Integral Sliding Mode Control Design for High Speed Tilting Trains

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***Abstract:* Active tilt technology has been widely used in high-speed railway vehicle. The used of 'precedence' tilt control design have been accepted most manufacture around the world. This technology has it on drawback due to the complexity and the control design parameter is depending on train velocity and track route to achieve satisfactory ride quality. This research is the extension work on investigating the local loop control scheme whereby the sensor not depending on vehicle in front, which reduce the complexity of control design. A proposed integral sliding mode control strategies aim to improved body roll responses in curved track while reducing the effect of straight track irregularities.**

Keywords: sliding mode control, high speed tilting trains.

I. TILTING HIGH SPEED RAILWAY VEHICLE

The use of tilting technologies in high speed train running on conventional tracks decreases the journey time between two places. Non-tilting conventional trains operated at slower speeds on curved track due to the lateral force acting on the vehicle. By leaning the vehicle train inwards on curved sections, will reduces the lateral force, thus allowing an increase in speed of the train while maintaining appropriate passenger lateral accelerations.

Figure 1 shows the complex structure of the railway car body. It consists of an arrangement of interconnections between the vehicle body, two bogies and four wheel sets located at each end of the vehicle. The railway systems are dynamically complex systems characterized by a significant coupling between the lateral and roll motion and often referred to as the 'sway modes' (see Fig. 2). The mathematical model of the system is based upon the end-view of a railway vehicle, to incorporate both the lateral and roll degrees of freedom for both the body and the bogie structures. A pair of air springs represents the secondary suspension, whilst the primary suspension is modeled via pairs of parallel spring/damper combinations. The model also included the stiffness of an anti-roll bar (ARB) connected between the body and the bogie frames. Active tilt is provided via a rotational displacement actuator, included in series with the roll stiffness. The active anti-roll bar is assumed to cater for up to a maximum tilt angle of 10°. The advantages of active ARB result from their relative simplicity, low cost and easily fitted [1].

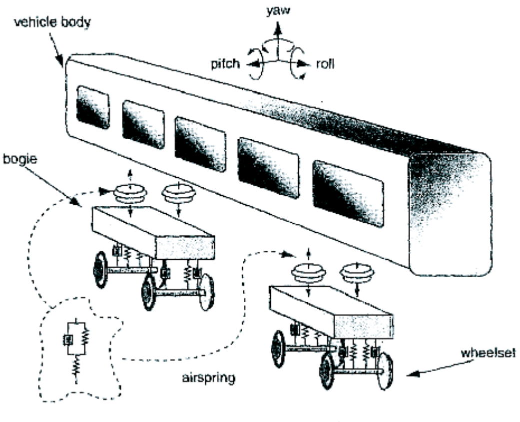

Figure 1. Mechanical arrangement of railway vehicle.

The tilt model system can be represented in state space by:

$$\begin{aligned} \dot{x} &= Ax + Bu + \Gamma w \\ y &= Cx \end{aligned} \tag{1}$$

where $A \in \mathbb{R}^{n\times n}, B \in \mathbb{R}^{n\times m}$ and $C \in \mathbb{R}^{p\times n}$. *n,m* and *p* represent the number of states, control inputs and output respectively.

Consider ω as constant external disturbance matrix given by;

$$w = [\,\frac{1}{R}\,\theta_o\;\dot{\theta}_o\;\ddot{\theta}_o\;y_o\;\dot{y}_o\,]^T \tag{2}$$

where $\frac{1}{R}$ is curvature, $\theta_o, \dot{\theta}_o, \ddot{\theta}_o$ are the elevation track components and $y_v,\ \dot{y}_v$ are the lateral track irregularities components and $u = [\delta_a]$ is the vehicle actuator angle and the state vector *x*,

$$x = \begin{bmatrix} y_v & \theta_v & y_b & \theta_b & \dot{y}_v & \dot{\theta}_v & \dot{y}_b & \theta_r \end{bmatrix} \tag{3}$$

