H_∞-based control system and its digital implementation for the integrated tilt with active lateral secondary suspensions in high speed trains

Ronghui Zhou[1], Argyrios Zolotas[2], Roger Goodall[3]

1. Real-Time Controls and Instrumentation Lab., Global Research Center, GE, Shanghai, China, 201203,
E-mail: ronghui.zhou@ge.com

2. School of Engineering and Informatics, University of Sussex, Sussex House, Brighton, BN1 9QT, UK,
E-mail: a.zolotas@sussex.ac.uk

3. Control systems group, Department of Electronic and Electrical Engineering, Loughborough University,
Loughborough, LE11 3TU, UK,
E-mail: r.m.goodall@lboro.ac.uk

Abstract: In this paper, H_∞-based control system and its digital implementation for the integrated tilt with active lateral secondary suspensions in high speed railway vehicles are discussed, in which mixed-sensitivity H_∞ control is designed for the tilting suspension, while skyhook damping control is employed for the active lateral secondary suspensions. Compared with classical decentralized control, the proposed control system can well attenuate the strong coupling between the roll and lateral dynamic modes of the vehicle body. Particular emphasis is also on the digital implementation of the reduced order H_∞ tilt controller in an embedded control unit. Proposed digital controllers are validated via a FPGA-based Hardware-In-the-Loop system.

Key Words: H_∞ Control, Active Suspensions, Railway System Control, Integrated Tilt With Active Lateral Suspensions, HIL Digital Implementation

1 Introduction

High speed trains which are able to operate at $200km/h$ and faster are nowadays widely spread in the world, i.e. France, Germany and United Kingdom in Europe, Japan and China in Asia. The recent world rail speed record is $578km/h$ achieved by French V150 TGV [1]. However, in order to develop the TGV, new rail ifrastructures are needed, i.e. new rail tracks. One type of high speed train is tilting train, which can operate at increased speeds without the need to upgrade the rail infrastructure. The idea is to tilt the vehicle body inwards on the curved sections of the track to compensate the large lateral acceleration perceived by passengers at higher speeds. Early passive tilting trains completely relied upon the natural pendulum motion laws which caused safety issues, i.e. vehicle body over turning [2], and a tilt mechanism (tilting bolster in most cases) in conjunction with an actuator to tilt the vehicle body was introduced, which has become a standard technology used in trains worldwide.

The active Anti-Roll Bar (ARB) is one of the tilting system mechanical configuration, as shown in Fig. 1. It is configured by a transversely-mounted torsion tube on the bogie with vertical links to the vehicle body, except that one of the links is replaced by a tilt actuator, and thereby applies tilt via the torsion tube.

Control systems for the tilt actuator can be designed based on either the local vehicle body measurements or the bogie-mounted sensors from the front vehicle. The control system based on the local measurements (named as "Nulling Control") however cannot well address the strong interaction between vehicle roll and lateral dynamic modes, the industrial sector nowadays adopts a control structure called "precedence control", in which, the bogie-mounted lateral accelerometer from the front vehicle is used to provide "precedence" information which minimises the lateral and roll dy-
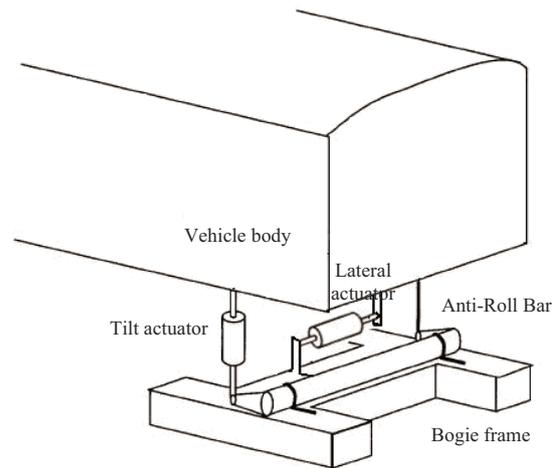

Fig. 1: Tilting train with tilt and lateral actuator

namic interaction problem. Appropriate low pass filters are employed to attenuate the high frequency signal caused by the track irregularity response of the bogie. The delay introduced by the filter is compensated by the carefully designed precedence control strategy [3]. Although precedence control is an accepted commercial solution for the tilting train, research on local tilt control still has practical benefits which make the system simpler and more straightforward in terms of detecting sensor failure.

