Making mathematics and science integration happen: key aspects of practice

DOI:
10.1080/0020739X.2015.1078001

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Received: 8 Jan 2015
Published online: 21 Aug 2015

Abstract

The integration of mathematics and science teaching and learning facilitates student learning, engagement, motivation, problem-solving, criticality and real-life application. However, the actual implementation of an integrative approach to the teaching and learning of both subjects at classroom level, with in-service teachers working collaboratively, at second-level education, is under-researched due to the complexities of school-based research. This study reports on a year-long case study on the implementation of an integrated unit of learning on distance, speed and time, within three second-level schools in Ireland. This study employed a qualitative approach and examined the key aspects of practice that impact on the integration of mathematics and science teaching and learning. We argue that teacher perspective, teacher knowledge of the ‘other subject’ and of technological pedagogical content knowledge (TPACK), and teacher collaboration and support all impact on the implementation of an integrative approach to mathematics and science education.

Keywords

mathematics,
science,
integration,
practices,
second-level education

1. Introduction


However, challenges have been identified in previous research in implementing curriculum integration, such as time, resources, lack of experience of collaboration, lack of teacher education/professional development and seeing technology as aids and additional devices.[11. Czerniak CM. Interdisciplinary science teaching. In: Abell SK, Lederman NG, editors. Handbook of research on science education. New York (NY): Routledge; 2007. p. 537–560.

The literature indicates that there has not been a great deal of research in recent years that has focused on classroom implementation of science and mathematics involving collaboration between in-service teachers.[11. Czerniak CM. Interdisciplinary science teaching. In: Abell SK, Lederman NG, editors. Handbook of research on science education. New York (NY): Routledge; 2007. p. 537–560.


It is suggested this is as a result of the pressure on teachers to teach to standards and to prepare students for external examinations that are discipline-based.[4–7

Therefore, this research project specifically focused on the integration of mathematics and science at classroom level, with in-service mathematics and science teachers collaborating within a given school to develop mutually dependent lessons, and undertaking the integration within the ‘normal’
school context, e.g. prescribed central syllabi, school resources, timetabling, etc. Accordingly, a central aspect of this research project was the development of an integrated unit of learning on distance, time and speed, with integration between the mathematics and science classes and the teachers facilitated by the use of the TI-Nspire™ handheld graphic calculator and data logging equipment.

Prior to commencement of the project, the authors hypothesized that there are key aspects of in-service practice that impact on the successful curriculum integration of mathematics and science at a classroom/school level. Second-level education in the Irish context is premised on a culture of individualism and segregated subject delivery. Therefore, the aim of this paper is to describe the first integrative mathematics and science teaching and learning case study undertaken in the Irish second-level education context, with a focus on classroom implementation. The project involved three second-level schools with mathematics and science teachers working in collaboration to implement a unit of learning. In particular, we address the following research question: What aspects of practice are of importance to facilitate the integration of mathematics and science teaching and learning at second-level education in Ireland?

Although this case study is set in the Irish context and specifically in mathematics and science disciplines, these findings can contribute to developing integrative in-service subject delivery in other contexts and in other subject combinations. The findings of this research project are intended to be used to inform the design and implementation of a longitudinal integrative project at second-level education with mathematics and science teachers. However, for the purpose of this paper, the focus is on developing our understanding of the key aspects of practice that need to be taken into consideration when planning such an initiative with in-service teachers in the future.

This research paper begins with a review of relevant literature on the integration of mathematics and science, integration facilitated by technology, and benefits and challenges of each. Following this is a description and background of the research project in order to provide the readers with an understanding of the second-level context in Ireland and a description of the design of the integrated unit of learning. The authors then describe the research project undertaken and methodology employed to gather and analyze data in relation to key aspects of practice. Finally, the authors discuss the importance of these findings for informing future interventions to support an integrative approach to mathematics and science teaching and learning.

2. Integrating mathematics and science teaching and learning

Many of the rationales for integrating STEM (Science, Technology, Engineering and Mathematics) subjects are based on the fact that in a complex, challenging globalized world, citizens need to draw on multidisciplinary knowledge in order to understand and address the multifaceted issues and concerns they face. School curricula, however, usually compartmentalize knowledge into isolated disciplines.[88. Breiner JM, Harkness SS, Johnson CC, et al. What is STEM? A discussion about conceptions of STEM in education and partnerships. Sch Sci Math. 2012;112:3–11.]
This section examines the literature on mathematics and science and technology integration, the benefits for students, and also the challenges faced by teachers in implementing integrative approaches into predominantly disciplinary curriculum.

2.1. Benefits of integrating mathematics, science and technology


Science can provide students with concrete examples of abstract mathematical ideas, while mathematics can enable students to achieve deeper understanding of science concepts by providing ways to quantify and explain science relationships.[1111] McBride JW, Silverman FL. Integrating elementary/middle school science and mathematics. Sch Sci Math. 1991;91:285–292.


report that teachers observed several benefits of integration for their students, including increased motivation, engagement and application of mathematics and science concepts. Research evidence also indicates that curriculum integration and mathematics and science integration specifically also have a positive effect on student achievement.[55. Vars GF. Can curriculum integration survive in an era of high-stakes testing? Mid Sch J. 2001;33:7–17.


However, it is important to note that integration of science and mathematics can take many forms, a point underlined by the fact that defining science and mathematics integration is a prominent theme in the literature.[11. Czerniak CM. Interdisciplinary science teaching. In: Abell SK, Lederman NG, editors. Handbook of research on science education. New York (NY): Routledge; 2007. p. 537–560.


17–20
In this project, it was recognized that the content to be integrated would have to be based on making connections between the centrally mandated syllabuses of the two disciplines, as teachers are unlikely to adopt integrative strategies that will not speak directly to the concepts students have to learn for subject-specific examinations.

Given the move towards increased use of online and information technologies in all aspects of life in the twenty-first century, education too must incorporate an increased use of technology within different subject areas.[21] Niess ML. Preparing teachers to teach science and mathematics with technology: developing a technology pedagogical content knowledge. Teach Teach Educ. 2005;21:509–523.

Technology can give teachers tools that permit their students to perform complex tasks that are similar to those in the real world, for example, when they use technology to integrate data from science experiments and fieldwork into their mathematics class. Additionally, it can promote student-centred pedagogies.[22] BECTA. What the research says about using technology in Maths. England: BECTA Technology Research; 2003.


In this study, the curricular imperative for the use of the data logging technology was to facilitate integration between science and mathematics classes through the collection of real-life data and examination of key concepts associated with speed, distance and time. Therefore, the intention was that both mathematics and science integration and technology integration would be mutually facilitated.