The output measurement, the *effective cant deficiency* (e.c.d), θ_{ecd} is the combination of body lateral acceleration ($\ddot{y}_{vm}$) and secondary suspension roll angle (θ_{2sr}), which give 60% tilt compensation to the tilt angle and it given by:

$$\theta_{ecd} = -0.615\frac{\ddot{y}_{vm}}{g} - 0.385\theta_{2sr} \tag{4}$$

Moreover, the variety of track inputs also contributes to the complex system which can be categorized into deterministic and stochastic signals Deterministic inputs refer to the curved track which is carefully designed by civil engineers to meet the requirement of the passenger comfort index. The curved track is leaned inward (elevated) around 6° rising linearly over a period of 2-3 seconds at the transition of the start and end of the curve. The stochastic track input represents the deviations of the actual track from the intended alignment, irregularities which occur in the vertical, lateral and cross-level directions. The secondary suspension of the vehicle is designed to reduce the effect of track irregularities, expressed in RMS acceleration levels in the body of the vehicle. In principal, the design of the tilt controller is to provide a fast response related to the transition to and from the curves, but at the same time does not affect the responses on track irregularities,i.e. the ride quality on straight track Early tilt system used a local feedback control from a lateral accelerometer mounted on the body of the vehicle. However, it proved difficult to achieve fast response on curve transition without suffering substantial ride quality degradation on straight track. Current tilting railway vehicle now use ‘precedence’ tilt control strategies [2]. In this scheme, a bogie-mounted accelerometer is used to develop a tilt command signal by measuring the curving acceleration on non-tilting part of the vehicle However, because the accelerometer also measures frequency movement associated with lateral track irregularities, it is necessary to filter the signal. This filtering action creates a detrimental performance on the transition from the straight track to curve section. The usual solution is to use accelerometer signal from the vehicle in front to provide ‘precedence’, carefully designed so that the delay introduced by the filter compensates for the preview time corresponding to a vehicle length. Since the signal interconnected between vehicles, it increases complexity of the system [2].

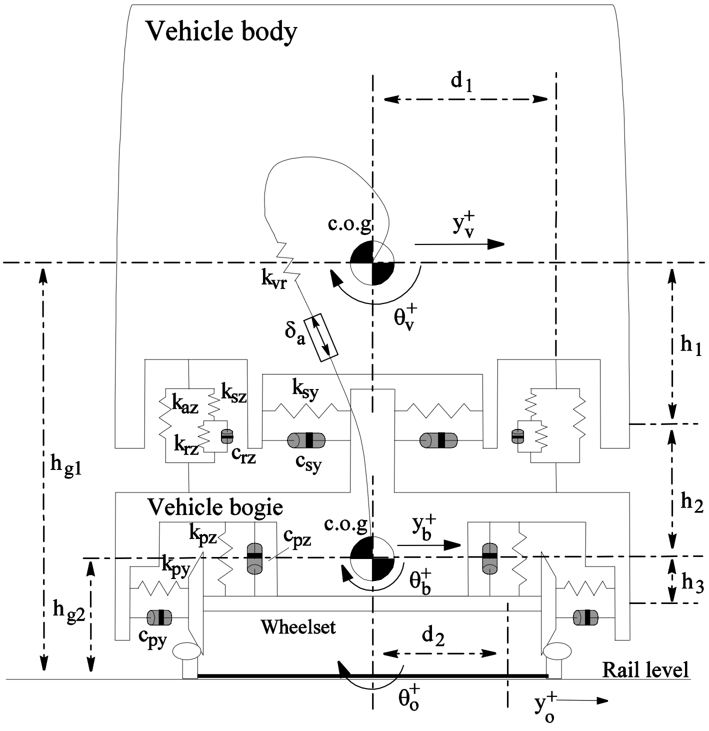

Figure 2. End-view schematic representation of tilting vehicles.

