurements and false alarms in the position Cartesian coordinate system.

estimation errors (RMSEs) averaged over the 100 simulations at different time instances were calculated. For a comprehensive evaluation, we considered the various of parameter settings as did in [11], including different detection probabilities P_D and the standard deviations of range rate measurement $\sigma_{\dot{r}}$. From Figures 5 and 6, we can that the proposed method achieves smaller position and velocity RMSEs at most time instances thus more accurate tracking performance is obtained.

We particularly focus on a scenario when the vehicle was standstill (sample indexes 80-92) and hence was within the Doppler blindness region. This is a challenging scenario because no target measurements was recorded during the vehicle stopping interval. A comprehensive evaluation was performed under different parameter settings to compare different approaches. The time averaged RMSEs of both positions and velocities are calculated from 100 Monte-Carlo simulations, which are shown in Tables I and II.It can be seen that the proposed approach outperformed the other methods with the smallest tracking errors in both position and velocity estimations, due to its capability of incorporating the Doppler blindness region information for vehicle tracking.

TABLE I
THE POSITION RMSES AVERAGED OVER 100 EXPERIMENTS (IN METERS) WHEN THE TARGET STOPS.

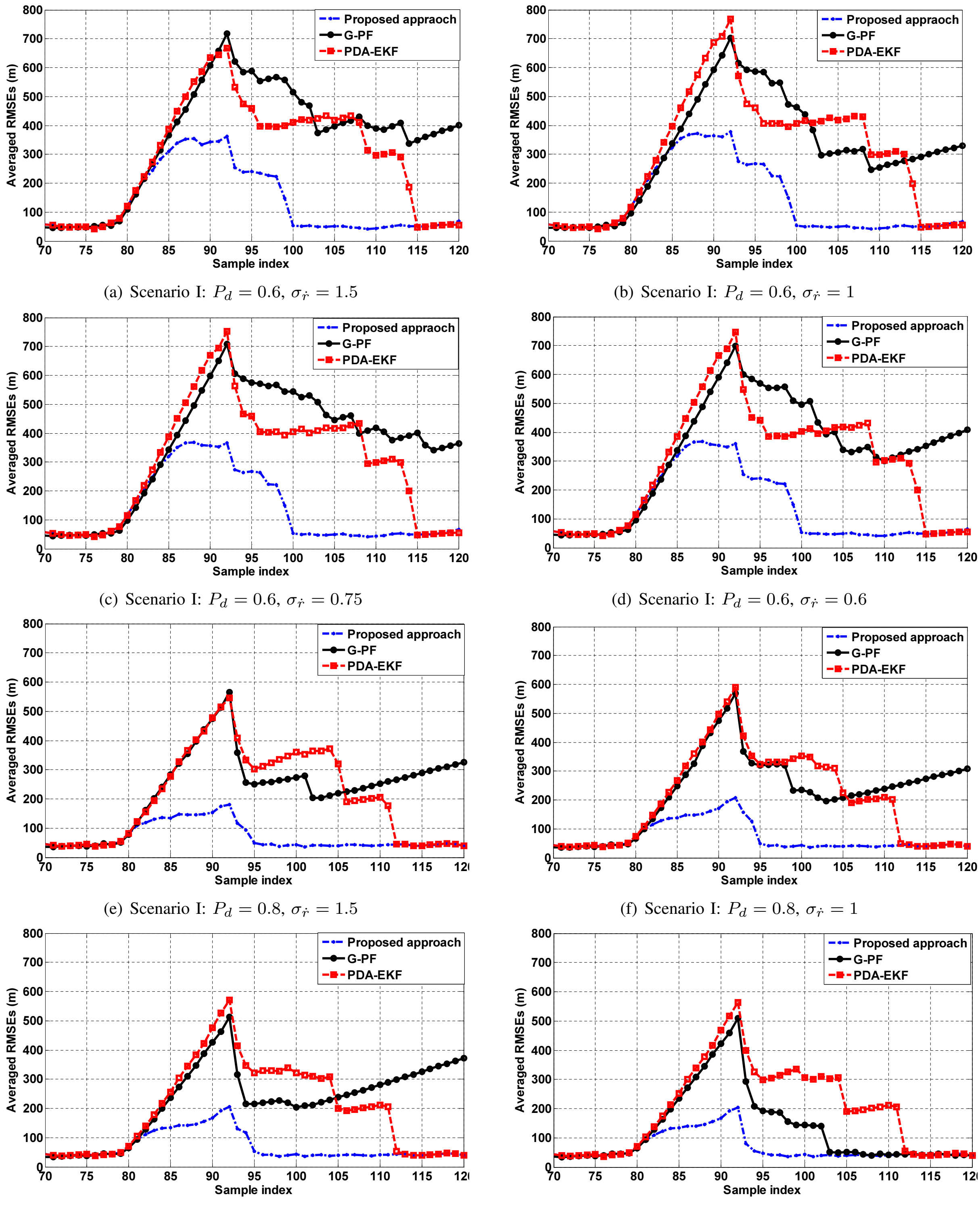
Scenario	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**
P_d	0.6	0.6	0.6	0.6	0.8	0.8	0.8	0.8
$\sigma_{\dot{r}}(m/sec)$	1.5	1.0	0.75	0.6	1.5	1.0	0.75	0.6
$\kappa/\sigma_{\dot{r}}$	2	3	4	5	2	3	4	5
PDA-EKF	426.69	451.84	442.38	439.33	317.98	320.52	307.27	302.70
G-PF	411.54	391.06	395.86	389.37	318.97	302.10	277.52	275.84
Proposed method	289.80	301.58	297.16	295.68	139.16	143.72	139.67	139.02