Tilt control based on local vehicle body signals with H_∞ and Fuzzy logic controllers were studied in [4][5], but due to the dynamic interaction between roll and lateral modes of the railway vehicle body, there is further research potential of improving the overall transient performance. In addition, the high speeds associated with tilting trains result in worse ride quality on straight track. In the work of [6], a lateral actuator

was proposed to be installed between the vehicle body and bogie in parallel with (or to replace) the original secondary damper, as shown in Fig. 1. The control system design for this dual-actuator system (tilt and lateral) was carried out in both decentralised and centralised way, in which, the Classical Decentralised (CD) control, and LQG centralised control were investigated. Genetic Algorithm was employed to optimize the controller parameters due to the multiple design requirements. In this paper, H_∞-based Decentralised (HD) control is investigated, mixed-sensitivity H_∞ control is designed for the tilting suspension, while skyhook damping control [7] is employed for the active lateral secondary suspension. Compared with CD control, the proposed control system can further attenuate the strong coupling between the roll and lateral dynamic modes of the vehicle body.

The proposed HD control strategy is validated via a FPGA-based Hardware-In-the-Loop (HIL) system. Controller reduction technique is also employed for the H_∞ tilt control before its digital implementation. The remainder of this paper is organized as follows: Part II presents railway vehicle model and controller performance assessment approaches, Part III refers to the basics of CD control, while Part IV gives the details of the HD control system design. It is followed by the HIL implementation and validation. Conclusions are discussed in the last part.

2 Railway Vehicle Model And Controller Performance Assessment

2.1 Railway Vehicle Model

The simplified mechanical configuration of the integrated tilt and lateral system is shown in Fig. 1.
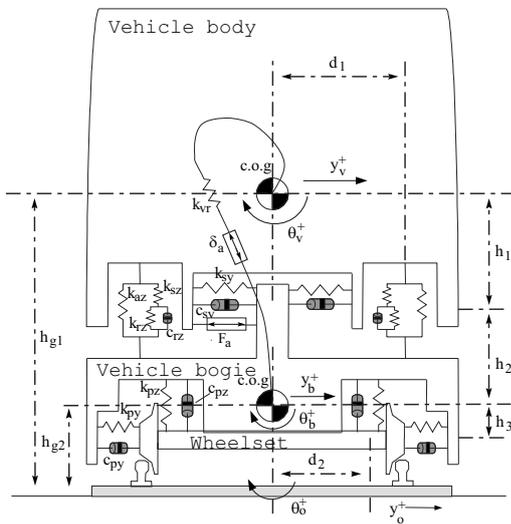

Fig. 2: Integrated ARB with lateral actuator

The end-view model consists of a four Degree-Of-Freedom (DOF) dynamic system, illustrated in Fig. 2. The lateral and roll degrees of freedom for both the body and bogie systems are included. A rotational displacement actuator shown by δ_a is included in series with the roll stiffness. Moreover, a lateral actuator shown by F_a is installed in parallel with the original lateral damper between the bogie and the body. Further details about the model can be found in [6]. The equations of motion are:

Body lateral dynamics:

$$
\begin{aligned}
m_v \ddot{y}_v =\ & -2k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) \\
& -2c_{sy}(\dot{y}_v - h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b) \\
& -\frac{m_v v^2}{R} + m_v g\theta_0 - h_{g1}m_v\ddot{\theta}_0 + F_a \quad (1)
\end{aligned}
$$

Body roll dynamics:

$$
\begin{aligned}
i_{vr}\ddot{\theta}_v =\ & 2h_1k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) + 2h_1c_{sy}(\dot{y}_v \\
& -h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b) - k_{vr}(\theta_v - \theta_b - \delta_a) \\
& +m_v g(y_v - y_b) - 2d_1^2 k_{az}(\theta_v - \theta_b) \\
& -2d_1^2 k_{sz}(\theta_v - \theta_r) - i_{vr}\ddot{\theta}_0 - F_a h_1 \quad (2)
\end{aligned}
$$