2.2. Aspects of practice that impact on the implementation of mathematics and science integration

Although mathematics and science integration is theoretically desirable for a number of reasons, there are a number of difficulties associated with its implementation. Previous studies have identified barriers to integration including poor teacher content knowledge and pedagogical content knowledge, teachers’ beliefs and attitudes, school structural factors, such as lack of time for planning with other teachers, and the impact of standards and subject-based examinations.[7] Meier SL, Nicol M, Cobbs G. Potential benefits and barriers to integration. Sch Sci Math. 1998;98:438–447.


These are a set of interrelated factors and are often reported as ‘perceived’ barriers to integration [2626. Czerniak CM, Johnson CC. Interdisciplinary science teaching. In: NG Lederman, SK Abell, editors. Handbook of research on science education. New York (NY): Routledge; 2014. p. 395–411.]


Generally speaking, teachers have positive views about the benefits of integration for student's learning and motivation, but second-level teachers in particular have concerns that it will take too much time from their teaching of the curriculum and would be more difficult because of lack of materials and lack of staff development.[2626. Czerniak CM, Johnson CC. Interdisciplinary science teaching. In: NG Lederman, SK Abell, editors. Handbook of research on science education. New York (NY): Routledge; 2014. p. 395–411.]

However, one of the more significant barriers to enacting mathematics and science integration is lack of, or insufficient, teacher content knowledge and pedagogical content knowledge.[77. Meier SL, Nicol M, Cobbs G. Potential benefits and barriers to integration. Sch Sci Math. 1998;98:438–447.]
This particularly refers to content knowledge in the other subject. Most teachers are subject specialists, and will not have the necessary knowledge and confidence required to integrate the language, methods, concepts or content of another discipline. \[11\] Czerniak CM. Interdisciplinary science teaching. In: Abell SK, Lederman NG, editors. Handbook of research on science education. New York (NY): Routledge; 2007. p. 537–560.

A number of studies have examined how teacher content and pedagogical content knowledge can be enhanced and developed across disciplines through teacher collaboration. Frykholm and Glasson \[2727\]. Frykholm J, Glasson G. Connecting science and mathematics instruction: pedagogical context knowledge for teachers. Sch Sci Math. 2005;105:127–141.

propose that teachers collaborate to explore authentic and situated connections between science and mathematics, as a way of developing their ‘pedagogical context knowledge’ for integration. Morrison and McDuffie \[2929\]. Morrison J, McDuffie AR. Connecting science and mathematics: using inquiry investigations to learn about data collection, analysis, and display. Sch Sci Math. 2009;109:31–44.
suggest collaboration is the way to overcome limitations in content-area expertise, while Orton and Roper [10].


indicate the importance of liaison between mathematics and science teachers, so that students are not presented with conflicting language and mathematical techniques for overlapping concepts. On the other hand, the literature indicates that school structures often impede teachers’ opportunities to collaborate and plan integrated materials together.[11. Czerniak CM. Interdisciplinary science teaching. In: Abell SK, Lederman NG, editors. Handbook of research on science education. New York (NY): Routledge; 2007. p. 537–560.


2.3. Aspects of practice and technology

Efforts to implement technology have not had a major impact on teaching and learning, possibly because teachers have had little experience of using technology as learners, and little experience of designing and developing technology implementation plans.[2121. Niess ML. Preparing teachers to teach science and mathematics with technology: developing a technology pedagogical content knowledge. Teach Teach Educ. 2005;21:509–523.


found that barriers to the integrating of technology in education can be classified as school-level barriers and teacher-level barriers. School-level barriers include lack of effective training in solving technical problems, lack of technical support, lack of access to resources and lack of time to resolve these issues. Teacher-level barriers include lack of confidence, lack of competence and resistance to change in integrating technology. The notion of technological pedagogical content knowledge (TPACK) as a framework to identify teachers’ knowledge requirements for effective technology integration in education is a useful construct through which to consider teacher-level barriers that will impact upon practice.\cite{Mishra2006}

TPACK includes both teacher’s technological knowledge (TK) and their technological pedagogical knowledge (TPK). A deficiency in either TK or TPK will impact upon teachers’ integration of technology into teaching and learning. Within the TPACK framework, particular emphasis is placed on the connections between technologies, curriculum content and pedagogical approaches in order to facilitate effective teaching and learning.

As the literature review demonstrates, the implementation of these integrated science and mathematics lessons, mediated via the use of technology, is likely to be influenced by related, but also somewhat distinct, factors at the level of both the teacher and the school. Our research project is specifically focused on identifying what aspects of practice are of importance in facilitating the integration of science and mathematics teaching and learning at second-level education. In particular, we are examining this within centralized, mandated syllabi, teachers collaborating (as opposed to an individual teacher implementing integration on their own) and within the normal working conditions of a school. Very few studies have reported such findings within the context of day-to-day, science and mathematics in-service teachers’ teaching and learning\cite{Czerniak2014} and accordingly our findings provide rich insights into this area of science and mathematics integration, through the use of technology.

3. Background to the study

In 2008, the EPI·STEM: National Centre for STEM Education\cite{EPI-STEM} was established as an acknowledgement of the importance of the teaching and learning of mathematics and science for Ireland’s economic future. EPI·STEM is a centre for research on national priority issues, conducting best practice, evidence-based research into the teaching and learning of mathematics and science. It was in light of the remit of EPI·STEM (in which two of the authors were then working) that this project was undertaken. It was topic-driven and evidence-based. This section
will outline the background to this particular study, the Irish education system, the participating teachers and integration in the Irish context.

3.1. Second-level education in Ireland

Education is compulsory for children in Ireland from the ages of 6–16 or until children have completed three years of second-level education. There is a state examination, the Junior Certificate, at the end of the first three years of second-level education. Students typically enter second level at the age of 12 and the majority of students complete the Junior Cycle (ages 12–15/16) and then progress onto the Senior Cycle (ages 15–17/18) for further two years. There is a state examination at the end of the Senior Cycle known as the Leaving Certificate.[3434. Department of Education and Skills [Internet]. Ireland: Dublin [cited 2014 Oct 15]. Available from: www.gov.ie

3.2. Irish curriculum

The Irish education system operates on a centralized education model and contains very descriptive syllabi. The Irish curriculum is centrally devised by the National Council for Curriculum and Assessment (NCCA) [3535. National Council for Curriculum and Assessment [Internet]. Ireland: Dublin [cited 2014 Oct 15]. Available from: www.ncca.ie

and is determined by the Minister for Education and Skills. Irish, English and mathematics are compulsory subjects for all students in lower second-level education. Science is not a compulsory subject but it is taught in most schools in Ireland and it is assigned compulsory status in some schools, depending on the type of school.[3535. National Council for Curriculum and Assessment [Internet]. Ireland: Dublin [cited 2014 Oct 15]. Available from: www.ncca.ie

In this project, mathematics and science were compulsory subjects in all the participating schools. Traditionally, in Irish schools, mathematics and science departments work in isolation from each other and there is little or no planning done in terms of overlapping topics in science and mathematics. A culture of collaboration across disciplines does not exist. Integration of science and mathematics at the level implemented in this project has never been undertaken in an Irish context prior to this. Opportunities for in-service teachers to participate in collaborative professional development at classroom level are also very limited in the Irish education system.

