Hybrid Bond Graphs for Contact, using Controlled Junctions and Dynamic Causality

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Abstract

Controlled junctions with dynamic causality can be used to generate mixed-Boolean mathematical models. Several variations on the Switched or Hybrid Bond Graph have already been proposed, with none reaching common usage. The motivation for suggesting this approach was to develop a general method for adoption by practicing engineers, which is intuitive, adheres to the principles of idealized physical modelling and facilitates both structural analysis and efficient simulation. This paper revisits the classical example of a bouncing ball in order to discuss the advantages and disadvantages of such an approach with reference to the body of literature on hybrid models and nonsmooth dynamics. A switching model (as opposed to an impulse model) is generated which is both stiff and contains kinematic constraints, making it problematic to simulate. However, the method facilitates model simplification and the derivation of a coefficient of restitution, allowing Newton's collision law to be applied. The resulting model simulates efficiently and well, without the need for parasitic elements or state reinitialization algorithms.

1. INTRODUCTION

Hybrid Bond Graphs – those which contain both continuous and discontinuous behaviour - are an active area of research, but no single method has reached common usage. Instead, several variants of hybrid bond graph exist which have been developed for different applications and are generally suitable for either analysis or simulation. There are ongoing discussions about the treatment of impacts and the use of dynamic vs. static causality. In parallel with developments in the bond graph community, there have been developments in the field of hybrid and nonsmooth dynamics in general.

Margetts et al [1] have proposed a method for constructing hybrid bond graphs which is suitable for both analysis and simulation. This paper discusses some of the issues encountered in hybrid system models, and investigates a bouncing ball in order to demonstrate how this method can be applied to give an accurate and efficient simulation.

1.1. Background to Hybrid Bond Graphs

Discontinuities arise where highly nonlinear behavior is abstracted to a piecewise continuous model. They are also used to describe the case where a model has variable topology, e.g. contact. The resulting models can be classified as *switched* and *impulse*. For bond graphs in particular, there is a clear distinction between causally static and causally dynamic methods too.

1.2. Switched bond graphs

The vast majority of hybrid bond graph methods yield *switched* models [2] i.e. they describe continuous equations and some form of binary switching device which selects the active continuous equations at a given time or operating condition (often described as *mode* of operation). The terms 'hybrid bond graph' and 'switched bond graph' are used almost interchangeably in the literature.

1.2.1. Causally Static Methods

The simplest forms of switching devices in Bond Graphs do not affect the overall causal assignment, and are referred to as causally static (i.e. the causality assignment does not change with commutation). The Modulated Resistance element can be used to describe an ideal diode or hydraulic valve using a Boolean modulation signal which imposes zero effort or flow when OFF [3, 4], or a Boolean-modulated Transformer element connected to a Resistance [5] behaves in a similar way. Controlled storage elements [6] likewise commutate between being an ideal null source and a regular element. There are also a number of ways to make causally dynamic hybrid bond graphs static, by adding *causality resistance* to 'absorb' changes in causal assignment [7], or using an alternative causality assignment procedure [8].

These methods tend to be used because they are relatively easy to implement in computer simulation packages. However, they do present some problems. They can result in overly complex stiff models, especially where parasitic causality resistance has been added [9], which yields additional high frequency dynamics and can slow a simulation. They can also restrict insight into the model, and it has been suggested that dynamic causality is preferable as it reveals something of the model's properties [10].

1.2.2. Causally Dynamic Methods

Several switching methods exist which effectively disconnect regions of the bond graph with commutation. These include the switched bond [11], the switching or Boolean-modulated transformer element [12, 13] and various junctions (the time-dependent junction [3], controlled junction [14] and switched power junction [15]). Of these methods, the controlled junction is arguably the most widely used with a large supporting body of work on semantics and simulation [16-21]. The controlled junction is a regular bond graph junction when ON and a source of zero flow or effort on each bond when OFF. It was selected for the author's work on hybrid bond graphs [1] because it was physically intuitive and can be used to yield mixed-Boolean state equations describing all possible modes of operation.

