DEVELOPMENT OF A GENERAL FISHERIES PRODUCTION FUNCTION: THE ROLE OF EFFORT INDICES AND SEPARABILITY

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ABSTRACT

This paper discusses the connection between the concepts of fishing effort and separability of the variables in a general fisheries production function of the Translog form. It compares the results of the discussion with the Cobb-Douglas and Constant Elasticity of Substitution forms. It concludes that to assume unconditionally that some or all inputs in the production function can be aggregated into one effort index, frequently done simply by multiplication of the separate input variables, is often erroneous. First, the variables may not be separable, and secondly, if some input variables are separable, the structure of the effort index will depend on the specification of the production function employed, and will not be a simple product.

Keywords: Fisheries Economics, Fisheries Production Function, Fishing Effort, Separability

Introduction

Since fisheries scientists constructed a production function for their own purposes with only two explanatory variables, the fish stock and an index of all other inputs called fishing effort, fishery managers have been encouraged to believe that effort is a tangible intermediate variable that can be controlled. Legislation has been written in that belief. For example, European Union Council Regulation 2371/2002 empowered the Council to achieve its management objectives by "limiting fishing effort", defining fishing effort as "the product of the capacity and the activity of a fishing vessel; for a group of vessels it is the sum of the fishing effort of all vessels in the group"[1].

Controlling fishing effort has become an important factor in the management of fisheries. It is usually part of a combination of input controls including vessel and gear restrictions. Serious problems defining fishing effort were set out by Hildebrandt (1975) and Huang and Lee (1976) followed by questioning the concept itself.

The purpose of this paper is to consider whether a single index of the non-stock factors of production can be constructed and to examine the requirements. If this is to be done the non-stock inputs have to be separable from the fish stock as a single group. It is shown that this is not always the case.

The aim is achieved by investigating separability in the Transcendental Logarithmic (Translog) function using established mathematical theory (Christensen, Jorgenson and Lau 1971). The

[1] Council Regulation (EC) No 2371/2002 of 20 December 2002 *on the conservation and sustainable exploitation of fisheries resources under the Common Fisheries Policy*, Article 3, para (h) and Article 4, para 2(f).

Translog function was chosen because of its general functional form. The Cobb-Douglas (Cobb and Douglas 1928) and the Constant Elasticity of Substitution (CES) (Arrow *et al* 1961) functions which are reduced forms of the Translog are included for comparison.

The paper is composed of 5 sections. Section 2 gives a short review of the concept of separability and its connection with the concept of fishing effort. Section 3 discusses the economics and the mathematics, Section 4 discusses separability conditions for the Cobb-Douglas, the CES, and the Translog functional forms. Finally, Section 5 presents the discussion and conclusions from the analysis.

The Concepts of Separability and Fishing Effort

Biological and economic research defines fishing effort in different ways. Andersen (1999) gives a review of the different approaches and states the basic distinction between them, that "In the traditional biological approach the level of catch determines the level of effort. The causality is reversed in the economic approach… where the level of effort is the determinant of catch".

The biological approach distinguishes between effective fishing effort and nominal fishing effort. The former is equivalent to fishing mortality while the latter refers to non-fish stock resources employed in fishing through a specified aggregate effort index, usually measured as the product of fishing time and engine power.

In the economic approach, nominal fishing effort is normally still composed of fishing time and fishing power (which may, however, consist of several different factors), but without assuming a pre-defined aggregate index of these. It is, on the contrary, recognised that it is often not possible to construct a consistent aggregate effort index composed of activity and engine power (Del Valle, Astorkiza and Astorkiza 2000).

It is more than a quarter of a century since Hildebrandt (1975) expressed his doubts about the complexities of trying to measure fishing effort in a way that enabled a sensible comparison to be made across fleets which vary technically. He identified fishing effort as a biological item but after examining a wide range of the component variables of fishing power, he concluded that fishing effort is very difficult to measure consistently.

Hannesson (1993) also considered the problem of defining fishing effort and concluded that it "seems impossible to derive any generally valid relationship" between fishing effort as a strict counterpart to fishing mortality and fishing effort as an index of inputs in a production process.

Squires (1987) on the other hand used sophisticated methods to calculate consistent indices for the capital, labour, and fuel inputs in fisheries at the level of the vessel production function and then combined them in an effort index. The size of the fish stock was excluded on the grounds that it is costless to the firm. Nevertheless, he concluded that if effort is not separable any attempt to construct an index from the inputs will lead to one which varies with the quantity and mix of landings.