The development of tilting control strategies based on local sensors (i.e no need precedence) has shown progress in recent years. In the work reported by [3], the design of a classical controller was improved by using the estimated 'true' cant deficiency to provide partial tilt based on the combination of vehicle body lateral acceleration and a gyroscope to measure the body roll speed. Unfortunately, the oscillations due to interaction with the lateral suspension still exist. Investigation of capability using fuzzy control in local loop tilt control scheme have been done by Zamzuri *et al.* [4]. The researchers in [5-7] introduction the fuzzy mechanism as ‘add-on’ to classical and model based control scheme to improve the performances both on the stochastic and deterministic responses. Therefore, this paper presents the extension work on local loop tilt control strategies using sliding mode control scheme. A comparison and performance analysis between proposed control scheme and current technology used is also being presented in the last chapter.

II. COMMAND DRIVEN WITH PRECEDENCE SCHEME

Current technology used in tilting train, known as ‘precedence’ tilt control strategies [2] used a accelerometer mounted in bogies of a non-tilting part of the vehicle to measure the curving acceleration. However, the accelerometer also measured high frequency signals due to the bogies are directly contacted with the track. Therefore, it is necessary to filter the signal. The filter action creates time delays which lead to detrimental performance on the transition from straight track to a curve section. The usual solution is to use accelerometer signal from the vehicle in front to provide ‘precedence’, as shown in Figure 3, carefully designed so that the time delay introduce by the filter compensates for the preview time corresponding to a vehicle length. Normally a single command signal front or the 1st vehicle would be used to generate the tilt signal and transmitted to all vehicle train with appropriate time delay. However, due to the delay signal, the velocity and direction of travel are important factor for the correct operation. This strategy proved to be successfully. However is more complex-sensitive scheme with the signal connection between vehicle and tilt system parameter need to be optimized based on the train route. Moreover, leading vehicle has inferior performance due to the lack of precedence. This approach is not strictly optimizes in term of control design point of view, while achieving a satisfactory local sensor loop tilt control strategy remain an important research topic due to the system simplification and more straight forward system design.

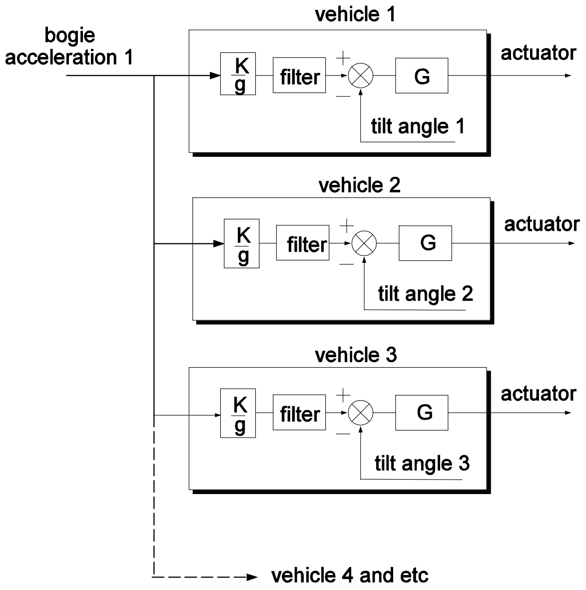

Figure 3. Command driven precedence tilt control scheme used in current tilting high speed railway vehicle.

III. INTEGRAL SLIDING MODE TILT CONTROL STRATEGY

This section will introduced the design of Sliding Mode controller on tilt control system for railway vehicle. The design is based on local null control scheme whereas the feedback signal will locally feed to the controller. The aims of the controller design it to minimize the influence of the straight track irregularities while improving fast responses on curved track. The controller approach used integral compensator to achieve steady-state zero.