Another causally dynamic method which has been used frequently is the switched source (sometimes called switched element) [7, 9, 22, 23]. The switched source is a null effort source in one state, and a null flow source in the other. This effectively yields a complementarity condition, such as that used to describe ideal diodes (equation 1).

$$0 \le i(t) \perp v(t) \ge 0 \quad (1)$$

The hybrid bond graph with switched sources yields equations which are discontinuous on the input. These types of mathematical model have, themselves, been used in electrical applications [24, 25]. Although hybrid bond graphs with switched sources have been used to model a variety of applications including mechanical ones [26], they can be less intuitive to use and simulate.

1.2.3. Other Methods

A hybrid model can be thought of as a collection of continuous models with some kind of mechanism to select the appropriate one at a given time or event. A hybrid model can therefore be constructed from continuous bond graph models describing individual modes of operation and a petri-net [27]. A Quantized Bond Graph has also been proposed, which is inherently discrete and solved using DEVS [28]. These methods make sense from a computational point of view, but lose some of the graphical advantages of the hybrid bond graph.

1.3. Impulse bond graphs

There is another subset of hybrid model known as the *impulse* model [2] where the state changes impulsively and there is an impulse loss on commutation. These have been tackled explicitly by the Impulse Bond Graph [29]. The treatment of collisions and the issue of reinitializing state variables after such an impact motivated the development of HyBrSim [30, 31] and subsequent work on reinitializing state variables in SIMULINK models based on bond graphs [32]. This paper aims to demonstrate that explicit representations of impulse loss and reinitialization of states is not necessary.

1.4. Equation Generation from the Causally Dynamic Hybrid Bond Graph

Earlier work described the construction and structural analysis of a causally dynamic hybrid bond graph in detail [1]. This work had the following salient points:

- The controlled junction as described by Mosterman and Biswas was adopted, assuming a Boolean control signal.
- The controlled junction is causally dynamic i.e. at least one bond changes its causal assignment with commutation.
- A notation of dashed causal strokes was proposed to aid in visualising a reference mode (most storage elements in integral causality) and deviations from it. This allows the user to identify regions of dynamic causality and generate equations describing localised behaviour.
- The controlled junction can be represented in the Junction Structure Matrix by a Boolean variable. The JSM therefore contains Boolean expressions, showing that inputs and outputs are connected in some modes of operation but not others.
- Where storage elements are in dynamic causality, pseudo-state variables can be used to describe the derivative causality case.
- The Junction Structure Matrix can be used to derive state equations in much the same way as for a regular bond graph. The resulting state equations are also mixed-Boolean and can be implicit where storage elements are in dynamic causality.

The end result is a state equation which is discontinuous in the coefficients of both states and inputs. i.e., if the model is linear time-invariant in all modes, there are discontinuities manifesting as Boolean terms in the **A** and **B** matrices (eqn. 2). The Boolean terms are denoted κ in this paper, to differentiate them from the response on contact λ used later.

$$\mathbf{E}(\kappa)\dot{\boldsymbol{x}} = \mathbf{A}(\kappa)\boldsymbol{x} + \mathbf{B}(\kappa)\boldsymbol{u} \qquad (2)$$

1.5. Simulation of Hybrid Models

The bulk of work on simulating hybrid bond graphs has concentrated on constraining causality using *causality resistance,* which (in the authors' experience) is comparable to what a practicing engineer will often do using dynamics simulation packages: adding parasitic resistance. This approach is open to abuse, as well as yielding stiff models which can simulate inefficiently. In order to improve simulation, Newton's Collision Law is often used on systems such as the bouncing ball considered here (equation 3). The hybrid bond graph developed here can be used to derive a value for the coefficient of restitution, instead of relying on estimated or experimental values

$$\Delta v^{+} = -\varepsilon\left(\Delta v^{-}\right) \qquad (3)$$

There is a body of literature on the simulation of Hybrid and Nonsmooth Dynamical Systems which has developed in parallel with the Hybrid Bond Graph. A number of issues merit consideration, as outlined by Acary and Brogliato [33].