Bogie lateral dynamics:

$$
\begin{aligned}
m_v \ddot{y}_b =\ & 2k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) + 2c_{sy}(\dot{y}_v - h_1\dot{\theta}_v \\
& -\dot{y}_b - h_2\dot{\theta}_b) + 2k_{py}(y_b - h_3\theta_b - y_0) - 2c_{py}(\dot{y}_b \\
& -h_3\dot{\theta}_b \dot{y}_0) - \frac{m_v v^2}{R} + m_v g\theta_0 - h_{g1}m_v\ddot{\theta}_0 - F_a \quad (3)
\end{aligned}
$$

Bogie roll dynamics:

$$
\begin{aligned}
i_{br}\ddot{\theta}_b =\ & 2h_2k_{sy}(y_v - h_1\theta_v - y_b - h_2\theta_b) + 2h_2c_{sy}(\dot{y}_v \\
& -h_1\dot{\theta}_v - \dot{y}_b - h_2\dot{\theta}_b) - 2h_3(k_{py}(y_b - h_3\theta_b - y_0) \\
& +c_{py}(\dot{y}_b - h_3\dot{\theta}_b \dot{y}_0)) + k_{vr}(\theta_v - \theta_b - \delta_a) \\
& +2d_1^2(k_{az}(\theta_v - \theta_b) + k_{sz}(\theta_v - \theta_r)) \\
& -2d_2^2(k_{pz}\theta_b + c_p z\dot{\theta}_b) - i_{br}\ddot{\theta}_0 - F_a h_2 \quad (4)
\end{aligned}
$$

for the additional air-spring state:

$$
\dot{\theta}_r = -\frac{k_{sz} + k_{rz}}{c_{rz}}\theta_r + \frac{k_{sz}}{c_{rz}}\theta_v + \frac{k_{rz}}{c_{rz}}\theta_b + \dot{\theta}_b \quad (5)
$$

The vehicle model and control system are tested with specific track inputs including both deterministic (low frequency signals) and stochastic (high frequency signals) features. The deterministic track input was a curved track with a radius of 1000m and a maximum track cant angle (θ_{0max}) of 6^0, with a transition (150(m)) at the start and end of the steady curve. The stochastic track inputs represent the irregularities in the track alignment on both straight track and curves, and these were characterised by an approximate spatial spectrum equal to $(2\pi)^2 \Omega_l v^2 / f_t (m^2/(cycle/m))$ with a lateral track roughness (Ω_l) of $0.33 \times 10^{-8}(m)$ [4].

2.2 Controller Performance Assessment

Two main design criterion for the dual-actuator system controller are summarized below, which needs to meet both tilt performance and lateral suspension requirements [8].

(i) Provide a fast response on curved track (deterministic criterion) which is divided into two aspects:

• P_{ct} value for the curve transitions: this is a criterion on quasi-static lateral acceleration and lateral jerk perceived by the passengers and was suggested by a British Rail research study, see [9]. It indicates the percentage of passengers who will feel uncomfortable as a result of the transition onto the curve, calculated via a non-linear formula.

• Investigation of the transitional dynamic suspension effects based upon the "ideal tilting" approach [10], a technique which essentially quantifies how closely a particular control solution fits to the ideal response.

(ii) Maintain good ride quality in response to track irregularities on straight track (stochastic criterion). The root Mean Square (R.M.S.) value of the body lateral acceleration on straight track in response to the track irregularities is traditionally utilized to assess the straight track performance. More information about tilting train control assessment can be found in [10]. Associated with ride quality improvement is the constraint on lateral suspension deflection, which should not exceed the maximum available before bump stops are reached, i.e. ± 60 (mm) is used in this study.