Theoretically the problems with the concept of an aggregate index of fishing effort are closely connected to the concept of separability of production functions. An economic production function is defined to be separable if the input factors can be divided into two or more groups in such a way that the marginal rate of technical substitution of the factors in one group is not affected by the factors in another group; that is, the rate at which the factors in the one group must be substituted for each other in order to keep production (catch) constant is not affected by the price of the factors in the other groups.

Assume, for example, that the input factors are fish stock, fishing time and maximum vessel horsepower, and that the fishing time is constrained to a reduced level by new regulations. For the product of fishing time and fishing power to constitute a consistent effort index, the rate at which the fisherman must exchange the declining number of fishing days with a larger engine must be independent of the size and spatial distribution of the fish stock.

In practice, it is difficult not to group variables in a production function since the number of inputs in a production process is generally high. Grouping, say, the different labour and capital variables into two indices reduces the mathematical risk but at a trade-off with theoretical purity (Solow 1955, Fisher 1969).

However, the question of separability depends on the form of the production function employed. The Cobb-Douglas and CES production functions are globally separable by nature. For these functions, fishing time and fishing power are separable from the fish stock and can as such be combined into an aggregate effort index, the form of which depends on the functional form chosen. The Translog production function, which may be seen as a second-order Taylor approximation to any general production function (Heathfield and Wibe 1987), is on the other hand only separable given specific restrictions on the functional form.

Note from the above, that the specification of an effort index consistent with separable forms of the production function generally depends on the structure of the production function itself. Thus no global aggregated effort index can be constructed.

The Relationship between Economic Theory and Mathematical Practice

Selection of the type and quantity of factors of production used in fishing is firmly based in the micro-economic theory of choice. For example, it is possible to fish either by labour-intensive or capital-intensive methods. The mix of capital and labour arrived at to produce a given level of output will be governed by cost-minimisation, since this maximises the profitability of individual firms in a competitive environment. The relative unit price of the inputs and whether they are substitutes, complements, or separable, will determine the particular mix of inputs chosen. When the price of one input changes relative to that of another, the input quantities employed will move in opposite directions if they are substitutes, and in the same direction if they are complements. If they are separable, a change in the relative price of one input will have no effect on the proportions of the quantities used of the separable inputs.

Inputs to the production process are manifold and may include not only the fish stock, but also fishing gear, vessel type and size, fuel consumed, crew and so forth. Separability implies that the fisheries production function may be reduced to a function of stock and one or more effort index functions of the remaining input variables.

Normally in microeconomic theory, the level of output *y* of a production process is considered to be determined by an industry production function explained by the three factors of production; land, labour, and capital (Samuelson 1947). Measured in physical units of each employed, these may in fisheries be translated into resource (fish stock) *S*, labour *L*, and capital *K*, such that:

$$y = f(S, L, K) \tag{1}$$

Solow (1955) discusses the appropriateness of using an index to represent capital in a production function that incorporates labour and two or more forms of capital. He concludes that a necessary and sufficient condition for the forms of capital to be collapsible into a single index is that the marginal rate of technical substitution of one kind of capital good for the others must be independent of the amount of labour used. Thus the production function must satisfy the Leontief theorem on separable functions (Leontief 1947).

In fisheries production theory, the capital and labour inputs have habitually been collapsed into a single effort index that is assumed to be sufficient to explain output given the stock abundance. Such functions are usually of the following form (Cunningham, Dunn and Whitmarsh 1985), where it is assumed that the variables are homogeneous:

$$y = h(S, E) \quad ; \quad E = \phi(L, K) \tag{2}$$

This is however only a valid representation of the data generation process if capital and labour are actually separable from the fish stock. In this connection Huang and Lee (1976) consider the separability conditions to be a rather severe restriction of a general fishery production function and conclude that “the concept of a single index of fishing effort stands on a weak foundation”.

Huang and Lee (1978) identified two additional difficulties. First, important aspects of production cannot be analysed when all inputs are aggregated into a single effort factor. Secondly, the crowding externality is excluded because fishing effort emphasises the biological aspect of the resource and especially the stock externality.

The evidence from empirical testing of the usefulness of an aggregate fishing effort index is mixed, which is perhaps not surprising given that the concept may be theoretically suspect. However, the basis for any discussion of an index of fishing effort should always be the question of the separability of the specific form of the production function chosen. This is therefore the starting point below.