- Switching must always occur at the end of a time-step, in order to be captured. This usually motivates an *event-driven* method. However, this can be impractical where there is a large number of switching instants or it is not known where they occur.
- *Chattering* may occur where a sliding mode cannot be reached due to numerical approximation.
- A procedure for accurately finding the location of events may be required, along with some method for reinitialising states after the event.
- Where there are a number of events, there may be a finite *accumulation point* past which the event-driven method cannot progress.
- There may be an *impulsive term* on commutation giving a Dirac or Steltjes measure. For example, the differential measure of velocity which manifests on impact between bodies.

There are numerous ways to represent nonsmooth dynamic systems. The linear complementarity problem, comprising a continuous equation (such as a state equation) and complementarity condition (eqn. 4) is perhaps the most widely used [34]. These contain an external signal λ which can be thought of as a Lagrange multiplier and commutates between zero and a value which must be calculated. This model can be transferred to a single inclusion or a variational inequality, which have unique continuous solutions.

$$\begin{cases} w = M\lambda + q \\ 0 \le \lambda \perp w \ge 0 \end{cases} \qquad (4)$$

Acary and Brogliato [33] suggest the use of discrete-time Moreau's second-order sweeping process for solving problems such as the bouncing ball discussed here. There are no detection times, and hence no accumulation point. Nonsmooth measures are treated rigorously with no 'jump' in acceleration, and hence no impulse losses or need to reinitialize states.

A mathematical model derived from any variant of hybrid bond graph could be simulated in these forms, but this paper aims to show that the mixed-Boolean model derived by Margetts et al [1] lends itself to this type of analysis by facilitating derivation of the coefficients.

2. THE BOUNCING BALL MODEL

2.1. The Hybrid Bond Graph

The classic bouncing ball problem is described in Figure 1, with the associated bond graph in Figure 2.

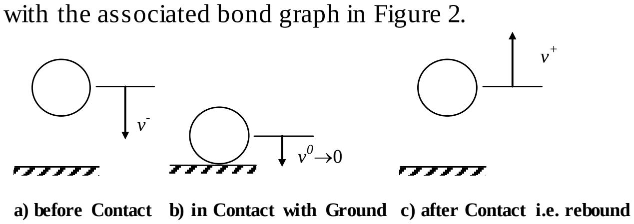


Figure 1: Motion of a Bouncing Ball

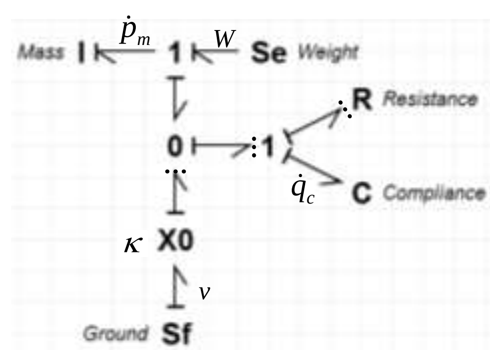


Figure 2: Hybrid Bond Graph of a Bouncing Ball

The inputs and outputs of the model are related by the junction structure equation (5). In addition to the mixed-Boolean Junction Structure Matrix, note that an additional diagonal matrix (de)activates flow or effort outputs from the resistance elements. The state equations can be derived from the JSM (6).

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \kappa & 0 \\ 0 & 0 & 0 & \bar{\kappa} \end{bmatrix} \begin{bmatrix} \dot{p}_m \\ \dot{q}_c \\ f_r \\ e_r \end{bmatrix} = \begin{bmatrix} 0 & -\kappa & -\kappa & 0 & 1 & 0 \\ \kappa & 0 & 0 & -\bar{\kappa} & 0 & \kappa \\ \kappa & 0 & 0 & 0 & 0 & \kappa \\ 0 & -\bar{\kappa} & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} f_m \\ e_c \\ e_r \\ f_r \\ W \\ v \end{bmatrix} \quad (5)$$

$$\begin{bmatrix} \dot{p}_m \\ \dot{q}_c \end{bmatrix} = \begin{bmatrix} -\kappa/mR & -\kappa/C \\ \kappa/m & -R\bar{\kappa}/C \end{bmatrix} \begin{bmatrix} p_m \\ q_c \end{bmatrix} + \begin{bmatrix} 1 & -\kappa/R \\ 0 & \kappa \end{bmatrix} \begin{bmatrix} W \\ v \end{bmatrix} \quad (6)$$

Consider each mode of operation in isolation, as shown in Table 1. Before contact, the ball falls under its own weight. The velocity associated with the compliance (i.e. the velocity of deformation) $\dot{q}_c$ is zero because the ball has not yet been compressed, i.e. q_c (the deformation of the ball) is zero. Note that the velocity of the ball itself is not explicitly expressed by the state equation.