3 Classical Decentralized Control

Details of the CD control can be found in [6], refreshing here to provide a comparison object for the HD control. Fig. 3 shows the overall system configuration.
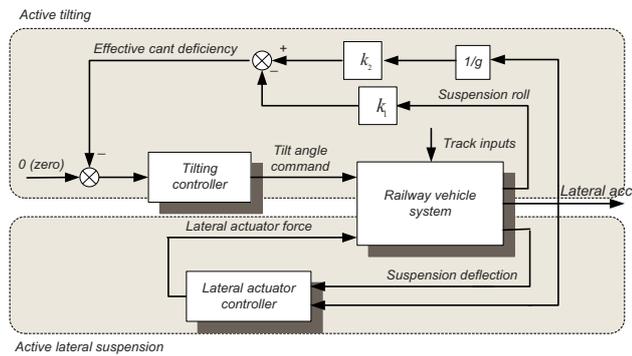

Fig. 3: Classical decentralized control system configuration
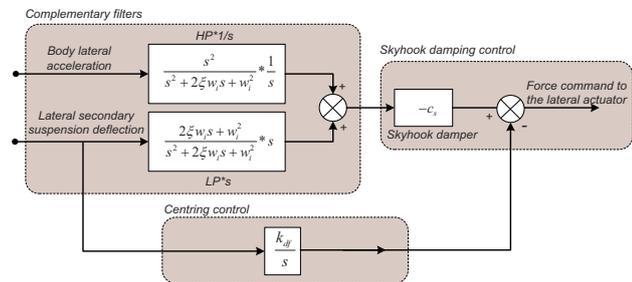

Fig. 4: Skyhook damping lateral actuator control with body lateral centering control

Sequential design process is adopted because the lateral actuator control loop is a high bandwidth strategy (to attenuate high frequency lateral irregularities) that is intended to respond faster than the tilting action. The complementary filter skyhook damping control [7] with centring loop is used for the lateral actuator control driven by the measured body lateral acceleration and lateral secondary suspension deflection (shown in Fig. 4). Effective cant deficiency (e.c.d.) [6] is used to drive the tilt actuator with approximate PID control.

4 H_∞-based Decentralised Control

H_∞ based Decentralised (HD) control is introduced in this section. It was found that the H_∞ tilting control combined with the intuitive skyhook damping lateral actuator control can meet all the design requirements, which simplifies the controller design. This is because the H_∞ tilting

provides a faster response compared to PID tilting when the train starts to negotiate the curve transition, hence reducing the interaction between the tilting response and lateral suspension. Centring control loop is still used.

4.1 Intuitive Skyhook Damping Lateral Actuator Control

The configuration of intuitive skyhook damping control with centering loop for lateral actuators is illustrated in Fig. 5. The actuation force is proportional to the absolute body velocity. A High Pass filter (HP) is used to eliminate the integrator drifting due to zero-offset and also to reduce the low frequency velocity signal, which in turn reduces the suspension deflection for the deterministic inputs.
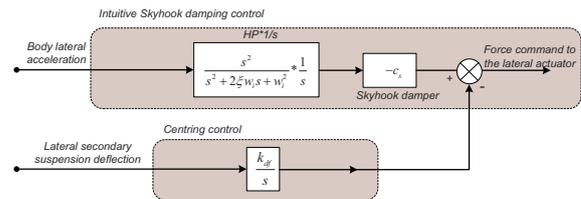

Fig. 5: Intuitive skyhook damping control with centring loop

The parameters for the lateral actuator control in this design are listed as follows:

c_s: $59000 N \cdot s/m$; w_i: $0.7 rad/s$; k_{df}: $590000 N/m$;

4.2 Mixed-Sensitivity H_∞ Tilting Control

Research on H_∞ control started in early 1980s with the objective to compensate the weakness of LQG control to deal with good robustness properties [11]. The design process involves the minimization of the H_∞ norm of the transfer function from exogenous signals (such as disturbances and input commands) to the signals which are to be minimized to meet the control objectives. Mixed-sensitivity H_∞ control, signal-based H_∞ control and H_∞ loop-shaping are three basic types of H_∞ control [12].