TABLE II
THE VELOCITY RMSES AVERAGED OVER 100 EXPERIMENTS (IN METERS/SECONDS) WHEN THE TARGET STOPS.

Scenario	**I**	**II**	**III**	**IV**	**V**	**VI**	**VII**	**VIII**
P_d	0.6	0.6	0.6	0.6	0.8	0.8	0.8	0.8
$\sigma_{\dot{r}}(m/sec)$	1.5	1.0	0.75	0.6	1.5	1.0	0.75	0.6
$\kappa/\sigma_{\dot{r}}$	2	3	4	5	2	3	4	5
PDA-EKF	11.20	11.47	11.20	11.11	8.72	8.47	8.09	7.97
G-PF	10.93	10.25	10.27	10.16	8.88	8.14	7.53	7.46
Proposed method	7.69	7.75	7.61	7.56	3.91	3.67	3.54	3.51

V. CONCLUSIONS

In this paper, we develop a novel algorithm for solving the GMTI tracking problem. The proposed algorithm is based on the exact Bayesian inference framework, with related distributions being modelled as a Gaussian mixture model. Various of filtering algorithms (including both the extended Kalman filter and its generalization version for interval measurement) are applied to update the target posterior distribution for consecutive time instances. The



(a) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 1.5$

(b) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 1$

(c) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 0.75$

(d) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 0.6$

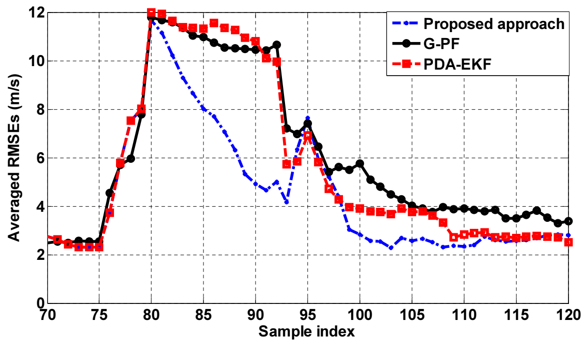
(e) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 1.5$