On contact, the force on the ball (row 1 of equation (6)) changes from simply the weight of the ball to become a function of deformation and inertia. The ball deforms with a velocity which is equal to the velocity of the ball (row 2). The initial momentum on contact is equal to the final momentum before, so the state does not need to be reinitialized.

Immediately after contact, the ball is travelling upwards as a result of the forces applied during the contact phase. The initial momentum of the ball is equal to the final momentum incrementally before losing contact.

Each mode of operation is adjacent to the last, with no impulsive losses. The initial state of one mode can be taken as the final state of the last[35]. However, the model is stiff and may run into difficulty when simulated. Inspection of the contact phase in more detail can yield the coefficient of restitution which amends the velocity (and hence momentum) of the ball after contact.

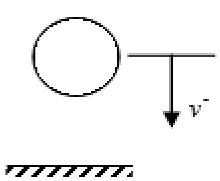
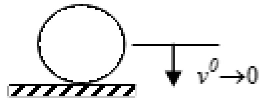
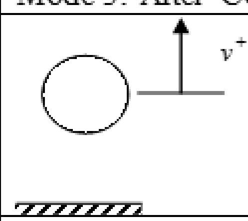
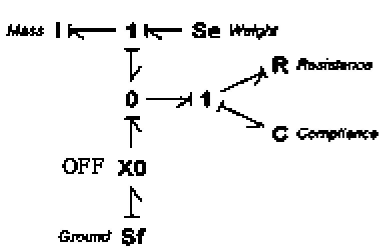
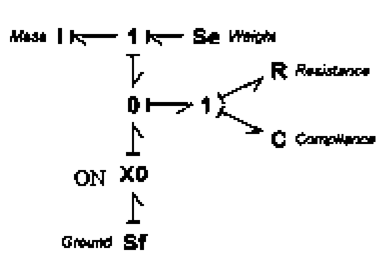
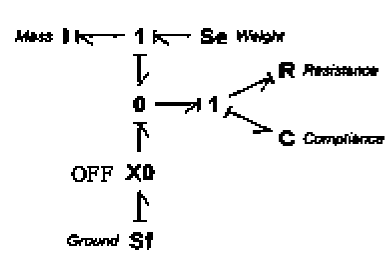
2.2. Newton's Collision Law with Restitution

In classical mechanics, Newton's collision law assumes two modes of operation - before and after collision – and a coefficient of restitution is used to give the difference in velocities (7).

$$\Delta v^{+} = -\varepsilon\left(\Delta v^{-}\right) \quad (7)$$

The hybrid bond graph developed here abstracts the collision differently, into *contact = TRUE* and *contact = FALSE*. This gives three sliding modes of operation: before contact, during contact and after contact. The model is a switching model as opposed to an impulse model [2]. The 'during contact' mode is so short as to be negligible.

Table 1: Static Bond Graphs and Equations for Each Mode of Operation

Mode 1: Before Contact	Mode 2: Contact	Mode 3: After Contact
		
		
$\begin{bmatrix} \dot{p}_m \\ \dot{q}_c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -R/C \end{bmatrix} \begin{bmatrix} p_m \\ q_c \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} W \\ v \end{bmatrix}$	$\begin{bmatrix} \dot{p}_m \\ q_c \end{bmatrix} = \begin{bmatrix} -1/mR & -1/C \\ 1/m & 0 \end{bmatrix} \begin{bmatrix} p_m \\ q_c \end{bmatrix} + \begin{bmatrix} 1 & -1/R \\ 0 & 1 \end{bmatrix} \begin{bmatrix} W \\ v \end{bmatrix}$	$\begin{bmatrix} \dot{p}_m \\ q_c \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & -R/C \end{bmatrix} \begin{bmatrix} p_m \\ q_c \end{bmatrix} + \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} W \\ v \end{bmatrix}$

However, during this mode, energy storage and dissipation may occur, which is what causes the ball to bounce until it reaches rest. Any model including this in-contact phase would need an integrator with either a very small time step or accurate event-location. Simplifying the model to a Newtonian impact with restitution gives faster simulation times than would be achieved by forcing a computer to calculate behavior during the contact phase. This bond graph can be used to calculate a coefficient of restitution.