Separability of Production Functions

This review of the theory of input separability in production functions follows the introduction given in Boisvert (1982) and discusses its meaning and implications in relation to fisheries. Separability is discussed first in general terms, summing up the basic mathematics necessary to test for its presence in a given production function. Secondly, and more specifically, it is discussed for the Translog, Cobb-Douglas and CES functions.

A production function specifies a parametric dependence between the product y of a given industry and the factors of production ($x_1,\dots, x_n$) such that $y \equiv f(x_1,\dots, x_n)$.
The function f is said to be separable if the explanatory factors can be divided into two or more different groups in such a way that the substitution properties within one group are not affected by the explanatory variables outside the group. Each group can thus be represented by an orthogonal index function or composite input of the factors in the group. A distinction is made between weak and strong separability.

Consider an input set X = ($x_1,\dots, x_n$), subject to the partition X = [$X_1,\dots, X_R$], for which each subset X consists of at least one element (a factor of production) and for which the subsets are non-overlapping; that is, they do not share input factors. Then f is defined to be weakly separable for the partition X if there exist individual index functions $g_1,\dots, g_R$, where each g_i is defined for the subset X_i, such that f can be represented by $f \equiv F[g_1(X_1),\dots, g_R(X_R)]$. If moreover the specific form of F is given by $f \equiv g_1(X_1) + \dots + g_R(X_R)$ then f is defined to be strongly separable. It is clear that strong separability implies weak separability.

Thus, a necessary and sufficient condition for a function f to be weakly separable is that the marginal rate of technical substitution between any two factors x_m and x_n within each of the R subsets is independent of every factor x_k from the other subsets.

The marginal rate of technical substitution is given by:

$$MRS_{nm} \equiv -\frac{\partial x_m}{\partial x_n} = -\frac{\left(\frac{\partial f}{\partial x_n}\right)}{\left(\frac{\partial f}{\partial x_m}\right)} \equiv -\frac{f_n}{f_m} \tag{3}$$

f is then weakly separable if and only if:

$$\frac{\partial}{\partial x_k}\left(\frac{f_n}{f_m}\right) = \frac{f_{nk} f_m - f_n f_{mk}}{\left(f_m\right)^2} = 0 \quad x_n, x_m \in X_N, x_k \in X_K, X_M \cap X_K = \{\ \} \tag{4}$$

where $f_i = \partial f/\partial x_i$ is the first derivative of f with respect to x_i, and $f_{ij} = \partial f/(\partial x_i \partial x_j)$ is the second derivative of f with respect to x_i and x_j.

(4) further shows that the function is weakly separable if and only if

$$f_{nk} f_m - f_n f_{mk} = 0 \qquad (5)$$

Likewise the function f is strongly separable, if and only if, for x_m taken from the group X_M and x_n taken from the group X_N, the marginal rate of technical substitution between these two factors is not affected by any factor x_k outside the subgroups for these two factors:

$$\frac{\partial}{\partial x_k}\left(\frac{f_n}{f_m}\right) = 0 \quad x_n \in X_N, x_m \in X_M, x_k \in X_K, X_N \cap X_K = \{\ \}, X_M \cap X_K = \{\ \} \qquad (6)$$

In the following, only weak separability is considered.

From (3) it is readily shown that the Cobb-Douglas production function:

$$y \equiv f(x_i, \ldots, x_n) = \alpha_0 x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdots x_n^{\alpha_n} \qquad \Leftrightarrow \qquad \ln(y) = \ln(\alpha_0) + \sum_{i=1}^{n} \alpha_i \ln(x_i) \qquad (7)$$

is separable in all the explanatory variables.

Consider an example with three inputs. In this setting the Cobb-Douglas function has the form:

$$Y \equiv f_{CD}(x_1, x_2, x_3) = \alpha_0 \cdot x_1^{\alpha_1} \cdot x_2^{\alpha_2} \cdot x_3^{\alpha_3} \qquad (8)$$

where Y is total landed quantity. It can be seen that, for example, x_1 can x_2 can be aggregated into one separable input index by the following operation:

$$x_1^{\alpha_1} \cdot x_2^{\alpha_2} = \left(x_1^{\frac{\alpha_1}{\alpha_1+\alpha_2}} \cdot x_2^{\frac{\alpha_2}{\alpha_1+\alpha_2}} \right)^{\alpha_1+\alpha_2} \equiv \left(E_{CD}(x_1, x_2)\right)^{\alpha_1+\alpha_2} \qquad (9)$$

where E_{CD} is the aggregate input index. With this notation the Cobb-Douglas function (9) is given by:

$$Y \equiv f_{CD}\left(E_{CD}(x_1, x_2), x_3\right) = \alpha_0 \cdot E_{CD}^{\alpha_1+\alpha_2} \cdot x_3^{\alpha_3} \qquad (10)$$

This is still a Cobb-Douglas function, but it is now one employing an aggregate input index for the two first inputs.