For disturbance rejection and to enhance the ability of the controller to achieved required tilt responses without steady state error, the states is included with the *integral of effective cant deficiency*, $x'_\theta = \int (-\theta_{dm})dt$ which give the overall state model,

$$\tilde{x} = \begin{bmatrix} x'_\theta & x \end{bmatrix}^T \tag{5}$$

and the system and input distribution matrices are

$$\tilde{\mathbf{A}} = \begin{bmatrix} 0 & -C \\ 0 & A \end{bmatrix} \quad \tilde{\mathbf{B}} = \begin{bmatrix} 0 \\ B \end{bmatrix} \tag{6}$$

To developed a sliding mode controller, compose Eq. (6) to a regular form as described below

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \tilde{B} \end{bmatrix} \tag{7}$$

where

$$\begin{bmatrix} \tilde{A}_{11} & \tilde{A}_{12} \\ \tilde{A}_{21} & \tilde{A}_{22} \end{bmatrix} = \begin{bmatrix} 0 & -C_1 & -C_2 \\ 0 & A_{11} & A_{12} \\ 0 & A_{21} & A_{22} \end{bmatrix} \tag{8}$$

Let the switching surface be define as $\sigma = S\tilde{x}$

$$\sigma = \begin{bmatrix} S_1 & S_2 \end{bmatrix} \begin{bmatrix} \tilde{x}_1 \\ \tilde{x}_2 \end{bmatrix} \tag{9}$$

$$S_1 \in \mathbb{R}^{m\times n+p}; \quad S_2 \in R$$

If sliding surface $\sigma = S\tilde{x} = 0$, then the equivalent system in eq 9 become,

$$\dot{x}_1(t) = (\tilde{A}_{11} - \tilde{A}_{12}S_2^{-1}S_1)\tilde{x}_1(t) \tag{10}$$

If $M = \frac{S_1}{S_2}$ and $S_1 = MS_2$, then the sliding motion, S become,

$$S = S_2[M \quad 1] \tag{11}$$

From Eq. (10) and Eq. (11), it can be concluded that the matrix S_2 is acting as a scaling factor for the switching function and parameter $\tilde{A}_{11}^s = \tilde{A}_{11} - \tilde{A}_{12}M$ must have stable eigenvalues. Since $(\tilde{A}_{11}, \tilde{A}_{12})$ is controllable, the LQR method is used to obtained matrix gain *M*.

The sliding mode control input is separated into linear μ_l and nonlinear μ_{nl} components. The linear μ_l can be obtained by the following equation,

$$\dot{x} = \tilde{A}\tilde{x} + \tilde{B}u$$

$$\dot{\sigma} = S\sigma = 0$$

which will give

$$u_l(t) = -(S\tilde{B})^{-1}(S\tilde{A} + \phi S)$$

and for nonlinear control law [8],

$$u_{nl} = -\rho(t,\tilde{x})(SB)^{-1}\frac{Ps(t)}{\| Ps(t) | + \delta |} \quad (12)$$

$$\text{for} \quad s(t) \neq 0$$

where σ is a small positive constant and depends on the magnitude of the uncertainty [8], and $P \in \mathbb{R}^{m \times m}$ is a symmetric positive definite matrix satisfying the Lyapunov equation, while ϕ is the range dynamic satisfying $P\phi + \phi^T P = -I$.

Figure 4 shows the integral sliding mode control scheme for the tilt control design. The feedback signals to the controller consists of the vehicle model states and the *integral of the effective cant deficiency*, $\int(\theta_{dm})$ to give $\tilde{x} = \left[x'_\theta \quad x\right]^T$.

IV. RESULTS AND ANALYSIS

For designing the regulator, which requires a state feedback gain *M* to be chosen so the $\tilde{A}_{11} - \tilde{A}_{12}M$ is stable, the quadratic minimization approach is used to seek an optimum value for *M*. In this approach, the cost functional is minimized, where t_s represent the time at which sliding first occurs and the cost function is,

$$J = \int_{t_s}^{\infty} \tilde{x}^T(t)Q\tilde{x}(t)dt \quad (13)$$

Figure 5 and Fig. 6 show the comparison of the vehicle body lateral acceleration and body roll gyro responses at speed of 210 km/h on curved track between current technologies used in high speed tilting railway vehicles (precedence control scheme) and proposed integral sliding mode control approaches. It can be shown the proposed control scheme offer an improvement compared to conventional precedence control scheme, due to the used of local feedback control scheme. Please note: precedence control scheme is more complex design scheme due to interaction between two vehicles to provide tilt information. Figure 5 also shown the responses improvement towards steady state zero at curved track due to the used of integral action of effective cant deficiency. Moreover, the improvement is also shown for the body lateral acceleration responses on straight track in Table 1.