3.3. Participating teachers
The teachers (Ann, Lee, Matthew, Norma and Rachel) who participated in this research project are highly regarded teachers and are well respected in terms of successfully meeting the demands of the Irish educational system (all teachers have been given pseudonyms to protect identity). These teachers operate within an education system that demands results due to its focus on state examinations, which accordingly impact on entry into third level education. Local innovation takes additional energy and in the majority of Irish schools there is a lack of support structures in terms of teacher professional development. [36] TALIS. Teaching and Learning International Survey Technical report OECD; 2008. Available from: http://www.oecd.org/education/school/44978960.pdf

Participating teachers in this research project were not allocated reduced teaching hours or any additional support within the school. Participation involved use of personal time and out-of-school hours. Within the active implementation of the classroom element of the research project, there was no flexibility in the timetable or class schedule. The schools and principals supported the project in principle but did not facilitate any additional supports for the teachers.

4. The design of the integrated unit of learning


[38] Mason J. Learning and doing mathematics. 2nd ed. York: QED; 1999.

These theoretical constructs were applied by the authors to the design and intended implementation of the integrated unit of learning, and not to the analysis of the research project presented in this paper (research methodology presented in Section 5).

4.1. Connected, situated and sequential integration

conceptual framework was designed originally to foster the development of pre-service teacher content and pedagogical context knowledge for integrating mathematics and science. Their ‘connected’ approach to integration is based on ‘situated’ learning, the concept of negotiating and constructing meaning through social interaction, drawn from the work of Lave [3939. Lave J. Cognition in practice: mind, mathematics and culture in everyday life. Cambridge (MA): Cambridge University Press; 1988.


Rather than attempting fully integrated or interdisciplinary learning, which would require teachers to have very strong content knowledge in both disciplines, teachers collaborate on the development of curricular units based on connections between mathematics and science. For the purpose of this research project, this was done in conjunction with the authors and emphasis was placed on connections being authentic with respect to both mathematics and science, and suitable for student experiences.[2727. Frykholm J, Glasson G. Connecting science and mathematics instruction: pedagogical context knowledge for teachers. Sch Sci Math. 2005;105:127–141.

The process of collaborating together to identify themes, connections and activities for integration provides both the content and an important process for building the ‘situated’ experiences necessary for effective integration of science and mathematics. The lessons were designed in a sequential format.[1515. Hurley MM. Reviewing integrated science and mathematics: the search for evidence and definitions from new perspectives. Sch Sci Math. 2001;101:259–268.


4.2. Teaching for understanding

The integrated unit of learning and individual lesson plans were designed by the authors to promote a teaching for understanding approach that advocated providing students with rich and concrete
opportunities in order to develop their thinking and promote their higher order learning.[4141. Piaget J. The origins of intelligence in teaching. London: Routledge and Kegan Paul; 1952.

The science lessons were designed to engage and build students’ understanding of scientific concepts and skills through concrete experience/situations.[3737. Arons AB. Teaching introductory physics. New York (NY): Wiley; 1990.

The mathematics lessons employed rich mathematical tasks, with concepts initiated in the science lessons, in order to provide students with the opportunity of specializing (constructing particular examples to see what happens) and generalizing (detect a form; express it as a conjecture; then justify it through reasoned argument) in the mathematics class.[3838. Mason J. Learning and doing mathematics. 2nd ed. York: QED; 1999.

4.3. The integrated unit of learning

Central to this research project, and to assist in achieving the aim of integrating mathematics and science teaching and learning, was an integrated unit of learning on distance, speed and time. The authors were central in the design and development of the unit of learning. It consisted of seven lessons in total (three 70-minute science lessons and four 35-minute mathematics lessons) to be taught over a period of three weeks (Table 1). The purpose of the unit of learning was to facilitate the development of students’ mathematical and scientific concepts and skills through concrete real-life experience,[4242. Rosenquist ML, McDermott LC. A conceptual approach to teaching kinematics. Am J Phys. 1987;55:407–415.

, to allow students an opportunity of specializing and generalizing in mathematics,[3838. Mason J. Learning and doing mathematics. 2nd ed. York: QED; 1999.


Appendix 1 contains Mathematics Lesson Plan 3 and Science Lesson Plan 3 from the implemented integrated unit of learning. The lesson plans are underpinned by the theoretical frameworks
provided in Sections 4.1 and 4.2, while demonstrating the linkage, interconnection and development of speed, distance and time in mathematics and science lessons, through the incorporation of technology (TI-Nspire™ handheld graphic calculator and data logging equipment).

Table 1. Sequence and description of lessons.

5. Methodology for evaluating the research project

Disclosure statement

A case study approach was utilized in this research project, which presents a unique example of real people in real situations, enabling readers to understand ideas more clearly.[4343. Cohen L, Manion L, Morrison K. Research methods in education. 5th ed. London: Routledge and Falmer; 2000.

The methodological approach is qualitative in nature and the following sections provide key information in relation to the implementation and evaluation of the research project.

5.1. Participants and context of research

Three second-level schools in the southwest of Ireland participated in this research project. Selection of schools was determined by location and willingness to participate in the project. Table 2 provides a brief description of the three schools.

Table 2. Description of the schools.

The authors had intended to recruit one mathematics teacher and one science teacher from each of the participating schools. Two teachers took part in School B (Norma and Rachel) and School C (Ann and Lee). In School A, one teacher (Matthew) took part as he was both the science and mathematics teacher for the first year group participating in the project. Out-of-field teaching in mathematics is a concern in the Irish context with out-of-field teachers often assigned to the junior classes at second-level education.[4444. Riordán Mní, Hannigan A. Who teaches our students mathematics at post-primary education in Ireland? Irish Ed Stud. 2011;30:289–304.

It is important to note that only one qualified mathematics teacher took part in this study (School C – Ann). The teachers teaching mathematics in Schools A and B were qualified science teachers but were out-of-field mathematics teachers but had responsibility for teaching mathematics to the first year classes participating in this research project. Table 3 illustrates the number of years of teaching experience, subjects taught and experience of integration of the participating teachers in this research project.
This research project was one school year in duration. The authors facilitated professional development in the use of the technology (TI-Nspire™ handheld graphic calculator and data logging equipment) and in supporting integration in the implementation of the unit of learning. The following considerations applied:

Every student in the classes of the three selected schools and all of the participating teachers were provided with a TI-Nspire™ handheld graphic calculator and data logging equipment.

The integrated unit of learning on distance, speed and time was developed by the authors, with the support and feedback of the participating teachers.