Mixed-sensitivity is studied in this paper. It addresses the transfer function shaping problems in which the sensitivity function $S = (I + GK)^{-1}$ is shaped along with one or more other closed-loop transfer functions such as $R = KS$ or the complementary sensitivity function $T = I - S$. The objective of Mixed-sensitivity design is to minimize the H_∞ norm of the closed-loop transfer function:

$$\left\| \begin{bmatrix} W_1 S \\ W_2 R \\ W_3 T \end{bmatrix} \right\|_\infty$$

The norm is usually required to be below a level γ, where W_1, W_2 and W_3 are weighting filters for sensitivity transfer function (S), complementary sensitivity transfer function (T) and control inputs sensitivity (R) respectively. The returned values of S, R and T should satisfy the following loop shaping inequalities:

$$\overline{\sigma}(S(jw)) \le \gamma \underline{\sigma}(W_1^{-1}(jw))$$
$$\overline{\sigma}(R(jw)) \le \gamma \underline{\sigma}(W_2^{-1}(jw)) \quad\quad (6)$$
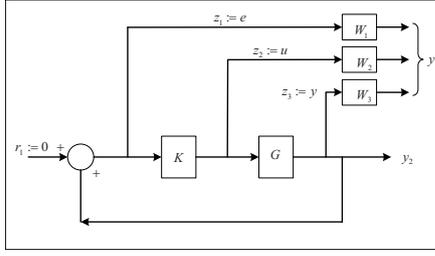$$\overline{\sigma}(T(jw)) \le \gamma \underline{\sigma}(W_3^{-1}(jw))$$

Fig. 6: Mixed-Sensitivity control formulation

Fig. 6 illustrates the general control problem configuration for tilting control, r represents a set-point zero reference command, and the regulated outputs are z_1 (the weighted e.c.d. error signal), z_2 (the weighted control signal u) and z_3 (the weighted e.c.d. output signal). Note that regulating z_1 to zero will provide the required 60% tilt compensation, the regulation of z_2 will satisfy control limitation and noise attenuation at high frequencies, while regulation of z_3 is for system robustness and modelling uncertainty. The usual difficulty in H_∞ control design is choosing the weighting filters, normally based on rule of thumb choice or designer's experience. Examples of weighting filter choice can be seen in [12]. The filters employed in this paper: W_1 was chosen to be a low-pass filter with a very low cut-off frequency essentially to enforce integral action on z_1. In contrast, W_2 and W_3 were chosen as high-pass filters with pole and zero cut-off frequencies. The weighting filters for the tilting control are chosen as:

$$W_1 = 1100\frac{s/30 + 1}{s/0.001 + 1}$$

$$W_2 = 0.0032\frac{s/0.1 + 1}{s/30 + 1} \qquad (7)$$

$$W_3 = 0.00032\frac{s/0.008 + 1}{s/300 + 1}$$

The design can be done in a straightforward way, i.e. using Matlab's Robust Control Toolbox capabilities[13]. In fact, using function mixsyn(G,W_1,W_2,W_3) to shape sigma plots of S and T to conform to GAM/W_1 and GAM*G/W_2 respectively, as shown in Fig. 7. G is the plant transfer function.

4.3 Simulation Results

The proposed HD control system is tested with the 4 DOF vehicle model and track data presented in section 2.1. The assessment values are presented in Table 1. The Nichols chart for e.c.d. is illustrated in Fig. 8, time domain simulation results are illustrated in Fig. 9 and Fig. 10. The simulation results of HD control show the improvement of the performance and system robustness compared to the CD control. The P_{ct} value for seated passengers is reduced to 14%, which is very close to the value for Precedence Tilt (PT) control (13.5%) (Refering to [4] for the PT and Nulling Tilt (NT) control assessment). The R.M.S. value of the lateral acceleration on straight track is less than 3.778%, which illustrates the good ride quality can be guaranteed on the straight track. The Gain Margin for the tilting control system (with closed lateral actuator control loop) now is $5.6dB$ and Phase Margin is $58.9deg$.
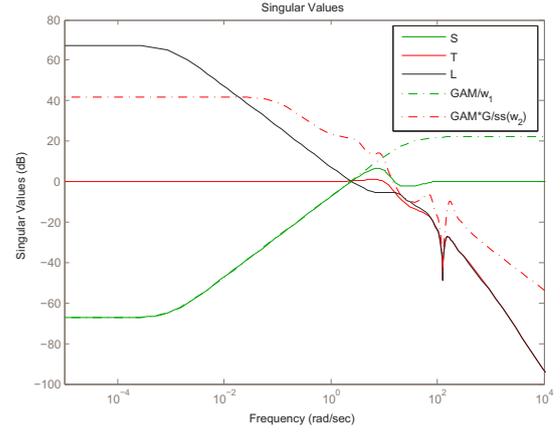