The ride quality degradation shown in the table is difference in root means square (rms) values taken between the passive (without the controller: rms value = 0.381 %/g) and active system traveling at high speed (210 km/h) which is increases 30% compared to non-tilting vehicle speed.

V. SUMMARY

The paper reveals the potential of proposed sliding mode controller scheme on local feedback tilting control design. The used of output information of *integral cant deficiency* guarantee zero steady state error and definitely reduce/reject external disturbance present in most practical system. Since, sliding mode is mostly used for an unmatched parameter and non-linear system, the result shows the nominal plant give performance equal to current precedence control strategies. The next proposed scheme is to include observer in the integral sliding tilt control strategies since not all system parameter state can be measured directly by the sensors.

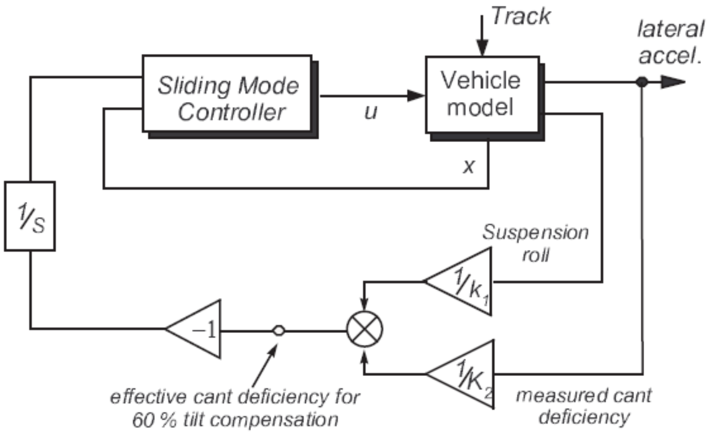

Figure 4. Integral sliding mode tilt control scheme.

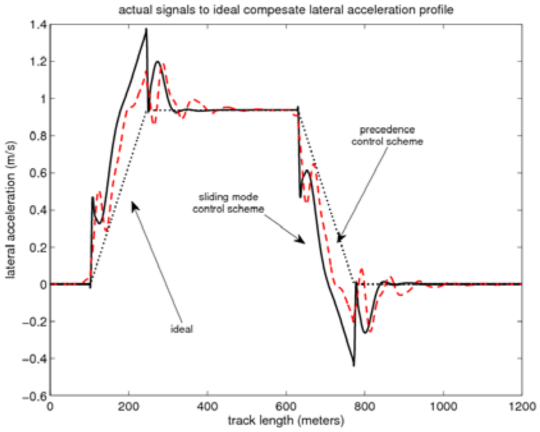

Figure 5. Body lateral accelerations responses.

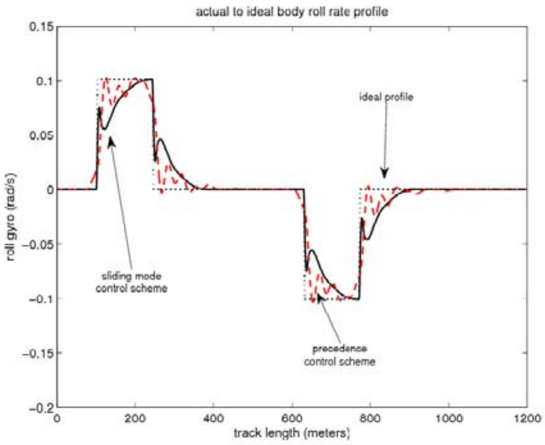

Figure 6. Body roll gyro responses.

Table 1: Body acceleration on straight track.

	rms values (%g)	Ride quality degradation (%g)
Precedence control scheme	0.332	-12.143
Integral sliding mode	0.309	-19.00

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