Continuous support was provided for all teachers involved in the research project.

The teachers participated in in-house professional development (two days – September/October), where the focus of the professional development was on introducing the teachers to the technology and to the sequence of mathematics and science lessons. The teachers had an opportunity to read the lesson guidelines and the authors also did a ‘walk through’ of the lessons, detailing the focus and the purpose of each lesson. This was followed by in-school support by the authors in the individual schools, as required, in order for teachers and students to become familiar with the technology (November–February). The teaching approaches and linkage to the syllabi were discussed with the teachers but the emphasis was on the content, sequence and technology integration. The active research of the integrated mathematics and science lesson plans took place during March and April of the school year. Table 1 provides a description of each of the lesson plans within the unit of learning and the order in which they were taught.

5.3. Data collection

This study is qualitative in nature and centred on eliciting the teachers’ experiences of participating as active researchers in the classroom. Table 4 provides a list and description of key data collection tools employed in this research project.

Table 4. Description of data collection tools.

5.4. Data analysis

The authors utilized a three-stage process in order to analyze all of the qualitative data collected throughout the duration of the project. For the purpose of this paper, ‘transcript’ refers to any of the data generated from the teachers’ reflective journals, lesson observations and interviews.


The authors independently read all of the transcripts and highlighted themes that were relevant to the research question. Together, the authors then compared and compiled the themes and these are listed in Table 5. During this phase, emphasis was put on identifying indicators of concepts and categories that fit the data. Repeatedly appearing categories and concepts helped to construct themes based on what aspects of practice impacted on the implementation of the integrated unit of learning. The viability of the construction of themes was then tested against all relevant data sets (i.e. observations, reflections and interviews).

Table 5. Initial codes emerging from transcripts.

Subsequently, extracts from the transcripts were categorized into the identified themes. The authors examined these data in detail and identified 17 codes that related to the aspects of practice that are important in facilitating the integration of mathematics and science (see Table 6). The codes were created through an iterative cycle of code development and analysis.

Table 6. Codes summarizing the aspects of practice important in facilitating mathematics and science integration.

Triangulation of data was employed to ensure the validity or trustworthiness of our interpretations of the data. Lincoln YS, Guba EG. Naturalistic inquiry. Newbury Park (CA): Sage Publications; 1985.

By reviewing the data with the teachers collaboratively, the validity and reliability of the data sources were tested, as well the authors’ interpretations of the data. Miles MB, Humberman AM. Qualitative data analysis: an expanded source book. Thousand Oaks (CA): Sage Publications; 1994.

Different data sources were triangulated by comparing and contrasting different descriptions (observations, reflections, interviews) of the situation. Lincoln YS, Guba EG. Naturalistic inquiry. Newbury Park (CA): Sage Publications; 1985.

The final phase of analysis involved the continued interpretation of each source of data with particular attention to the experiences the teachers shared throughout the course of this research project. Through construction and reconstruction of the teachers’ accounts,\[^{4949}\] Clandinin J, Connelly M. Teachers’ professional knowledge landscapes. New York (NY): Teachers College Press; 1995.

narratives grounded in the data were created that makes sense of the aspects of practice that impacted on the teachers implementing the integrated unit of learning.

6. Findings

Four key themes (teacher perspectives, teacher content knowledge of the ‘other subject’, teachers’ TPACK, and collaboration and support) emerged from the qualitative data on the aspects of practice that are of importance in order to facilitate the integration of mathematics and science at second-level education in Ireland. Each of these themes is discussed in the following sections.

6.1. Teacher perspectives

Teacher perspectives influenced the implementation of the unit of learning, their perceptions of their students’ ability and their conceptions of integration. Prior to implementation in the classroom, the teachers raised concerns in relation to the length of time being spent on this topic, with Lee stating ‘it is only a tiny piece of the course you know you would get it done in a double class more or less’ (Focus Group, Pre-Interview). During the implementation of the unit of learning, the teachers did not take ownership (although they were encouraged to) of the unit of learning and viewed the sequence and lessons as being rigid and inflexible, while putting pressure on themselves to implement the lessons as presented in the sequence. The teachers constantly referred to not getting lessons finished, ‘I kind of found that the lesson plans were very long for a double class’ (Matthew, Interview), running out of time and feeling under pressure to have key elements completed for the next lesson. However, teacher adaption of the unit of learning occurred infrequently.

The data indicate some underestimation of students’ ability. For example, the teachers stated that they felt that the students would have difficulty building the balloon rocket car, ‘so that is why you would have to give them everything spot on, so that all they would have to do is slot it in, so building of the car should not be a big thing’ (Norma, interview). A significant portion of the teacher discussions during the pre-focus group meeting centred around the difficulty of the mathematics questions in particular, and the teachers perceived that the level might not be accessible to their students. The teachers’ concerns centred on their students’ perceived struggle with drawing and
interpreting graphs and that their students needed to develop skills in hand-drawing graphs either before introducing the technology or at some point thereafter. The teachers’ focus on hand-drawn graphs links with expectations of state examination in the Irish education system. However, the focus in the integrated unit was on the interpretation of graphs and application to the concept of speed, distance and time.

In general, the teachers perceived mathematics and science as distinct subjects and serving separate purposes. For example, when asked in the interview how the lesson helped integrate mathematics and science teaching, Rachel (Interview) stated that:

I think that was probably more of a responsibility of the maths because in the science it was all about getting the data not particularly drawing it and then interpreting graphs but the numbers were not important, ... so probably maths had a harder part in doing that because they were the one who had to record data and probably come up then with the concept of speed...I think the science were probably the lucky ones.

Teachers’ perceptions of science included ‘you probably get more ability to break away from the norm you know...the science I think it's just great enjoyment’ (Rachel, Interview), whereas mathematics was not perceived in a similar manner (see section 6.2). All participating teachers expressed the belief that making the link between mathematics and science was good for them as teachers, and good for their students’ learning. Ann talked about the sense of achievement in participating in the project ‘we've met a new challenge and I would like to think we succeeded’.

6.2. Teacher content knowledge of the ‘other subject’

Two of the three teachers teaching mathematics in this study were out-of-field teachers (qualified science teachers but do not have specific qualification in mathematics). Teacher identity was very much rooted in their subject area and this impacted on the integration process. Norma stated that ‘I would see myself more as a science teacher’ – even though she was teaching the mathematics component of the unit of learning (Norma, Interview). She expressed that she thought that the topic of speed was ‘a small enough topic’ and that it was ‘stretched out’ in the mathematics lessons, more than ‘what it would be in the chapter in the textbook’ (Norma, Interview). From her lesson observations (and others’), she frequently missed key concepts and lacked depth in terms of discussion of concepts and development of the topic with her students. Similarly, from the science perspective, teachers reported that certain aspects were more of a ‘responsibility’ for the mathematics teachers and that they did not have a role in addressing the mathematical concepts in their science lessons (see Rachel Interview in Section 6.1). Rachel perceived mathematics as ‘numbers’ and ‘formulae’ and a lack of a role for mathematics in science teaching and learning.