Fig. 7: S and T to conform to GAM/W_1 and $GAM*G/W_2$, respectively ($GAM := \gamma$)
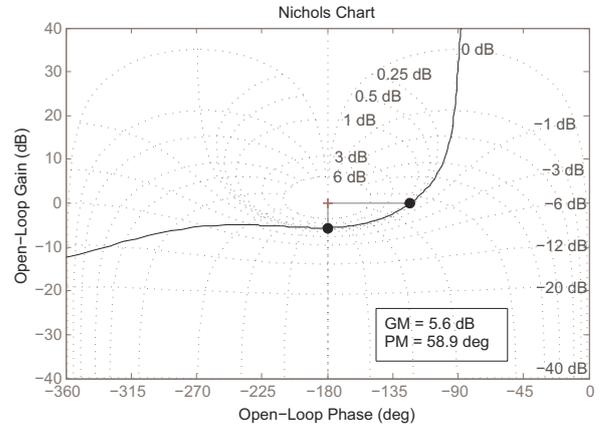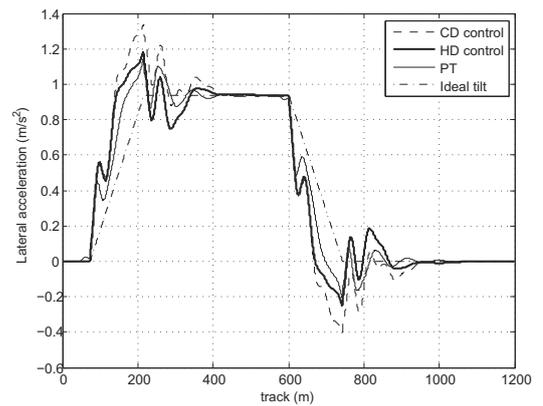

Fig. 8: Nichols plot for e.c.d

Fig. 9: Measured body lateral acceleration

5 Digital Implementation of H_∞-based Decentralised Control

The proposed HD control is further investigated with the consideration of the controller practical implementation. A HIL simulation system is setup.

Table 1: Control system assessment for HD control @ 58(m/s)

| Deterministic(CURVED TRACK)					
		HD	CD [6]	NT [4]	PT [4]
Lateral acceleration.	-Steady-state(%g)	9.530	9.530	n/a	9.530
	-R.M.S. deviation error(%g)	1.800	4.576	5.555	1.54
	-Peak value(%g)	12.144	13.714	19.510	12.18
Roll gyroscope	- R.M.S. deviation(rad/s)	0.020	0.021	0.032	0.018
	-Peak value(rad/s)	0.111	0.104	0.086	0.104
	-Peak jerk level(%g/s)	7.349	7.687	10.286	6.80
P_{ct}(P-factor)	-standing(% of pasengers)	50.548	53.846	71.411	47.62
	-seated(% of pasengers)	14.214	15.674	22.640	13.455
Stochastic(STRAIGHT TRACK)					
passenger comfort	- R.M.S. passive(%g)	3.778	3.778	3.778	3.778
	- R.M.S. active(%g)	3.569	3.568	3.998	3.31
	- degradation (%g)	-5.553	-5.558	5.802	-12.12
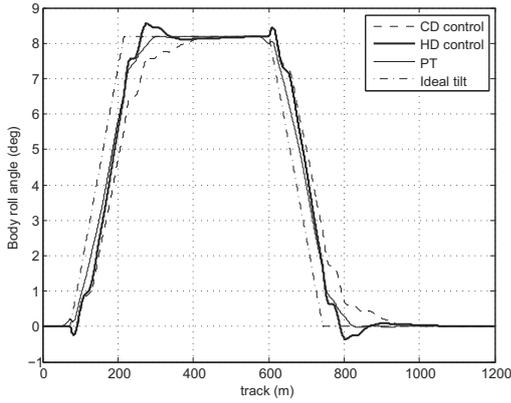
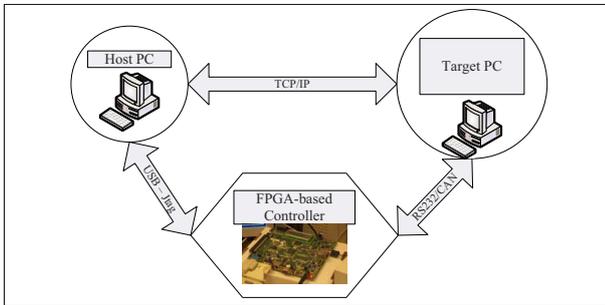
Fig. 10: Actual body roll angle