(f) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 1$

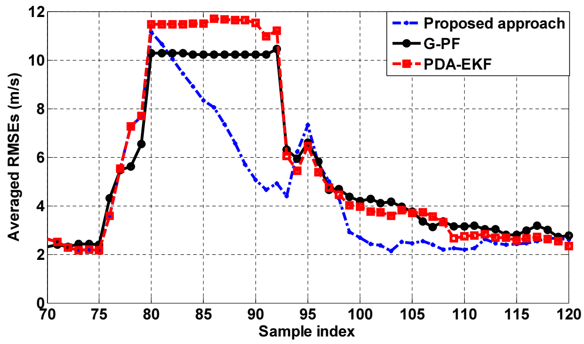
(g) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 0.75$

(h) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 0.6$

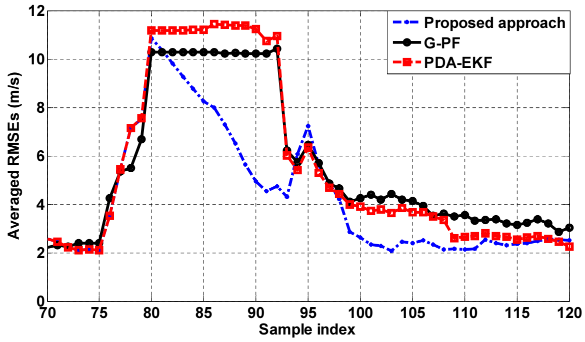
Fig. 5. The comparisons of position RMSEs for different tracking algorithms on different scenarios.



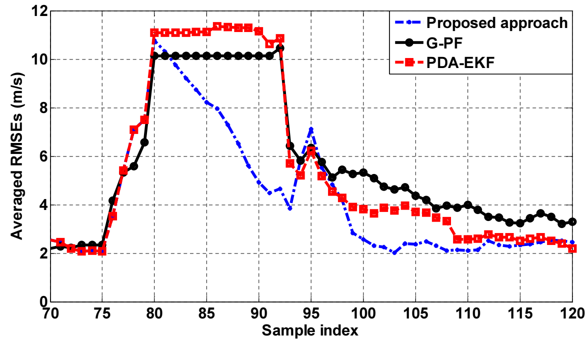
(a) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 1.5$



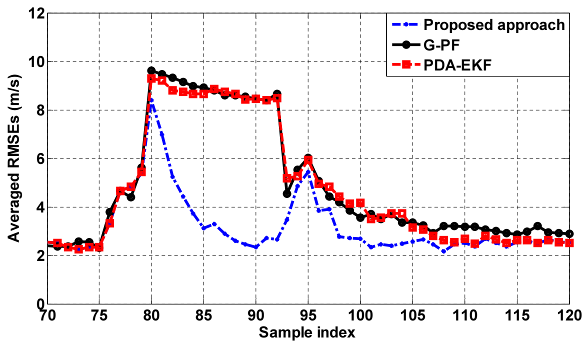
(b) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 1$



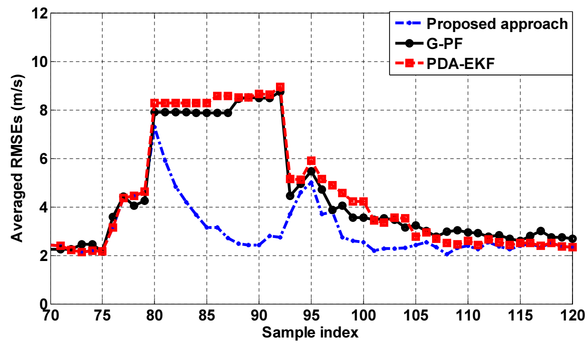
(c) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 0.75$



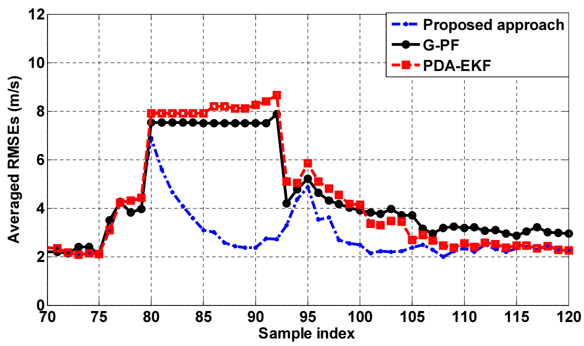
(d) Scenario I: $P_d = 0.6$, $\sigma_{\dot{r}} = 0.6$