All teachers in their interviews and reflections emphasized that integration is important and beneficial for student learning. From participating in this research project, they recognized the potential for developing their students’ understanding and helping them to see the relevance in what they are learning. As stated by Matthew, 'They could see, even taking the stuff we did in science and the maths drawing the graph, they could see a really close link between the two. So that was very good'. From the lesson observations, it is evident that some tasks within the lesson plans
were done admirably by individual teachers and positive aspects can be utilized for future planning. The observations also noted that the students responded confidently to these tasks when done well. ‘One of the students, on leaving the classroom stated it was ‘the best lesson ever sir’ (School A, Science Lesson 3). The teachers’ reflections portray students as engaged, interested, responding well and enjoying the mathematical and science activities.

6.3. Teachers’ TPACK

The notion of TPACK as a framework to identify teachers’ knowledge requirements for effective technology integration in education has gained significant interest in recent years.

Of the five teachers participating in this study, only one demonstrated good TK. The remainder lacked confidence in its use. This was corroborated from all three perspectives: it was reported by themselves in their reflections ‘I felt flustered as I do not feel confident enough’ (Ann, Reflective Journal), frequently noted in the lesson observations ‘The TI-Nspire™ and the probes were not an integral part of the lesson’ (School C, Science Lesson 1), and discussed during the interviews ‘but you know in terms of when there was a question I could feel my own head go panic, panic’ (Rachel, Interview). All teachers indicated that they needed further support and training in developing their own competence before bringing technology into the classroom. The teachers lacked an understanding of the pedagogical affordances and constraints of the TI-Nspire™. Only one teacher demonstrated forward-looking and open-minded seeking of the application of the TI-Nspire™ and software. ‘What I found very useful was the software on the laptop and having the calculator on the screen, I thought it was brilliant’ (Matthew, Interview). The other teachers, perhaps due to their own TK, struggled in adapting and using the technology to support learning. It was often perceived as an ‘add-on’ or additional constraint to the implementation of the lesson. The difference between the teachers may be attributed to Matthew having greater exposure to the use of technology within his school.

All teachers acknowledged that the students enjoyed the technology and its use in the classroom and recognized that technology needs to be incorporated into teaching and learning. However, the practicality aspect of utilizing technology in the classroom was also a significant factor in the (un)successful integration of science and mathematics. The teachers were very vocal about the technology being ‘time-consuming’ and the need for ‘extra hands’ in the classroom to manage distribution of technology, setting up the technology (e.g. motion probes), replacing equipment parts (e.g. flat batteries) and dealing with student queries (Focus Group Interview). In general, the teachers emphasized that the time required for implementing such lessons developed in the unit of learning requires more time than ‘you would have in the real world of teaching!’ due to the technological requirements (Norma, Reflective Journal).

6.4. Collaboration and support
This was the participating teachers’ first experience of collaborating in a community of practice that focused on an integrative approach to mathematics and science at classroom level. The teachers’ prior experience of collaboration emerging from the data supports the subject-based approach. For example, when Lee was asked how he found working closely with a mathematics teacher, he stated ‘I had no problem what so ever, we have four or five of us in the science department anyway, we work together, we share notes’ (Lee, Interview). Matthew did state that his school encourages cross-curricular links. However, there was no indication of any specific planned links with other teachers in other departments. Overall, there was little or no evidence of any planned activities or tasks on the integration of mathematics and science, or no evidence of cross-departmental discussions, in the schools participating in this project. In general, the teachers were very happy with the level of support they received, with Rachel (Interview) stating that she felt supported ‘by the teachers (the authors) and the group of peers’. They stated how they enjoyed being part of a community of practice and that it was good to have the opportunity to share ideas and develop with other teachers. All teachers expressed a desire to participate in further projects similar to this.

The support provided for teachers at school level was at a minimum throughout this research project. The principal granted teachers permission to participate in the project but no further support was provided. The teachers participated on their own time and out-of-school hours, when attending the professional development sessions. During the active research element of the research, collaborating teachers within the schools generally met with each other to discuss the project during their break times or after school. Interestingly, the teachers did not discuss the lack of support provided at school level but the issue of time was mentioned regularly with Norma stating ‘time...like you hadn’t had enough...you didn’t get a chance to test it out, to try it out ... because of a busy school life and home life’ (Norma, interview). It was the teachers’ willingness to give up their own time to participate in this project that ensured its implementation. The issue of syllabus planning and timetabling emerged early on in the research project. The topic of speed, distance and time features in both the science and mathematics syllabi but yet in all three schools they were being taught at different times of the year. As Ann reiterated in her interview, ‘science cover speed and distance maybe in the first term and we weren’t doing it until maybe the second term. So if you could become more coordinated so that you ended up doing it around the same time period’ (Ann, interview). Further to this, Ann emphasized that ‘if you were on a particular topic, you would be running it right through for a week and I think that would have helped...I think it would have been very hard trying to fit everything in and that is just the way the timetable works’ (Ann, interview). The teachers saw the benefits of integrating both subjects but felt that school structures and supports need to be flexible in terms of accommodating and achieving integration, particularly in relation to timetabling and subject planning.

7. Discussion

Change in pedagogical practices involves complex processes functioning at individual, school and systemic levels, and entails an element of ‘risk-taking’. Le Fevre DM. Barriers to implementing pedagogical change: the role of teachers’ perceptions of risk. Teach Teach Educ. 2014;38:56–64.
mathematics and science, while being acutely aware of the teacher being the most central in influencing educational outcomes for students.[5151. Hattie J. Visible learning: a synthesis of over 800 meta-analyses relating to achievement. London: Routledge; 2009.

Findings from this study reveal a number of key elements of practice that are of significance in supporting an integrative approach to the teaching and learning of mathematics at second-level education. For the teachers participating in this study, this was their first time undertaking such an initiative in their practice and the authors would like to commend and acknowledge the commitment and openness demonstrated throughout the project. These are innovative teachers who were prepared to take risks with new technologies in their teaching and to implement integrated units of learning in an educational context that is not encouraging of such efforts. Insights emerging from the data presented in this paper demonstrate that students enjoyed the linkage between science and mathematics lessons and it assisted with engaging the students in both curricular subject areas. Thus, there is value in teachers collaborating and planning cross-curricular linkage between both subject areas to improve the teaching and learning experiences of the students at second-level education. However, what this study demonstrates is that this is a difficult task, and that support structures, and the teachers themselves, are core to successful integration. Nevertheless, the significance in terms of criticality, relevance and connectivity for student learning is an important consideration and justification for the pursuit of integration of both subjects.[88. Breiner JM, Harkness SS, Johnson CC, et al. What is STEM? A discussion about conceptions of STEM in education and partnerships. Sch Sci Math. 2012;112:3–11.