Fig. 11: HIL system configuration

5.1 HIL System Configuration

The MATLAB/xPC-Target [14] is employed to provide the real-time environment for the 4 DOF tilting railway vehicle model. As shown in Fig. 11, the railway vehicle model is developed in MATLAB/SIMULINK in the host PC. It is downloaded into the Target PC via the TCP/IP link. High speed RS232 serial communication (Baud rate is configured as $115200bit/s$) is adopted for the data transmission between xPC-Target and FPGA-based controller. Digital controllers designed in MATLAB m code are compiled to C code via Embedded MATLAB, then downloaded into the Microblaze soft processor in FPGA.

5.2 Digital Controller Design

The controllers designed in s domain are converted to z domain using Tustin transformation. The equations below

are the intuitive skyhook damping control with centring loop for the lateral actuator:

$$HP * 1/s = \frac{-726.4z^2 + 726.4}{z^2 - 1.974z + 0.9742};$$

$$Centring\ loop = \frac{-7247z^2 + 7247}{z^2 - 2z + 0.9998} \qquad (8)$$

H_∞ controller design usually results to rather higher order controller structures (relative to the original size of the design model). Model reduction based on Schur method is applied to reduce the controller order down to five order for further efficient embedded implementation. The frequency responses of the original controller and reduced order controllers (9th order controller is also studied here) are illustrated in Fig. 12. The equation for H_∞ tilt controller (5th order):

$$T_t = \frac{0.1626z^4 - 0.6422z^3 + 0.952z^2 - 0.6277z + 0.1553}{z^5 - 4.771z^4 + 9.124z^3 - 8.745z^2 + 4.201z - 0.8099}$$
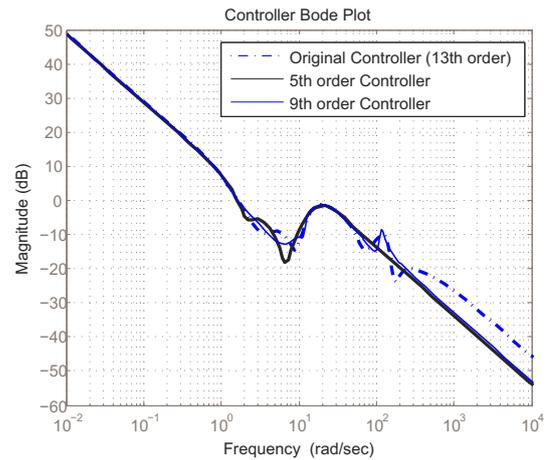