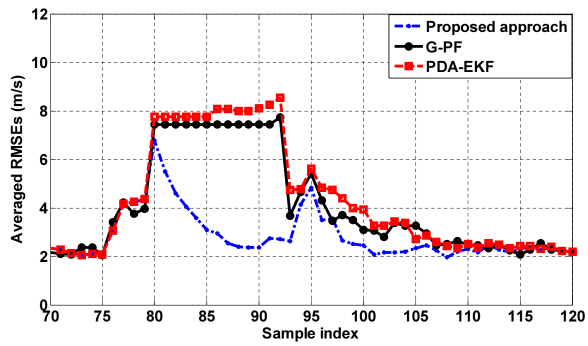
(e) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 1.5$



(f) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 1$



(g) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 0.75$



(h) Scenario I: $P_d = 0.8$, $\sigma_{\dot{r}} = 0.6$

Fig. 6. The comparisons of velocity RMSEs for different tracking algorithms on different scenarios.

proposed algorithm can not only deal with miss detection and false alarms; besides, it can effectively exploit the negative information for achieving more accurate tracking when a target stops. The simulation results show that the proposed algorithm can achieve more accurate tracking performance when compared with other approaches. To better cope with the target manoeuvring, multiple state models are needed. And we will integrate the proposed approach with a multiple model scheme in the future work.

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APPENDIX

We summarise the results on the mathematical expectation and variance conditional on some interval-censored measurements below; see [18] for details.

Consider a measurement m given by $m = \mathbf{q}^T\mathbf{x} + w$ with $w \sim \mathcal{N}(w|0, \sigma^2)$. Assume that the prior distribution of the state vector $\mathbf{x}$ is Gaussian, $\mathcal{N}(\mathbf{x}|\hat{\mathbf{x}}_0, \mathbf{P}_0)$. Let A denote an interval $A = [a, b]$.

Let $\mu = \mathbf{q}^T\hat{\mathbf{x}}_0$. Given that m falls into A, the conditional expectation and covariance are:

$$E(\mathbf{x}|m \in A) = \hat{\mathbf{x}}_0 + \mathbf{K}[\hat{m}_A - \mu], \tag{34}$$

$$cov(\mathbf{x}|m \in A) = \mathbf{P} + \mathbf{K}\mathbf{V}_A\mathbf{K}^T, \tag{35}$$

where

$$\mathbf{K} = \mathbf{P}_0\mathbf{q}(\mathbf{q}^T\mathbf{P}_0\mathbf{q} + \sigma^2)^{-1}, \tag{36}$$

$$\mathbf{P} = \mathbf{P}_0 - \mathbf{K}\mathbf{q}^T\mathbf{P}_0, \tag{37}$$

$$\hat{m}_A = E[m|m \in A] = c^{-1}\bar{\sigma}^2[\mathcal{N}(a|\mu, \bar{\sigma}^2) - \mathcal{N}(b|\mu, \bar{\sigma}^2)] + \mu, \tag{38}$$

$$\mathbf{V}_A = cov[m|m \in A] = c^{-1}\bar{\sigma}^2[(a+\mu)\mathcal{N}(a|\mu, \bar{\sigma}^2) - (b+\mu)\mathcal{N}(b|\mu, \bar{\sigma}^2)] + (\mu^2 + \bar{\sigma}^2) - \hat{m}_A^2. \tag{39}$$

The parameter $\bar{\sigma}^2$ is estimated as $\bar{\sigma}^2 = \mathbf{q}^T\mathbf{P}_0\mathbf{q} + \sigma^2$, and c in (39) is a normalizing constant ensuring that the corresponding probability density integrates to unity, i.e. $c = \int_a^b \mathcal{N}(m|\mu, \bar{\sigma}^2)dm$.