Consider the local minima where velocity tends to zero and deformation is at its greatest. Assume that this occurs at point t_i. I.e. The contact phase is assumed to be taking place over a time period (t_{i-1} to t_{i+1}), during which the body compresses linearly between t_{i-1} and t_i, and decompresses linearly between t_i and t_{i+1}.

At the point of maximum compression t_i, $p_m=mv_m$, ground velocity v is zero, and $v_m \rightarrow 0$ at the local mimina. Looking at the state equations for mode 2 (Table 1).

$$(m\dot{v})_i = -C^{-1}(q_c)_i \qquad (8)$$

where $(q_c)_i$ is the maximum deformation of the ball.

Now consider the point incrementally before losing contact, at point t_{i+1}: the deformation $q_c \rightarrow 0$ but the ball is still just in contact. The momentum of the ball at this point is the initial value of $\dot{p}_m$ in the next mode of operation. Recall that $p_m=mv_m$, ground velocity v is zero and $v_m \neq 0$. At this point, the first state equation for mode 2 (Table 1) becomes:

$$(m\dot{v})_{i+1} = \frac{mv_{i+1}}{mR} - mg \qquad (9)$$

Now v_{i+1} is the change in displacement over a time step $v_{i+1}=((q_c)_{i+1}-(q_c)_i)/\Delta t$, and the displacement $q_c \rightarrow 0$ at the point where the ball is about to lose contact (i.e. t_{i+1}). Also, g is equal to the acceleration before contact:

$$\dot{v}_{i+1} = \frac{-(q_c)_i}{mR\Delta t} - \dot{v}_{i-1} \qquad (10)$$

Equation (8) can be substituted into (10):

$$\dot{v}_{i+1} = \frac{C\dot{v}_i}{R\Delta t} - \dot{v}_{i-1} \qquad (11)$$

Assume that acceleration decreases linearly on initial contact while compressing (t_{i-1} to t_i) and increases by the same rate in the other direction while extending (t_i to t_{i+1}) i.e. $\dot{v}_{i+1} = -\dot{v}_i$ hence (11) becomes:

$$\dot{v}_{i+1} = \frac{C\dot{v}_{i+1}}{R\Delta t} - \dot{v}_{i-1} \qquad (12)$$

This is an expression for $\dot{v}_{i+1}$ in terms of velocity and acceleration prior to contact. It can be rearranged to give Newton's collision law. Since $\dot{v} = \Delta v/\Delta t$:

$$\dot{v}_{i-1} = \left(1 - \frac{C}{R\Delta t}\right)\dot{v}_{i+1} \qquad (13)$$

Comparing (13) to Newton's collision law with restitution yields:

$$\varepsilon = \left(\frac{-C}{R\Delta t} + 1\right) \qquad (14)$$

This result could be used to simplify the mathematical model for simulation, with the benefit of having obtained the coefficient of restitution systematically from the model instead of using generic or estimated values.

2.3. Discrete-time Moreau's second-order sweeping process

Compare the mathematical model of the bouncing ball to that obtained by Acary and Brogliato[33] (eqn. 15):

$$\begin{cases} m\ddot{q}(t) + f(t) = -mg + \lambda \\ 0 \le q(t) \perp \lambda \ge 0 \\ \dot{q}(t^+) = -e\dot{q}(t^-) \text{ if } q(t) = 0 \text{ and } \dot{q}(t^-) \le 0 \\ q(0) = q_0 \ge 0, \dot{q}(0^-) = \dot{q}_0 \end{cases} \qquad (15)$$

Recalling that $p = m\dot{q}$, the velocity of the ground v is zero, and no external force is being applied (i.e. $f(t)=0$), a like set of equations (16) may be obtained from the bond graph. The first line of (16) is similar to the first line of (15).