Fig. 12: Controller frequency response

5.3 HIL Simulation Result

Fig. 13 shows the HIL validation result for the proposed digital HD controller for integrated tilt and active lateral secondary suspension in high speed railway vehicle (Measured body lateral acceleration) on curved track. The performance of the digital controllers with reduced order H_∞ controller
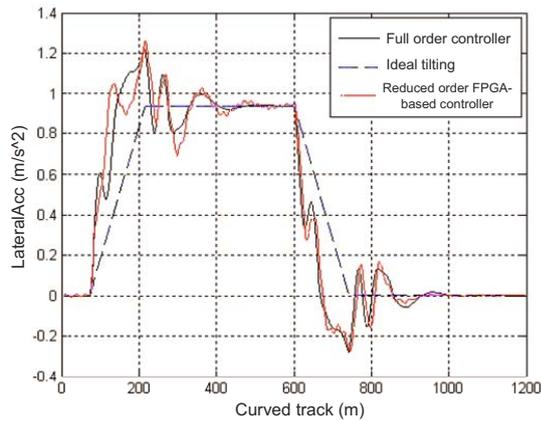
Fig. 13: Measured body lateral acceleration

(5th order, in FPGA) is similar to the full order continuous controller (in simulation). Further work will focus on the upgrade of the HIL system and investigation of the digital controller performance in the straight track case (high frequency excitation).

6 Summary

In this paper, the integrated mixed-sensitivity H_∞ tilt control with skyhook damping lateral actuator control in high speed railway vehicle is firstly discussed. It aims to further overcome the control loop interactions in the decentralized control and improve the performance of using the local integrated suspension control. The simulation results show that the proposed HD control system can meet both the tilt and lateral active suspension design requirements. The performance of the HD control is closer to the precedence control compared to the CD control.

The HIL digital implementation of the proposed HD control is also investigated. The digital controllers are discreted based on Tustin transformation and implemented into a FPGA-based electronic control unit (with reduced order H_∞ tilting control), which are validated in a xPC-Target-based HIL system.

References

[1] Alstom. http://www.transport.alstom.com, 2007.

[2] R. Persson, *Tilting trains Technology, benefits and motion sickness*, Licentiate Thesis, Railway Technology, Stockholm, Sweden 2008.

[3] R. M. Goodall, "Tilting trains and beyond - the future for active railwaysuspensions: Part 1 Improving passenger comfort", *Computing and Control Engineering Journal*, vol.10, no.4, 1999, pp 153-160.

[4] A. C. Zolotas, R. M. Goodall, "Modelling and control of railway vehiclesuspensions", *Lecture notes in control and information sciences*, vol.367, 2007, pp 373-412.

[5] H. Zamzuri, A. C. Zolotas, and R. M. Goodall, "Tilt control design for high-speed trains: a study on multi-objective tuning approaches", *Vehicle System Dynamics*, vol.46, no.1, 2008, pp 535-547.

[6] R. Zhou, A. C. Zolotas, R. M. Goodall, "LQG control for the integration of tilt and active lateral secondary suspension in high speed railway vehicles", *proceedings of the 8th IEEE international conference on control automation (ICCA'10)*, Xiamen, China, 9-11 June 2010.

[7] H. Li, R. M. Goodall, "Linear and non-linear skyhook damping control laws for active railway suspensions", *Control Engineering Practice*, vol7, no7, 1999, pp 843-850.

[8] R. M. Goodall, T.X. Mei, *Active suspensions*, Chapter 11 in Handbook of Railway Vehicle Dynamics, (Taylor and Francis). 2006.

[9] CEN: (2007), *Railway applications-Ride comfort for passengers Measurements and evaluation*, enquiry version pr EN12299.CEN. Brussels.

[10] R. M. Goodall, A. C. Zolotas, and J. Evans, "Assessment of the Performance of Tilt System Controllers", *The Railway Conference a Railtex 2000*, NEC Birmingham, UK, November 2000.

[11] G. Zames. "Feedback and optimal sensitivity: Model reference transformations, multiplicative seminorms and approximate inverse", *IEEE Transactions on Automatic Control*, vol26, no2, 1981,pp 301-320.

[12] S. Skogestad, I. Postlethwaite, *Multivariable feedback control: Analysis and design*, Wiley, 2000, Reprinted Version.

[13] The MathWorks. *http://www.mathworks.co.uk/products/robust*, 2013.

[14] The MathWorks. *http://www.mathworks.com/products/xpc-target*, 2013.