It is also readily shown from (3) that the CES production function:

$$y = \gamma \cdot \left(\sum_{m=1}^{n} \beta_m x_m^{-\rho} \right)^{-\nu/\rho} \quad ; \quad \sum_{m=1}^{n} \beta_m = 1 \tag{11}$$

is separable in all the explanatory variables.

The CES function including three inputs has the form:

$$Y \equiv f_{CES}(x_1, x_2, x_3) = \beta_0 \cdot \left(\beta_1 \cdot x_1^{-\rho} + \beta_2 \cdot x_2^{-\rho} + \beta_3 \cdot x_3^{-\rho} \right)^{-\nu/\rho} \tag{12}$$

ρ is the substitution parameter and ν gives the returns to scale. For this function a separable aggregate input index of x_1 and x_2 can be determined by the operation:

$$\begin{aligned} \beta_1 \cdot x_1^{-\rho} + \beta_2 \cdot x_2^{-\rho} &= (\beta_1 + \beta_2) \cdot \left[\frac{1}{(\beta_1 + \beta_2)^{-1/\rho}} \left(\beta_1 \cdot x_1^{-\rho} + \beta_2 \cdot x_2^{-\rho} \right)^{-1/\rho} \right]^{-\rho} \\ &\equiv (\beta_1 + \beta_2) \cdot E_{CES}^{-\rho} \end{aligned} \tag{13}$$

where E_{CES} is the aggregate input index. Thus, the CES function (13) is given by:

$$Y \equiv f_{CES}(x_1, x_2, x_3) = \beta_0 \cdot \left((\beta_1 + \beta_2) \cdot E_{CES}^{-\rho} + \beta_3 \cdot x_3^{-\rho} \right)^{-\nu/\rho} \tag{14}$$

This is again a CES function, but now including an aggregate index for the first two inputs.

It is apparent from these two examples that aggregate effort indices are not just the products of separable variables, but that, on the contrary, the specificaton of effort indices depends on the form of the production function in which they are to be present, be it a Cobb-Douglas or a CES function.

The Translog function, which is defined as:

$$\ln(y) = \alpha_0 + \sum_{i=1}^{N} \alpha_i \ln(x_i) + \frac{1}{2} \sum_{i=1}^{N} \sum_{j=1}^{N} \alpha_{ij} \ln(x_i) \ln(x_j) \tag{15}$$

is, on the other hand, only separable if certain restrictions on the Translog parameters are fulfilled.

Boisvert (1982) showed that the Translog function is separable into the group X_M containing at least two parameters if and only if, for the Translog parameters related to any (x_i, x_j) in X_M and for the Translog parameters related to any x_k outside X_M, it holds that:

$$\left(\alpha_{ki} \alpha_j - \alpha_{kj} \alpha_i \right) + \sum_{m=1}^{n} \left(\alpha_{ki} \alpha_{mj} - \alpha_{kj} \alpha_{mi} \right) \ln(x_m) = 0 \tag{16}$$

(16) shows that the Translog function is globally separable (that is, independent of the explanatory variables) if and only if the following conditions are fulfilled simultaneously:

$$\alpha_j \alpha_{ki} - \alpha_i \alpha_{kj} = 0 \quad \text{and} \quad \alpha_{ki}\alpha_{mj} - \alpha_{kj}\alpha_{mi} = 0 \tag{17}$$

If these conditions are fulfilled for all m = (1,…, N) for the Translog parameters related to (x_i, x_j) together and for the Translog parameters related to any $x_k \neq (x_i, x_j)$, the marginal rate of substitution between x_i and x_j is independent of any other x_k in the sample of inputs.