$$\begin{cases} m\ddot{q}_m(t) = -mg + \frac{-\kappa}{mR}p_m - \frac{-\kappa}{C}q_c \\ \dot{q}_c = -\kappa\dot{q}_m - \bar{\kappa}\frac{R}{C}q_c \end{cases} \qquad (16)$$

Acary and Brogliato must calculate the value of λ, which describes the reaction on contact. The hybrid bond graph gives an expression in terms of Boolean input (17):

$$\lambda = \frac{-\kappa}{mR}p_m - \frac{-\kappa}{C}q_c \qquad (17)$$

The model in equation can be rearranged to give a compact differential inclusion, and then discretised to give a scheme which is an example of discrete-time Moreau's second-order sweeping process. For the bouncing ball modelled here with no external force *f(t)* the model becomes:

$$\begin{cases} q_{k+1}-q_k=hv_{k+1} \\ m\left(v_{k+1}-v_k\right)+mgdt=-\mu_{k+1} \\ -\mu_{k+1}\in\partial\Psi_{T_{\mathrm{R}^+}(q_k)}\left(\frac{v_{k+1}+ev_k}{1+e}\right) \end{cases} \qquad (18)$$

Where $\mu_{k+1}=d\lambda\left(\left(t_k,t_{k+1}\right]\right)$ is a measure of the interval and $h(f_{k+1}+mg)$ is the impulse, as an integral of external force and weight. Again, all coefficients are known or can be derived from λ which is given by the hybrid bond graph.

3. SIMULATION RESULTS

This paper shows that hybrid bond graphs can be useful in deriving the coefficients for known hybrid models. However, simulation of these models requires other factors such as use of integrator, event-detection, etc. be taken into account. The following results are for illustrative purposes only.

A model based on Newton's collision law was constructed in Matlab. There are several ways to do this, using the first or second order integrator blocks in SIMULINK or a script using Stateflow [36]. Here a SIMULINK model using a Second-Order Integrator was selected, with a coefficient of restitution given by eqn (14). Using equation 18 necessitated a fixed-step solver, and a 4^{th} Order Runge-Kutta integrator with step size of 1×10^{-3}s was chosen.

A compliance of 0.001 m/N (i.e. spring stiffness of 1000 N/m) and linear resistive coefficient of 10 m/Ns were used, giving a coefficient of restitution of 0.91. The ball was dropped from a height of 3m. The resulting plot is shown in Figure 3.

4. DISCUSSION AND CONCLUSIONS

The hybrid bond graph proposed by Margetts et al [1] generates mixed-Boolean state equations. Like most mathematical models, these can be manipulated into forms suitable for simulation. They have an important advantage in that they can be used to rigorously generate coefficients which would otherwise need to be estimated or derived.

This paper shows how a coefficient of restitution ε can be obtained in terms of compliance, resistance and time step, and used to simulate a bouncing ball. The value is sensitive to the time step used, which should be small since Newton's collision law assumes a negligibly short contact phase. Too large a time step gives an unrealistically low loss on contact.

Likewise, the hybrid bond graph can be used to generate an expression for reaction on contact λ as used in the Linear Complementarity Problem. Solving these types of models is outside the scope of this paper, but further work will center on using discrete-time Moreau's second-order sweeping process to simulate hybrid bond graphs.

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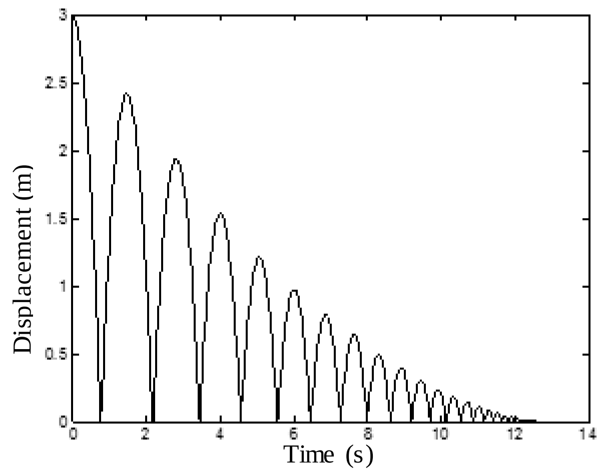

Figure 3: Simulation of a Bouncing Ball

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