Given that the success (or lack of success) is significantly influenced by the context in which it is embedded, we sought to understand what aspects of practice most influence the implementation of an integrative approach to the teaching and learning of mathematics and science.[5050. Le Fevre DM. Barriers to implementing pedagogical change: the role of teachers’ perceptions of risk. Teach Teach Educ. 2014;38:56–64.

In particular, teacher perspective and ‘knowledge’ has an important impact on the implementation of such an initiative and is a core component of successful implementation at classroom level. Teacher perspectives on the unit of learning and its implementation centred on the length of time spent on the topic and having the materials prepared for students in advance, as opposed to considering its purpose and the type of student learning being promoted. The teachers were consulted on the unit of learning and their feedback was implemented where appropriate, but the theoretical concepts underpinning the lesson plans remained.[1515. Hurley MM. Reviewing integrated science and mathematics: the search for evidence and definitions from new perspectives. Sch Sci Math. 2001;101:259–268.
In terms of teacher implementation, it became clear that there was a disconnect between a teaching for understanding approach and teachers’ classroom practice. The authors would recommend that greater emphasis and time need to be given to familiarizing teachers with pedagogical approaches before implementing such an initiative and to support them in taking greater ownership of the unit of learning and its implementation in their context of learning. [66. Hargreaves A, Earl L, Moore S, et al. Learning to change: teaching beyond subjects and standards. San Francisco (CA): Wiley; 2002.]

In the context of this study, knowledge for teaching is a core element of practice necessary to facilitate full integration. In particular, teachers require knowledge of the ‘other’ subject. The teachers viewed mathematics and science as being very different, distinctive subjects. This is not surprising considering the Irish educational system is dominated, according to Gleeson by a ‘technicist’ approach to education, where subject culture is very strong, with little room for interdisciplinary teacher and learning. [5353. Gleeson J. Curriculum in contest: partnership, power and praxis in Ireland. Bern: Peter Lang; 2010.]

The technological challenges, and being out-of-field, compounded the difficulty in teaching mathematics for understanding. Equally, in the science lessons, a lack of mathematical knowledge restricted the full potential of student learning within the lessons. The technical nature of mathematics and science, and the intricate links within the subject areas, can create problems for teachers whose understanding of respective subjects is limited. Accordingly, the teaching and learning of an integrated unit of learning is affected. [1414. Pang J, Good R. A review of the integration of science and mathematics: implications for further research. Sch Sci Math. 2000;100:73–82.]
Implementation of such an initiative again would need to ensure that the teachers are given an opportunity to acquire knowledge of the content of the mathematics and science lessons. This would permit them to more easily teach, modify and adapt the content of the lessons according to students' needs.[14] Pang J, Good R. A review of the integration of science and mathematics: implications for further research. Sch Sci Math. 2000;100:73–82.


Moreover, there is a need to develop teachers’ awareness and appreciation of the need for complementary knowledge in respective subject areas and to deconstruct perceptions in relation to other subjects.


A core component of this framework is the teacher’s TK. This was the teachers’ first time using such technological equipment in their classrooms and the authors fully acknowledge that they had underestimated the time it would take for teachers to acquire the skills needed to confidently implement the technological requirements of integrated science and mathematics lessons. TPK, another central element of TPACK, was always in a state of flux, with integration into some tasks done well by some of the teachers, and other tasks done poorly or avoided by other teachers. Of significance for future studies is the need to develop teachers’ TPACK if technology is core to the integration process (as was the case in this research project). For example, it is important to consider how teachers are to acquire an operational understanding of the complex relationships among content, pedagogy, technology and context in relation to the implementation of an integrative approach to teaching and learning mathematics and science. The purpose of incorporating technology into the integration of mathematics and science teaching and learning in this research project was to help redress conceptual challenges associated with speed, distance and time. TPACK encompasses understanding and communicating representations of concepts using technology.[32] Mishra P, Koehler MJ. Technological pedagogical content knowledge: a framework for integrating technology in teacher knowledge. Teach Coll Rec. 2006;108:1017–1054.

The teachers needed more time and opportunities to develop this TPACK in order that they would reap the benefits of using technology to facilitate and support integrated learning. We hypothesize that there is a need to reform teachers’ perceptions of technology and to emphasize its use in

The importance of supporting the teachers at school level, and at a community of practice level, is a major contributing factor to the successful implementation of the integrative approach. The authors feel that many of the ‘time’ issues described by the teachers could be reduced or avoided if there were sufficient support structures, such as a reduced timetable with opportunities to meet and discuss cross-curricular and syllabi sequences, a more flexible adaptable timetable and opportunity to attend in-school time professional development sessions. Teacher collaboration is a key aspect of practice that impacts on the integration of mathematics and science teaching and learning.[4040. Lave J, Wenger E. Situated learning: legitimate peripheral participation. Cambridge (MA): Cambridge University Press; 1991.

For the authors, it is distinctly apparent that greater teacher support, motivation and development of teacher identity need to be facilitated through an established community of practice.[5454. Luehmann AL. Identity development as a lens to science teacher preparation. Sci Educ. 2007;91:822–839.


Naturally, the authors are concerned with how these findings generalize to other contexts and cultures. The research provides new insights into the challenges encountered in implementing an integrated approach to teaching and learning mathematics at second-level education. There is a lack of research in this area with in-service teachers due to lack of support and resources, and pressures to prepare students for external examination which is a strong feature of the Irish education system.[44. Berlin DF, White AL. A longitudinal look at attitudes and perceptions related to the integration of mathematics, science, and technology education. Sch Sci Math. 2012;112:20–30.

The significance in this study lies in identifying key aspects of practice that can be addressed and taken into account when pursuing such an initiative at post-primary education.

8. Conclusion

Mathematics and science will continue to be regarded as important subjects for post-primary pupils. As others have demonstrated, there are many benefits to integrating the teaching and learning of these subjects.[11. Czerniak CM. Interdisciplinary science teaching. In: Abell SK, Lederman NG, editors. Handbook of research on science education. New York (NY): Routledge; 2007. p. 537–560.

The aim of this study was to examine the key aspects of practice that need to be navigated in order to facilitate an integrative approach to the teaching and learning of mathematics and science. We were interested in learning more about what an integrative approach may look like for science and mathematics teachers and their professional development needs as teachers. We want to contribute to a fuller understanding of the complexity surrounding implementing an integrative approach to teaching and learning mathematics and science, and how working through such an initiative becomes a mediating strategy for knowledge production. Based on the findings from this study, it is clear that teacher perspective, teacher knowledge of the ‘other subject’ and of TPACK, and teacher collaboration and support, all impact on the implementation of an integrative approach to mathematics and science education.