When three inputs are present, the Translog function (15) has the form:

$$\begin{aligned}\log(Y) = f_{TL}(x_1, x_2, x_3) \equiv \alpha_0 + \alpha_1 \cdot \log(x_1) + \alpha_2 \cdot \log(x_2) + \alpha_3 \cdot \log(x_3) + \\ \frac{1}{2}\alpha_{11} \cdot \log(x_1) \cdot \log(x_1) + \frac{1}{2}\alpha_{22} \cdot \log(x_2) \cdot \log(x_2) + \frac{1}{2}\alpha_{33} \cdot \log(x_3) \cdot \log(x_3) + \\ + \alpha_{12} \cdot \log(x_1) \cdot \log(x_2) + \alpha_{13} \cdot \log(x_1) \cdot \log(x_3) + \alpha_{23} \cdot \log(x_2) \cdot \log(x_3)\end{aligned} \tag{18}$$

This function is separable into the groups (x_1,x_2) and x_3 if and only if the following conditions (19) on the Translog parameters are all fulfilled:

$$\begin{aligned}\alpha_1 \cdot \alpha_{23} - \alpha_2 \cdot \alpha_{13} &= 0 \\ \alpha_{13} \cdot \alpha_{12} - \alpha_{23} \cdot \alpha_{11} &= 0 \\ \alpha_{13} \cdot \alpha_{22} - \alpha_{23} \cdot \alpha_{12} &= 0\end{aligned} \tag{19}$$

If this is the case, some manipulation shows that (18) can be written as:

$$\begin{aligned}\log(Y) = f_{TL}(E_{TL}(x_1, x_2), x_3) \equiv \alpha_0 + \alpha_1 \cdot \log(E_{TL}) + \alpha_3 \cdot \log(x_3) + \\ \frac{1}{2}\alpha_{11} \cdot \log(E_{TL}) \cdot \log(E_{TL}) + \frac{1}{2}\alpha_{33} \cdot \log(x_3) \cdot \log(x_3) + \alpha_{13} \cdot \log(E_{TL}) \cdot \log(x_3)\end{aligned} \tag{20}$$

where

$$E_{TL} = \exp\left(\log(x_1) + \frac{\alpha_{23}}{\alpha_{13}}\log(x_2)\right) = x_1 \cdot x_2^{\alpha_{23}/\alpha_{13}} \tag{21}$$

This shows that when the Translog function is separable an effort index is not just the simple product of the two input variables but, on the contrary, depends on the Translog parameters.

The above examples emphasise the fact that when a production function is separable, in all cases the separability function will not only be the product of the separable variables, but will depend on the structure of the production function, be it Cobb-Douglas, CES or a Translog function.

Discussion

This paper examines separability of input variables in connection with the concept of fishing effort. Many studies of production in fisheries unconditionally assume that some or all inputs in the production function can be aggregated into one effort index, often by multiplication of some of the input variables. When such an effort index is constructed it is implicitly assumed that the input variables that form the effort index are separable from the remaining inputs; for example, from the fish stock.

It has been shown in this paper, however, that this is often an erroneous assumption for two reasons. First, the input variables may not be separable. It may not be possible to construct an aggregate effort index which is independent of the remaining inputs. Mathematically, whether it is, is reliant on the specification of the production function but this itself is dependent on how accurately the specification represents the real world. For the Cobb-Douglas and CES production functions all input variables are separable by definition, but for the Translog production function, existence of separability must, on the contrary, be indicated by testing for it. Secondly, even if some input variables are separable from the remainder, the internal structure of an effort index will depend on the specification of the production function employed, and will not be a simple product.

As such, care should always be taken when discussing separability of input variables, and, further, when discussing a fishing effort index composed of several variables. Separability is not an unequivocal concept but, on the contrary, depends on the assumed data generation process and the form of the production function chosen.
These findings raise a number of questions about the content and specification of production functions chosen for empirical estimation if they are accurately to represent the real world.

The first is of the role of the stock. Is it truly a factor of production? Leaving aside the impact of market failure, the stock level is not in the control of fishing enterprises (or the industry) at all in the short run, and only partly in the long run. In the short-run case the stock is more akin to the level of technology and separable on the grounds of metaphysics rather than empirical evidence.

In the long run case, however, the stock introduces an unwelcome element of partial autocorrellation. Environmental factors play a large part in the variability of stock size and these can only be aggravated or ameliorated by the activities of a fleet, so the present stock is only partly a function of previous catches and its intrinsic growth function.

Leaving aside the econometric havoc involved, separability depends, in such circumstances, on the true relationship between the shadow price of the stock and the other inputs. It has been mooted that the stock has a shadow price which will be positive when the level of the stock falls below that necessary to sustain the maximum rent from the fishery in the future and zero at other levels (Coppola 1995, Rodgers 1995). The second question is therefore whether the shadow price of the stock affects the mix of other inputs used. If it does, then separability does not hold and reducing the non-stock inputs to an effort index is not a valid action whatever the specification of the production function or effort index.

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