While this study deepened our understanding of the aspects of practices that impact on the integration of mathematics and science teaching and learning, we believe that further study is required on the key role of the school context and a community of practice in providing support to teachers to engage in such an initiative in their schools. Given the difficulty encountered in relation to timetabling and support structures at school level, and teachers’ knowledge and perceptions of respective subjects, research examining the teaching and learning of mathematics within science and/or science within mathematics warrants further investigation. Similarly, a key purpose of introducing such an initiative is to demonstrate the linkage and connection between the subjects. Within the Irish context, these subjects are taught in isolation, although significant overlap and connection of context is evident. Therefore, an examination of both curricula would be beneficial with the development and identification of key elements of connections where the integration of mathematics and science teaching and learning is possible for teachers to implement. A follow-up study with current participants would be valuable in order to evaluate whether engaging in such an initiative impacts on their current practice in their schools. Significant and long-term mathematics and science education reform is inexorably linked to the capabilities and willingness of teachers to
encompass and take action on the uncertainties that emerge, which is difficult given that they teach in a time of high accountability and focus on high-stake testing. We conclude that teachers need to be supported in taking risks in their teaching so as that they can embrace new pedagogical and content approaches in mathematics and science education. [5050. Le Fevre DM. Barriers to implementing pedagogical change: the role of teachers’ perceptions of risk. Teach Teach Educ. 2014;38:56–64.

Acknowledgements

The authors would like to acknowledge the support of EPI-STEM: National Centre for STEM Education and Texas Instruments, who supplied the technology equipment, in carrying out this research project. The authors would also like to thank the teachers and schools who participated in this research project.

Disclosure statement

No potential conflict of interest was reported by the authors.

Appendix 1. Mathematics and Science lesson plans

Mathematics Lesson Plan 3

Previous Knowledge:

Students will have encountered measurement, quantities and basic relationships in their first year mathematics and science classes.

Data and measure are key strands in the primary school mathematics curriculum.

They have some experience of drawing and interpreting graphs in the science class (e.g. distance and time) and ratios (e.g. speed). Students will have experience of graphs (bar graphs, pie charts, trend graphs, histograms) in their everyday lives, such as opinion polls, weather reports and breakdown of examination results.

Students have experience of motion (distance/speed/time) and travel graphs from previous science and mathematics lessons.

Developmental stage:

Mathematical Task 2: Who has the fastest average speed?
Timmy, Tammy and Tommy all have tummy ache!

They all set off separately to visit their doctor, leaving their homes at exactly the same time.

Timmy cycles 8 km to the doctors and it takes him 24 minutes to get there.

Tammy walks 1.2 km and arrives there after 18 minutes.

Tommy drives 16.5 km to the doctors and arrives there after 22 minutes.

had (e.g. sprinting, running, jogging and walking). How do we calculate her average speed? Allow students to make suggestions given the information they already have (i.e. they have information on the distance she travelled and how long it took her).

Generate formula:

Sarah's average speed = 4 km/hr

Developmental stage – further calculation of average speed:

Key student learning outcomes from this part of the lesson include:

⇒ Further development of the concept of average speed and its relationship with distance and time.
⇒ Developing students' knowledge of the use of units, i.e. km/hr.

Figure 1. Science Lesson Plan 3 – mapping the motion of speed.

JC = Junior Cycle, SC = Senior Cycle.

References


33. EPI-STEM [Internet]. Ireland: University of Limerick [cited 2014 Nov 21]. Available from: www.epistem.ie


Table 1. Sequence and description of lessons.

**Science Lesson 1** The first double lesson attempted to engage the students in the ideas and concept of motion. Students identified the quantities needed to measure speed, described how to measure speed, built and tested balloon rocket cars and demonstrated the ability to measure the speed of the balloon rocket car.

**Mathematics Lesson 1** The purpose of the first mathematics lesson plan was to provide students with key skills required for drawing graphs, e.g. drawing axes, labelling, plotting points, connecting points and interpreting a graph.

**Science Lesson 2** Using their hand-made cars they were asked to predict, analyze and test their ideas about motion. Through the aid of the motion probe and the TI-Nspire™, they tested their predictions and collected data on the handheld equipment and drew a distance–time graph. Students were challenged to apply their experience with the balloon rocket cars and their new knowledge to the interpretation and explanation of new graphs and generate the relationship between distance, speed and time from their experience.

**Mathematics Lesson 2** The purpose of the second mathematics lesson plan was to develop further students’ understanding of graphical concepts (units, slope, speed) in relation to travel graphs. It incorporated the use of data generated from the previous science lesson to draw distance–time graphs, while also encouraging discussion and explanation of variations in their findings in relation to the key concepts developed.

**Science Lesson 3** The final science lesson involved the students actively acting out their motion using the TI-Nspire™ and the motion probe. In the previous mathematics lesson, students examined questions in relation to the direction of the motion and the slope. The active experience of acting out this motion helped the student connect the graph on paper to actual motion. Thus, they developed further the relationship between distance, speed and time by predicting and acting out the motion of the graphs.

**Mathematics Lesson 3** The overall aim of the lesson was that students themselves would generate the average speed formula through completion of mathematical rich tasks concerned with speed, distance and time. These tasks incorporated real-life applications, thus making the material relevant for student learning.
Mathematics Lesson 4  The last mathematics lesson plan was concerned with furthering students’ understanding of the concept of average speed through engagement in the different sets of distance and time data they had collected in the science lesson

Table 2. Description of the schools.

School A  Rural co-educational vocational school located 20 minutes from city ‘A’ (population ≈ 100,000). There were 824 students enrolled in the school (413 boys; 411 girls). The participating first year mathematics and science class had 27 students. Technology was a key focus within the school with every student owning a laptop, and Wi-Fi and data projectors are available in each classroom

School B  This single-sex girl’s secondary school is situated in city ‘A’. There were 252 girls enrolled in the school. The participating first year mathematics and science class had 24 students. There was a lack of basic technology evident in the school, with three laptops available to share amongst all staff in the school

School C  Co-educational vocational school located in a large town 25 minutes from city ‘A’. There were approximately 450 students enrolled in the school. The participating first year mathematics and science class had 24 students. The school had basic technology, the science laboratory contained a data projector and teachers participating in the project owned their own laptops

Table 3. Key information in relation to participating teachers.

<table>
<thead>
<tr>
<th>School</th>
<th>Years</th>
<th>Teacher</th>
<th>Subjects</th>
<th>Self-reported experience</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Matthew</td>
<td>10</td>
<td>Science JC Science, SC Physics, JC Mathematics</td>
</tr>
<tr>
<td>School B</td>
<td>Norma</td>
<td>30</td>
<td>Science JC Science, SC Chemistry, JC Mathematics</td>
<td>Ad hoc links made, does not follow any specific plan or integrated syllabus</td>
</tr>
<tr>
<td>School C</td>
<td>Rachel</td>
<td>5</td>
<td>Science JC Science, SC Biology</td>
<td>Some unplanned links made about mathematics within science lessons</td>
</tr>
</tbody>
</table>
School C

Ann 30 Mathematics Mathematics, Geography Some unplanned links made about other subjects within mathematics lessons

Lee 40 Science JC Science, SC Chemistry Some unplanned links made about mathematics within science lessons

Table 4. Description of data collection tools.

Teacher reflective journal Teachers were asked to respond to specific questions on the lesson plans, organizational aspects, student learning and development, the use and integration of technology, and an overall reflection on the lesson as a whole. Teachers were asked to reflect on each relevant lesson during the period of the implementation of the integrated unit of learning (approximately three weeks). There are four reflections from each of the teachers who taught the mathematics component and three reflections from each of the teachers who taught the science component of the integrated unit of the learning, giving 21 teacher reflections in total

Observation of lessons All lessons, in each of the three schools, were observed by an independent EPI-STEM staff member. All observers were provided with specific templates for evaluating each of the lessons which included key questions in relation to learning outcomes, implementation of the lesson, technology, integration of subjects, the teacher’s role and students’ concept skills development. There are 21 independent lesson observations in total (12 mathematics observations, 9 science observations)

Individual teacher and focus group interviews All teachers participated in focus group interviews (pre- and post-project) and a semi-structured individual interview on completion of the project. The interviews were audio-recorded and transcribed, and were organized around questions about teachers’ perspectives on the integration of science and mathematics as a unit of learning, the use of technology and their participation in such an initiative. A semi-structured approach was utilized in order to probe participants’ perceptions more closely if needed. The individual interviews were approximately 30 minutes in duration and the focus group interview was approximately one hour in duration

Table 5. Initial codes emerging from transcripts.

1 Enjoyment and engagement
2 Real-life application
3 Student learning with technology
Table 6. Codes summarizing the aspects of practice important in facilitating mathematics and science integration.

1. Positive teacher perspectives on integration
2. Negative teacher perspectives on integration
3. Perspective of mathematics and science subjects
4. Adaptability in terms of technology and implementation of unit of learning
5. Linkage and integration of mathematics and science
6. Use of mathematical language
7. Teacher's knowledge of technology and use for pedagogical purposes
8. Teacher's competence in the use of technology
9. Physical demands of utilizing technology and time factor
10. Teacher's experience of collaboration
11. Evidence of levels of collaboration
12. The context supporting/not supporting collaboration
13. The time factor in collaboration
14. The unit of learning and facilitating collaboration
15. Support provided at a school level
16. Training and support for teacher professional development
17. Time factor requirement for supporting teachers

Pupil activity  Teacher guidelines
Introductory activity: Introductory activity:

Key student learning outcomes from this part of Mathematical Task 1: Practical run the lesson include:

Sarah was practicing a long-distance run she was going to do next week. This run would take her 1 hour to complete. ⇒ Speed is a measure of how fast an object is travelling.

In the first 10 minutes, she sprinted 2 km. ⇒ The speed of an object will determine how far it travels.

• In the second 10 minutes, she ran half the distance that she had sprinted. ⇒ Generate the formula:
  Averagespeed=totaldistancetotaltime.

• In the third 10 minutes she jogged just half as far as she had ran.

• In the fourth 10 minutes, she walked and only managed to cover half the distance as she had jogged. Possible teaching approach

• In the fifth 10 minutes, she was walking slower and only managed to cover half the distance that she had walked in the previous 10 minutes. • In the final 10 minutes, she sprinted the last 125 m.

How far was the race to be and how long did Sarah take doing it? Possible extension Learners could change the problem to ask what the figures would be if the race was exactly 2 km long. Possible support For those who are struggling, you could suggest starting at the end of the problem and working backwards. This is a problem which can be easily misread. It would therefore be a good idea for the whole group to read it together and then put it into their own words. These can then be compared and a discussion started on the best place to begin doing the problem itself. After this students could work on the problem in pairs so that they are able to talk through their ideas with a partner. Students need to use the lists and spreadsheets function of their TI-Nspire™ in order to make a table of their findings. You need to set up a table and demonstrate how to fill in the information as they are working through the problem. At the end of this part of the lesson, the group could be brought together again to discuss their findings and how they reached them.

Pupil activity Teacher guidelines

Solution (students can use the Lists and Spreadsheets application for this task) Key teacher questions

<table>
<thead>
<tr>
<th>Minutes</th>
<th>Minutes so far</th>
<th>Distance in metres</th>
<th>Distance so far in metres</th>
<th>What exact measurement do we know from the question?</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>10</td>
<td>2000.00</td>
<td>2000.00</td>
<td>How might you use a table to organize the information?</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td>How far was the race?</td>
</tr>
<tr>
<td>10</td>
<td>20</td>
<td>1000.00</td>
<td>3000.00</td>
<td>How long did it take Sarah to complete it?</td>
</tr>
<tr>
<td>10</td>
<td>30</td>
<td>500.00</td>
<td>3500.00</td>
<td></td>
</tr>
</tbody>
</table>
Possible teaching approach: What are their average speeds (in km/hr) and put them in order? The teacher should generate discussion around calculating average speed and help the students with the task. Perhaps demonstrate calculating Timmy’s average speed while questioning the students in relation to the steps being undertaken.

Who gets there first? It will be necessary to draw students’ attention to the importance of the units (km/hr) being used and the implications for the calculations being undertaken.

Does average speed make a difference to getting to the doctor’s? For example, Timmy’s average speed

\[
\text{Average speed} = \frac{\text{total distance (km)}}{\text{total time (hour)}}.
\]

We need Timmy’s speed in km/hr. So we need to change 24 minutes to hours.

Possible extension

Provide students with the distance and average speed and get them to calculate how long it took the person to get to the doctors. 24 minutes = \(\frac{24}{60} = 0.4\) hours.

So,

Possible support: Timmy’s average speed = 80.4 km/hr

In addition to talking more with those students who are finding the task challenging, it might be appropriate to simplify the problem by changing the units and using easier numbers for calculation. By engaging the students in looking at their solutions (average speed for each person) and comparing with who actually arrived at the doctor’s first should develop their understanding further of the relationship between distance–speed–time.

Solution

Timmy cycles 20 km/hr;

Tammy walks 4 km/hr; Key teacher questions

Tommy drives 45 km/hr. What do we need to know in order to calculate average speed? Who had the fastest average speed?
The order therefore is Tommy, Timmy and Tammy. Does average speed make a difference for getting to the doctors first?

Tammy gets there first but has the slowest average speed. She lives nearest the doctors and therefore distance is important. Tommy arrives next even though he has further distance to travel than Timmy but he is travelling more than two times his average speed.

Article metrics

Views: 4

Article has an altmetric score of 2

Twitter (2)